

# News from Nordic mathematics education

In this issue of *NOMAD*, the NoRME column reports on some activities within the field of mathematics education research in the Nordic region. Readers having information relevant for this column or for the NoRME web page ([www.norme.me](http://www.norme.me)) are encouraged to contact Mette Andresen, the chair of NoRME, by e-mail: [mette.andresen@uib.no](mailto:mette.andresen@uib.no)

## Proceedings of NORMA14

The proceedings of NORMA14 have now been published, thanks to the great work of the conference committee and the editors Harry Silfverberg, Tomi Kärki and Markku Hannula!

The reference of the proceedings is:

Silfverberg, H., Kärki, T. & Hannula, M. S. (Eds.) (2015). *Nordic research in mathematics education. Proceedings of NORMA14, Turku, June 3-6, 2014 (Studies in Subject Didactics 10)*. Department of Teacher Education, University of Turku.

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*Thank you* very much for all your efforts!

## NORMA17

NORMA17 will take place from May 30 until June 2, 2017 at Stockholm University. See First Announcement in this issue of *NOMAD* and on <http://www.mnd.su.se/om-oss/evenemang/norma-17>. The Conference Scientific Committee and the NoRME board are investigating different possibilities for publishing the proceedings of NORMA17.

Lovisa Sumpter and Paul Andrews at Stockholm University have started organising a PhD meeting May 30–31, 2016 in Stockholm as a "warm up" to NORMA17. For funding the activity, an application "Doktorander i matematikdidaktik workshop" has been submitted to Riksbankens Jubileumsfond. The grant type is Forskningsinitiering and the ref. no. is F16-1012:1.

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**Mette Andresen**

*University of Bergen*

## Bridges 2016, Finland

Interdisciplinary conference on Mathematics, Music, Art, Architecture, Education and Culture, at University of Jyväskylä, August 9-13, 2016. For more information: <http://bridgesmathart.org>

## Nordic GeoGebra network

Nordic GeoGebra network continues its activities; next and last conference will take place September 23–25 in Trondheim, Norway. The website will be ready very soon, as well as information about the conference on the Nordic and Baltic GeoGebra network project's website: <http://nordic.geogebra.no>

Besides arranging the conferences, the Nordic GeoGebra Network project group is obliged to survey the Network project's influence on the teachers' use of GeoGebra in school mathematics. Therefore, we invite everybody to answer new questionnaires as soon as it has appeared on the project's website or to contact the survey team with relevant information. The survey team consists of: Morten Misfeldt (Denmark), Maria Fahlgren (Sweden), Anna Kairma (Finland), Sirje Pihlap (Estonia) and Mette Andresen (Norway).

## New dissertations in mathematics education

The following list of titles and short descriptions of new dissertations may not be complete, I presume. Since each and every new dissertation in the field of mathematics education will be of interest for readers of *NOMAD*. I hereby invite everybody to contribute to the dissemination of information by sending me an email about doctoral dissertations that have not already been mentioned here. Thank you in advance!

### *Latvia*

Liene Briede (2015), *Saistība starp matemātikas skolotāju mācīšanas pieejām un 9. klašu skolēnu matemātisko "es"* [The relationship between mathematics' teachers teaching approaches and 9th grade students' mathematical self]. Daugavpils University.

About the thesis (from the abstract): The aim of the research was to identify the relationship between the teaching approach supported by the mathematics teachers and their 9th grade students' mathematical self. The study searched for the answers on three research questions, namely, about 1) the approaches prevailing in mathematics teachers' beliefs about effective teaching and self-reports about their classroom

practices, 2) the qualitative and quantitative features of students' mathematical self and 3) the relationships between the teaching approaches supported by mathematics teachers, the indicators of their 9th grade students' mathematical self, teachers' socio-demographic indicators, and students' socio-demographic indicators. The outcomes of the study show that because of the complex structure of the phenomena, it is difficult to classify mathematics teachers' beliefs about teaching and their self-reported practice into theoretically predefined groups. Though, the use of constructivism in a lesson has a more positive influence on students' mathematical self than mere support of the constructivist beliefs. There exist correlations between the students' mathematical self-efficacy, self-conception and anxiety, teacher's chosen teaching approach and students' socio-demographic indicators (gender, place of living) as well as teacher's socio-demographic indicators (age, education, place of living, education program realized at school and working experience). The constructivist teaching approach in minority classroom realised by teachers with long working experience as well as the traditional teaching approach in country schools tend to result in students' higher mathematical self-efficacy, more positive mathematical self-conception and lower mathematical anxiety. The teaching approach which incorporates elements of both the constructivist and the traditional teaching approach negatively influences the indicators of girls' mathematical self, whereas the traditional teaching approach has the most negative impact on the indicators of students' mathematical self if it is used by teachers with bachelor's degree in the age of 36–45.

Aļesja Šapkova (2015), *Matemātikas skolotāju uzskatu un prakses sistēmas saistība ar viņu skolēnu sasniegumiem matemātikā* [The relationship between the system of beliefs and practice of mathematics teachers and their students' achievements in mathematics]. Daugavpils University.

About the thesis (from the abstract): The aim of the research was to explore the system of mathematics teachers' beliefs on teaching and learning (mathematics) and practice and clarify the relationships between this system and students' achievement in mathematics. The study was mainly designed as to describe the features characterizing the system of Latvian mathematics teachers' beliefs on teaching and learning (mathematics) (BTLM) and practice, to determine what external factors and in what way affect this system, and to identify the relationships between the mentioned system and students' achievements in mathematics. The system of mathematics teachers' BTLM and practice can be regarded as the complex adaptive system. The system under consideration has a nesting structure – it contains several interrelated and interdependent

subsystems that can be characterized by compatibility instead of dichotomy. The subsystem of teachers' formalist beliefs affects the subsystem of their traditional/constructivist beliefs: 1) the larger the teachers' support for the formalist ideas, the less they are driven toward the traditional approach, and 2) the larger inclination toward constructivism, the less they are driven toward formalist approach. The higher the teachers' support to the traditional or constructivist beliefs, the lower their students' achievements, though, the traditional instructional practice has a positive correlation with the students' achievements. The higher the teachers' support to the formalist beliefs concerning the use of precise proofs, the higher their students' achievements. The system of mathematics teachers' beliefs and practice is adjusted to the impact of external factors of teachers' environment. Not all constructivist beliefs may be implemented in practice because many external factors directly facilitate traditional practice. External factors facilitating constructivist practice of mathematics teachers are, e.g. new educational standards, functioning projects, constructivist preferences of school administration and teaching staff, necessity to address students' individual needs and abilities, bilingual education. The impact of external factors promoting constructivist beliefs facilitates a change of teachers' deductive beliefs that prevails over the change of their inductive beliefs.

### *Finland*

Jonna Salminen (2015), *Response to computer-assisted intervention in children most at risk for mathematics difficulties*. University of Jyväskylä.

About the thesis (from the abstract): This thesis aimed to examine to what extent a theory-based computer-assisted intervention method (GraphoGame Math, GGM) can support early number skills development in children (6–7-year-olds) most at risk for mathematics difficulties (MD; defined as performance below the 10th percentile). The specific aims were threefold: to examine, first, the immediate and delayed condition-specific effects of an intensified GGM intervention; second, the potential effects of sample characteristics on gain scores; and third, the effects of two theory-based intervention sequences on basic addition skills. The thesis is based on three intervention data sets collected during the years 2007–2011. The results revealed: 1) positive, condition-specific, immediate and delayed intervention effects after intensified GGM practice on dot counting, verbal counting, composing, and basic addition skills; 2) individual differences in response to intervention among the children most at risk for MD, while the benefits for low-achieving

children were more general and transferable; and 3) specific effects of number concept training on basic addition skills. The findings suggest that, theory-based computerized methods with game-log data analyses can be successfully used in systematic progress monitoring, remediation of initial gaps, and ongoing identification of difficulties. The data sets were collected as part of the LukiMat-project coordinated by the Niilo Mäki Institute and funded by the Finnish Ministry of Education and Culture since 2007.

Lasse Eronen (2014), *Quasi-systematic minimalism within socio-constructivist learning of mathematics*. University of Eastern Finland.

About the thesis (from the abstract): The studies in this summary thesis were conducted in order to consider the effects of a minimalist instruction philosophy within a quasi-systematic model for mathematical concept building. The cognitive and affective development of high school students was studied within a collaborative, technology-based, socio-constructivist, self-guided environments, using a CAS calculator without traditional teaching or homework. Another research objective was to study the professional development of prospective teachers involved in the project. The studies utilized both quantitative and qualitative methods, including quasi-experimental pretest-posttest design and grounded theory. Results: The term "student-centered" should be redefined to minimize the role of the teacher as decision-maker with regard both to learning objectives and working methods. As this paradigm reflects how students work, communicate, and utilize technology outside the classroom, the focus should be shifted from well-tailored classroom lessons to students' free time activities. To use a motor racing analogy, the teacher becomes a team manager and the school serves as a pit stop. However, prospective teachers seem to be hesitant if this approach could replace the conventional teaching in school.

Jari Lakka (2014), *Building addition and subtraction strategies in the teaching of primary level mathematics: the different learning paths to effective strategies by a class of students* (in Finnish). University of Helsinki.

About the thesis (from the abstract): 1st and 2nd grade students' thinking strategies in addition and subtraction tasks are researched in this multiple case study. The base for building counting strategies is the development of number concept. Counting strategies are studied from the point of being inadequate and undeveloped until the task becomes automatized. At the same time mental strategies are touched. The research problem

was to see which kind of counting strategies primary students have and how these strategies develop through the year. Learning paths of the class were also studied. The method was phenomenographic and, accordingly, there should be only a few counting strategy categories which contain all the strategies used in the tests. The outcome space of counting strategies consists of four main categories. The first one I named "Undeveloped strategies". The student's one-to-one correspondence between number words and physical objects can be incorrect and so called double counting missing. The second category I named "Counting with number words". In this category a task is counted by counting with number words in one's mind or for example tracking number words with fingers. The third category I named "Own strategies". They often differ from those taught in mathematics lessons or in text books. The fourth category is named "Strategies based on real partitioning". In this category strategies are based on derived known facts, and numbers are understandable for children and they are easily divided into chunks. The finding of this study is that counting develops from counting with number words towards a real understanding of numbers. At the same time, the ten-based value system becomes clear to the student and the automation of tasks increases. They also conceptualized a Class Learning Path that includes a small number of different learning paths followed by students. Indeed in our class there were found a few different learning paths. Ten-based physical materials and models ought to be placed in classrooms. Teaching should take place from concrete models and concrete strategies linked to them towards mental strategies and furthermore towards abstract thinking.

Sonja Lutovac (2014), *From memories of the past to anticipations of the future: pre-service elementary teachers' mathematical identity work*. University of Oulu.

About the thesis (from the abstract): This study explored mathematical identity work by drawing on the cases of Finnish and Slovenian pre-service elementary teachers. All cases reported having had negative experiences with mathematics during their school years. These experiences were shown to have a central meaning for pre-service teachers' mathematical identities. However, identity also extends to the future. For this reason, pre-service teachers' anticipations of the future were also explored. The overall narrative perspective of the study enabled a psycho-social understanding of identity. The special interest of the study was confined to an understanding of the role that educational contexts play in pre-service teachers' mathematical identity work. The findings showed striking similarities in pre-service teachers' school-time memories. The

cases in question felt like victims of their own mathematical experiences. The anticipations of mathematics teaching were also underlined by the challenges rooted in their school-time experiences. However, a surprising finding was that the identity work in which the Finnish and Slovenian cases engaged during their teacher education differed substantially. The main reasons for the differences in identity work seemed to stem from different emphases and pedagogical practices in mathematics education courses within the Finnish and Slovenian teacher education settings.

Riikka Mononen (2014), *Early mathematics interventions: supporting young children with low performance in mathematics*. University of Helsinki.

About the thesis (from the abstract): The purpose of this thesis was to investigate the effectiveness of early mathematics interventions for young children with low performance in mathematics. This thesis sought to complement and extend previous research in the field of early mathematics interventions, by reviewing early mathematics interventions, and investigating the effectiveness of two early mathematics intervention programmes.

According to the results of the review, in the majority of the interventions, the mathematics skills of the participating children improved more than the skills of the children in control groups, with effect sizes varying from small to large. Progress in mathematics learning was evident when instruction included one or more of the following instructional features: explicit instruction, peer-assisted instruction, applying a concrete-representational-abstract sequence, computer assisted instruction, or games. In general, the results indicate that rather than waiting for children to fail, mathematics interventions can be used successfully to promote the early mathematics skills of children with low performance in mathematics, already before the onset of formal schooling and in the early grades. Therefore, identifying low performance in mathematics and providing sufficient support should be emphasised already in early childhood education, in accordance with the Finnish three-tiered educational support system.

### *Estonia*

Indrek Kaldo (2015), *University students' view of mathematics in Estonia*. Tallinn University.

About the thesis (from the abstract): The aim of this research was to integrate the parts of the published instruments on mathematical-related



beliefs. This new instrument was then used to a) confirm its applicability in Estonia at the university level, b) compare science and non-science students' differences in their view of mathematics, and c) compare gender differences in the students' view of mathematics.

The results of the doctoral thesis have been published in five different peer-reviewed international science journals:

Article I focuses on the pilot study, which was carried out in the Estonian Business School in order to test compiling questionnaires and their applicability in universities. It can be concluded from the pilot study that students of the English curriculum are more motivated than their counterparts of the Estonian curriculum. The instrument was also found to be applicable in Estonian universities. Article II centers on the theoretical part of the concept of the thesis. As the current instruments on the students' beliefs lack the component of motivation, the concept/term "students' views of mathematics" has been taken into use. Article III deals with the differences between the science and non-science students' views of mathematics. Article IV gives an overview of gender differences in the students' view of mathematics. Article V presents the results of the separate issues of the research carried out in Estonia.

The results of the present thesis confirm that the instrument can be used in universities in order to study students' view of mathematics. It also concludes that science students have a more positive view of mathematics than non-science students. The comparison of gender differences proves that female students have a more positive attitude towards mathematics than male students.

### *Sweden*

Catarina Andersson (2015), *Professional development in formative assessment: effects on teacher classroom practice and student achievement: Effects on teacher classroom practice and student achievement*. Umeå University.

About the thesis (from the abstract): The potential of formative assessment, evident in several research reviews, has raised the interest in many countries to invest in reform initiatives to develop its use. However, implementation of formative assessment is not straightforward and there is a lack of knowledge about how to design appropriate professional development. The intervention study presented in this thesis aimed to see if a random selection of teachers, participating in a professional development program with many contact hours and substantial support of an expert, implemented formative assessment in a way that increased their



students' learning in mathematics. It also aimed to examine the reasons for the teachers' changes in their classroom practice.

The twenty-two year 4 teachers attended a professional development program in formative assessment in mathematics. A mixed methods approach used classroom observations, teacher interviews, questionnaire surveys and student mathematics tests to investigate the effects on teacher classroom practice and student achievement.

It was found that the teachers trained in formative assessment built on their previous formative classroom practice and added new formative assessment activities into their mathematics classroom practice to a level that had significant impact on student achievement in mathematics ( $p = .036$ ,  $d = .66$ ). The teachers developed their formative assessment practice in three dimensions: key processes in teaching and learning, agents in the classroom, and the length of the formative assessment cycle.

The reasons for teachers' implementation of new formative assessment activities were well explained by the expectancy-value theory of achievement motivation. Important aspects of the professional development program were: (1) A formative and process-oriented character; (2) Activities directly useable in classrooms; (3) Experience of using formative assessment activities; (4) Connection between theory and practice; (5) Time; and (6) Knowledgeable support.

The thesis shows that it was possible to provide sufficient support to a random selection of teachers for them to develop their formative assessment practice in a way that improved student achievement. However, this thesis also indicates that it can be expected that teachers would need substantial time and support to achieve such developments in their classroom practice.

Maria Larsson, (2015), *Orchestrating mathematical whole-class discussions in the problem-solving classroom: theorizing challenges and support for teachers*. Mälardalen University.

About the thesis (from the abstract): Promising teaching approaches for developing students' mathematical competencies include the approach of teaching mathematics through problem solving. Orchestrating a whole-class discussion of students' ideas is an important aspect of teaching through problem solving. There is a wide consensus within the field that it is very challenging for the teacher to conduct class discussions that both build on student ideas and highlight key mathematical ideas and relationships. Further, fostering argumentation in the class, which is important for students' participation, is also a grand challenge. Teachers need support in these challenges. The aim of the thesis is to

characterize challenges and support for mathematics teachers in orchestrating productive problem-solving whole-class discussions that focus on both mathematical connection-making and argumentation. In particular, it is investigated how Stein's et al. (2008) model with five practices – anticipating, monitoring, selecting, sequencing and connecting student solutions – can support teachers to handle the challenges and what constitutes the limitations of the research-based and widely-used model. This thesis builds on six papers. The papers are based on three intervention studies and on one study of a mathematics teacher proficient in conducting problem-solving class discussions. Video recordings of observed whole-class discussions as well as audio-recorded teacher interviews and teacher meetings constitute the data that are analyzed. It is concluded in the thesis that the five practices model supports teachers' preparation before the lesson by the practice of anticipating. However, making detailed anticipations, which is shown to be both challenging and important to foster argumentation in the class, is not explicitly supported by the model. Further, the practice of monitoring supports teachers in using the variety of student solutions to highlight key mathematical ideas and connections. Challenging aspects not supported by the monitoring practice are, however, how to interact with students during their exploration to actually get a variety of different solutions as a basis for argumentation. The challenge of selecting and sequencing student solutions is supported for the purpose of connection-making, but not for the purpose of argumentation. Making mathematical connections can be facilitated by the last practice of connecting, with the help of the previous practices. However, support for distinguishing between different kinds of connections is lacking, as well as support for creating an argumentative classroom culture. Since it is a great challenge to promote argumentation among students, support is needed for this throughout the model. Lastly, despite the importance and challenge of launching a problem productively, it is not supported by the model. Based on the conclusions on challenges and support, developments to the five practices model are suggested. The thesis contributes to research on the theoretical development of tools that support teachers in the challenges of orchestrating productive problem-solving whole-class discussions.

Åsa Harvard Maare, (2015), *Designing for peer learning: mathematics, games and peer groups in leisure-time centers*. Lund University.

About the thesis (from the abstract): Constrained by national tests and the mathematics curriculum, teachers have problems finding time for exploratory and hands-on mathematical activities, especially so in

classes with a reduced pace of progression, for example because of a large proportion of second-language learners. Could the leisure-time center, where time is not earmarked, provide such opportunities? The conclusion of this thesis is that this can be done, on the condition that designed activities build on the central premise of the leisure-time center: children have the right to choose which activities to engage with. The thesis is interdisciplinary, combining design research, situated cognition/embodied interaction, and pedagogy.

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The situated and embodied approach towards design is reflected in the analysis, which approaches visual artifacts as parts of multimodal communicative scenes with many co-present participants engaged in playing games or solving problems. It is shown that children learn the game through observation and participation, either as players or in non-playing roles. For many games, rules are written in a format that is inaccessible to children. One of the design tasks in the project has been to develop secondary artifacts related to games: graphic guides, conceptual maps, and paper-based exercises that can be used by children without adult support. The premise of the learners' right to choose has many consequences for the design of learning activities. One is that motivation changes from being a property of the learner to a property of the activity. In order to highlight this difference, this thesis proposes the notions of learnability and learnworthiness to describe those aspects of an activity and its context which make it motivating from the learner's perspective. The thesis concludes with a discussion of how design can increase the learnability and learnworthiness of a learning activity.

Watching the activity being practiced is the most important resource for potential participants to determine its learnability and

learnworthiness. The qualities determining the learnworthiness of an activity are reciprocity, mastery, and the potential for closure. Watching a peer successfully solving a task increases the learnability for the observers as well. If problem-solvers think aloud and use their hands to move or point at cards, collaboration and learning by observers is facilitated. Providing games with non-competitive side activities creates opportunities for deliberate practice, and offers a safe entry for children who are reluctant to engage as players.

Maria Fahlgren, (2015), *Designing for the integration of dynamic software environments in the teaching of mathematics*. Karlstad University.

About the thesis (from the abstract): This thesis concerns the challenge of integrating dynamic software environments into the teaching of mathematics. It investigates particular aspects of the design of tasks which employ this type of computer-based system, with a focus on improvement, both of the tasks themselves and of the design process through which they are developed and refined.

The thesis reports two research projects: a small initial one preceding a larger main project. The initial case study, involving two graduate students in mathematics, develops a task design model for geometrical locus problems. The main study constitutes the first iteration of a design-based study, conducted in collaboration with four upper-secondary school teachers and their classes. It seeks to identify task design characteristics that foster students' mathematical reasoning and proficient use of software tools, and examines teachers' organisation of "follow-up" lessons.

The findings concern three particular aspects: features of tasks and task environment relevant to developing a specific plan of action for a lesson; orchestration of a particular task environment to support the instrumental genesis of specific dynamic software tools; how to follow up students' work on computer-based tasks in a whole-class discussion.

Marita Lundström, (2015), *Förskolebarns strävanden att kommunicera matematik*. University of Gothenburg.

About the thesis (from the abstract): The aim of this study is to explore and describe preschool children's use of mathematics in their communication with others. This study is limited to examining situations in which children communicate with other children and adults. Moreover, the aim is to deepen the understanding of how preschool children use mathematics to convey mathematical meanings with other children and adults in preschool. The research questions is: How do

children communicate mathematics? In what kind of situations does mathematics occur in their communication? What mathematical content is communicated? The theoretical framework is selected from a sociocultural perspective, which is a collective term for theories which assume that language's original function is communicative and that it is a means of social interaction. From a socio-cultural perspective, language, culture and children's actions are seen as essential elements of childhood development and learning. Prerequisites for this development include children's participation in creating an interaction with their environment. In order to study preschool children's mathematical communication, this study is based in ethnographic methodological traditions. The results show that preschool children communicate mathematics through: linguistic expressions, semiotics, linguistic tools and bodily expressions. Children communicate mathematically in situations when: they are making comparisons, when they are comparing changes, and when they are trying to give descriptions about the world which surrounds them. It also shows that preschool routines, material support and activities stimulate mathematical communication. When teachers are supportive and engaged in children's communication their mathematical knowledge can also be developed and deepened.

Mats Brunström (2015), *Matematiska resonemang i en lärandemiljö med dynamiska matematikprogram*. Karlstad University.

About the thesis (from the abstract): Det övergripande syftet med avhandlingen är att undersöka hur dynamiska matematikprogram kan användas för att öka möjligheterna för elever/studenter att utveckla sin förmåga att föra matematiska resonemang. Detta görs dels genom att fokusera på design av uppgifter i en lärandemiljö med dynamiska matematikprogram och dels genom att studera och karakterisera de resonemang som utvecklas när elever/studenter jobbar med olika uppgifter i denna miljö. För att analysera resonemangen i en av studierna utvecklades ett nytt analysverktyg i form av en utökad version av Toulmins modell.

Resultat från en av studierna i avhandlingen visar att dynamiska matematikprogram i kombination med utforskande uppgifter kan stimulera till matematiska resonemang där hypoteser formuleras, undersöks och förfinas i en cyklisk process. Samtidigt visar samma studie att de resonemang som utvecklas i stor utsträckning saknar matematiskt grundade förklaringar. Detta resultat bekräftas till viss del av ytterligare en studie. Frågan hur uppgifter bör designas för att främja matematiskt grundade resonemang har därför varit central i avhandlingen. Två av artiklarna behandlar uppgiftsdesign, men utifrån olika utgångspunkter.

Helena Johansson (2015), *Mathematical reasoning – in physics and real-life context*. Chalmers University of Technology and University of Gothenburg.

About the thesis (from the abstract): This thesis is a compilation of four papers in which mathematical reasoning is examined in various contexts, in which mathematics is an integral part. It is known from previous studies that a focus on rote learning and procedural mathematical reasoning hamper students' learning of mathematics. The aims of this thesis are to explore how mathematical reasoning affects upper secondary students' possibilities to master the physics curricula, and how real-life contexts in mathematics affect students' mathematical reasoning. This is done by analysing the mathematical reasoning requirements in Swedish national physics tests; as well as by examining how mathematical reasoning affects students' success on the tests/tasks. Furthermore, the possible effect of the presence of real-life contexts in Swedish national mathematics tasks on students' success is explored; as well as if the effect differs when account is taken to mathematical reasoning requirements. The framework that is used for categorising mathematical reasoning, distinguishes between imitative and creative mathematical reasoning, where the latter, in particular, involves reasoning based on intrinsic properties. Data consisted of ten Swedish national physics tests for upper secondary school, with additional student data for eight of the tests; and six Swedish national mathematics tests for upper secondary school, with additional student data. Both qualitative and quantitative methods were used in the analyses. The qualitative analysis consisted of structured comparisons between representative student solutions and the students' educational history. Furthermore, various descriptive statistics and significance tests were used. The main results are that a majority of the physics tasks require mathematical reasoning, and particularly that creative mathematical reasoning is required to fully master the physics curricula. Moreover, the ability to reason mathematically creatively seems to have a positive effect on students' success on physics tasks. The results indicate additionally, that there is an advantage of the presence of real-life context in mathematics tasks when creative mathematical reasoning is required. This advantage seems to be particularly notable for students with lower grades.

Pernilla Mårtensson (2015), *Att få syn på avgörande skillnader. Lärares kunskap om lärandeobjektet*. Jönköping University.

About the thesis (from the abstract): Lärare som undervisar i matematik förväntas kunna mer avancerad matematik än vad de undervisar om. Men

formell matematikkunskap anses inte vara tillräckligt för att lärare ska kunna undervisa så att ämnesinnehållet blir begripligt för eleverna, de behöver även pedagogical content knowledge (PCK). Begreppet belyser en speciell form av ämneskunskap för undervisning och skiljer sig från den matematikkunskap som används av andra välutbildade vuxna. Det har föreslagits att olika arrangemang av kollegialt och praktikbaserat lärande kan utveckla lärares PCK. Ett exempel på ett sådant arrangemang är learning study. Den här avhandlingen handlar om den kunskap om lärande och undervisning i matematik som studiens lärare utvecklar då de deltar i learning studies och utforskar sin praktik utifrån ett variationsteoretiskt perspektiv. Det yttersta syftet med en learning study är att utveckla elevernas lärande om specifika lärandeobjekt, genom att undersöka vad som kan vara kritiskt för elevernas lärande. I ett samarbetsprojekt med fyra högstadielärare genomfördes två learning studies i matematik under ett år. Lärargruppen undersökte vad eleverna behöver lära för att de ska förstå i) varför en kvot kan vara större än talet i täljaren och ii) olika representationer av konstanterna  $k$  och  $m$  i räta linjens ekvation. Under learning study-arrangemangets olika steg samlades studiens empiri in och denna består av filmade lektioner, inspelade möten där lärargruppen planerade och analyserade undervisning och elevers lärande, skriftliga elevtest samt elevintervjuer. Studien har en variations-teoretisk utgångspunkt, vilket innebär att lärande förklaras ske när en person ser något på ett nytt och mer kvalitativt sätt, genom att personen urskiljer aspekter som han/hon inte tidigare har urskilt. Studien visar de två lärandeobjektens kritiska aspekter samt hur de kritiska aspekterna gradvis förändrades och specificerades. Förändringen var ett resultat av att lärargruppen fick syn på avgörande detaljer om på vilket sätt eleverna förstod ämnesinnehållet samt hur skilda sätt att förstå kunde användas i undervisningen för att utveckla elevernas lärande. Därav titeln att få syn på avgörande skillnader. Denna form av utvecklad kunskap om lärandeobjektet kan ses som ett bidrag om PCK och vad det kan vara.

### *Denmark*

Sif Ingibjörg Magnúsdóttir Skjoldager (2014), *The development of mathematics-related beliefs in Danish upper secondary school students*. Roskilde University.

About the thesis (from the abstract): This study aims at characterising students' interpretations of their experiences in A-level mathematics, the highest level in Danish upper secondary school, by addressing these research questions: #1: "How do the mathematics-related beliefs of students' develop during their three years in upper secondary school?"



and #2: “How are the students’ ideas for their choice of mathematics-related future study programmes influenced by their mathematics-related beliefs?”. The development of the four aspects of beliefs; Mathematics at School, Mathematics as a Discipline, Mathematics in Society and Mathematics & Me, and the influence from these aspects on the selection or deselection of mathematics related study programmes in students’ preliminary plans for tertiary education for answering the research questions, were investigated by means of a longitudinal research design consisting of a questionnaire ( $n = 147$ ) complemented by follow-up interviews (1st year:  $n = 24$ , 3rd year:  $n = 21$ ) in the 1st and the 3rd year of upper secondary school, respectively, it has been possible to trace the development in individual students over time. Among other things, the study shows a decline in the willingness to choose mathematics-related study programmes for tertiary education, especially as a result of experienced lack of understanding of the mathematics taught.

Steffen Møllegaard Iversen (2014), *Skrivning og skriveudvikling i de gymnasiale matematikfag*. University of Southern Denmark.

About the thesis (from the abstract): The subject of the dissertation is mathematical writing and writing development, and how students learn to write mathematically in the subject of mathematics in upper secondary education. In order to answer this question, a theoretical framework related to the sociocultural tradition of writing research is provided, and based on this a longitudinal, ethnographic study of eight students’ writing and their writing development in the subject of mathematics is planned and conducted. This field study took place over a two year period (2011-2013) and consisted of four separate case studies, one in each of the four different kinds of upper secondary education in the Danish educational system.

Initially, the data from the case studies was analysed in order to identify significant features characterising the writing cultures of the subject of mathematics in Danish upper secondary education. By this means, the overriding research interest was split in to three research themes – computer-based tools for writing, non-traditional task genres and mathematical writer identities – and each of these themes was subsequently analysed in greater detail.

In the first thematic analysis, the use of computer-based tools for writing is examined, and the analysis shows that the computer-based tools for writing partly regulate the content, form and use of the students’ mathematical texts. Furthermore, it is shown how students’ mathematical writing and writing development are tangled up with their use

of computer-based tools for writing. In addition to this, it is illustrated how the computer-based tools privilege certain semiotic resources, in this case two different forms of mathematical notation, in the students' mathematical texts, and in continuation hereof, it is argued that the use of computer-based tools for writing has a substantial impact, not only on the students' mathematical writing development, but also on the development of the subject of mathematics as such.

In the second analysis the so-called non-traditional task genres are brought into focus and it is demonstrated how the use of these task genres in authentic situations is able to facilitate a number of learning opportunities connected to the students' mathematical writing and writing development. At the same time it is pointed out, however, that part of these learning opportunities are linked to the ways students are asked to work with non-traditional task genres in the classroom, rather than to the task genres per se. Therefore, there is reason to believe that the identified learning opportunities can also be established when students are working with traditional task genres.

In the third and final thematic analysis it is examined how students' identity work has an impact on their mathematical writing and writing development. The concept of voice is operationalised initially and based on this two students' writing development, or writer development, is analysed. This analysis shows that the two students systematically construct different writer identities throughout their upper secondary education, and on the basis of this, it is argued that the students' understandings and interpretations of who they appear as, who they should appear as, and who they would like to appear as, in their mathematical texts is shaping and moulding their mathematical writing development. In particular, it is demonstrated that one of the student's writing development is marked by the institutional identity of being a student in a school, while the other student's writing development bear the mark of identification processes that involves constructing a more academic identity as a mathematician.

Finally, the dissertation is concluded with a reflection on the kinds of trustworthiness, generality and importance that characterise the findings in the dissertation, and in continuation hereof three educational points specific to writing and writing development in the subject of mathematics are offered.

Kind regards, Mette Andresen

