Methodological issues and challenges in research on mathematics textbooks

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The growing field of research on mathematics textbooks can be divided into three areas: 1) Research that focuses on the influences on textbooks; 2) Research that focuses on the mathematics textbook itself; 3) Research on the use of mathematics textbooks and its impact. Concentrating on research done in the Nordic countries, the paper provides an overview of methods used in all three areas. Based on an analysis of 24 papers, the findings are that whereas research related to area 1) and 2) usually relies on surveys or content analysis of textbooks – well-established methods from the social sciences –, research in area 3) has to face the methodological challenges of gathering valid data and of generalization of its results. Triangulation and typologies are presented as possible ways to tackle these challenges.

"Textbooks are the major resource for teaching mathematics. In TIMSS 2007, 65 % of Grade 4 teachers and 60 % of Grade 8 teachers internationally use textbooks as the main basis for mathematics lessons." (Askew et al., 2010, p. 34). This is one major result of a comparative study of international mathematics education sponsored by the UK Nuffield foundation. It clearly indicates how relevant textbooks are for the teaching and learning of mathematics in general education – even in the age of the introduction of digital information technology (in the following: "IT"). Somehow confirming this result, the Nuffield foundation even condensed this report into "Good textbooks are more important for high attainment in mathematics than factors such as setting or expensive IT equipment" (Nuffield foundation, 2010). Experience from various sources shows that textbooks are also the major resource used in the classroom and by the students learning mathematics. For the Nordic countries we can only refer to the Swedish Skolverket (2003, p. 21) stating

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"During the observed mathematics lessons in Grades 7–9 students work individually with book tasks and the teacher devotes time to give individual help. Joint reviews happen rarely or never" (translation RS, for the original Swedish citation see note 1).

The role of textbooks in the teaching and learning is often described as being the major curriculum material, e.g. in the framework of the *Third international mathematics and science study* (TIMSS) (Schmidt et al., 1997), textbooks are regarded as the "potentially implemented curriculum" between the "intended" and "implemented" curriculum. Various papers describe textbooks as the main source for teachers preparing lessons, not only suggesting exercises, activities, procedures and definitions, but globally defining the course of teaching in terms of structure and concepts to be taught (see various papers in the special issue of ZDM, vol. 45, issue 5, on textbook research, especially the overview in Fan, 2013).

For the authors of this paper, there is a deeper, epistemological reason for studying textbooks, their use and their impacts: We start from the assumption that mathematics is a non-material science. If users and/or learners want to get hold of it, they need some representation of mathematics (symbolic, textual, graphical or the like; for the various representations of school mathematics see Duval, 2000). In the mathematics classroom, teachers and learners need a representation of the matter to be taught. In fact, textbooks offer an intentional arrangement of representations of mathematics. Like a micro-cosmos, they enclose nearly all issues related to mathematics education. Textbooks are a focal point in mathematics education, because they offer a (hopefully) faithful, accurate image of mathematics to teachers and learners.

Facing the importance of mathematics textbooks it might be surprising that research on mathematics textbooks is a relatively small area within mathematics education with scattered and loosely connected studies. We agree with Fan (2013, p. 766) in that "the philosophical foundations, theoretical frameworks and research methods for disciplined inquiry on different issues in mathematics textbook research are still lacking or fundamentally underdeveloped". Our intention is to contribute to the development of research on mathematics textbooks by giving an overview of methods used in this field of research within mathematics education. This inventory might serve as starting point for the further development of methodological approaches in this field. As opposed to Fan (2013) who analysed methodological issues in papers presented at a selection of international conferences we focus on research in the Nordic countries, because a considerable number of studies on mathematics textbooks have been carried out in these countries in the past years.

Methodological issues

We used three sources in order to collect the sample of studies for our analysis. A starting point was the comprehensive overview of textbook research in the Nordic countries provided by Grevholm (2011). This was complemented by a search in the two databases 1) ERIC powered by the *Institute of education sciences* (http://eric.ed.gov) and 2) MathEduc powered by FIZ-Karlsruhe (http://www.zentralblatt-math.org/matheduc/) using the two terms "mathematics textbook" and "curriculum material" successively combined with "Nordic countries", "Denmark", "Estonia", "Finland", "Latvia", "Norway", "Sweden". Altogether a sample of 24 studies was identified and analysed according the research questions, theoretical frameworks and methods. Wherever appropriate, we also refer to studies in the international context in order to include further methodological approaches.

Our study on methodological issues of Nordic research on mathematics textbooks has to take into account the interrelatedness of the phenomena under study (here textbooks, its use and its impacts) as specified in the research questions, the methodology and the theoretical framework used within this research. Methodological issues clearly depend on the phenomena under study and they are inter-related with the theoretical framework. The choice of research methods and of the theoretical framework has to be appropriate for phenomena under study. Using the "socio-didactical tetrahedron" (see figure 1) as a theoretical lens in order to structure the field of research on mathematics textbooks, we distinguish three broad areas, i.e. three major groups of phenomena within textbook research, namely research on the *influence on textbooks*, which takes into

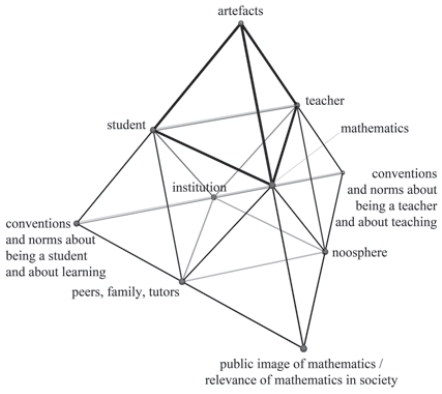

Figure 1. *Socio-didactical tetrahedron (from Rezat & Sträßer, 2012, p.648)*

account (oriented) relations between the truncated tetrahedron and the tip of the tetrahedron, research on the *textbook itself* (focusing on the very top of the tetrahedron), and research on the *use of textbooks and its impact* (taking into account the relations and processes within the socio-didactical tetrahedron as a whole (Rezat & Sträßer, 2012, p. 648).

By dividing the field of research on mathematics textbooks into these three areas, we offer a similar structure as Fan (2013), who distinguishes "1. Issues about the textbooks themselves [...]; 2. Issues about how different factors affect the development or production of textbooks or make textbooks [...]; and 3. Issues about how other factors are affected by the textbook" (p. 771). While Fan's structure seems to be grounded in a quantitative research paradigm that focuses on the unidirectional relations of different well-distinguishable variables, our perspective takes into account the interrelations within the systemic whole of the societal activities related to the artefact "textbook", which we described by the means of the socio-didactical-tetrahedron (Rezat & Sträßer, 2012).

By referring to the tetrahedron, we offer a specific way to reconstruct the inter-relations of variables and are able to put the objects of textbook research into a systematic structure. In this paper, the three broad areas of textbook research will be presented separately with a focus on the methods used to grasp the interrelations within the socio-didactical tetrahedron. Insofar, the paper elaborates on an article previously published as Rezat and Sträßer (2013).

Research on the influences on textbooks[2]

Gunnarsdottir and Palsdottir (2010) present the textbook author's perspective on the development of teaching materials. Because both authors themselves were – according to their own estimation – involved in the development of about 80 per cent of the teaching materials in Iceland they explain their own motives and goals in the process of development. This means they try to penetrate into their own beliefs about mathematics, the learning of mathematics, the curriculum and the use of curriculum materials. Thus, the method they apply is introspection which is a rarely used method in scientific research on beliefs. This methodology is afforded by the situation in a small country like Iceland since in this special case teacher, textbook author and mathematics education researcher are all unified in one person. This might explain why in other countries textbook authors hardly explicate their beliefs that guided their construction of textbooks in a scientific context.

Randahl (2012) also explores authors' views about their texts. Her investigation is based on a questionnaire that was sent to seven authors

of the most used calculus textbook for tertiary level in Norway. Thus, she infers the authors' beliefs about their texts from the authors written responses to the questionnaires based on content analytic methods. This can be regarded as one standard method in research on beliefs besides content analysis of other data sources such as interviews or the analysis of actions. Inferences are directly drawn from the responses.

According to the summary in MathEduc, Vassiltshenko (2004) seems to look into the evaluation of textbooks by students, using a questionnaire approach. Because of the language the text is published in, we are not in a position to categorise this text in a more precise manner.

Research on the textbook itself

Most widespread is research on (mathematics) textbooks which analyses the contents of textbooks. This strand of research is particularly relevant in the Nordic countries. We can find different topics that have been analysed (for the following list see Rezat & Sträßer, 2013, p. 472). Bremler (2003) compares the introduction of the derivative in Swedish textbooks from 1967 until 2002, while Randahl and Grevholm (2010) look into the way, the derivative is introduced in a university textbook. Jakobsson-Åhl (2006) investigates algebra in upper secondary mathematics textbooks. Bjarnadóttir (2007) focuses on the numbers one and zero in Northern European textbooks, and Keranto and Sarenius (2009) analyse the use of the number line as a teaching aid in grade one and two textbooks. According to the summary in MathEduc, Kudzma (2005) also belongs to this category, analysing textbook presentations of graphs of a specific function. Besides these mathematical aspects some studies focus on specific text types – in particular tasks: Brändström (2005) analyses tasks with regard to differentiation and Jakobsson-Åhl (2008) focuses on word problems, i.e. a specific type of task. A third strand of textbook analyses focuses on pedagogical and didactical aspects, as e.g. the teaching of modelling (Frejd, 2013), differentiation (Brändström, 2005) and heuristic approaches (Kongelf, 2012). The analysis of Lepik and Kaljas (2010) adds an additional aspect, namely the comparison of textbooks in different educational systems.

All these studies are, more or less explicitly, based on content analysis as their main research method. Krippendorff (2004, p. 19) defines this method as "a research technique for making replicable and valid inferences from texts (or other meaningful matter) to the contexts of their use" (p. 19)" (for a comprehensive description see Krippendorff, 2013). In this definition, content is seen as something that "emerge(s) in the process of a researcher analysing a text relative to a particular context"

(Krippendorff, 2013, p. 25). For the special case of research on mathematics textbooks the predominant aim of content analysis is to make inferences from the text to the impact of the text on students and teachers. The social sciences offer a whole plethora of techniques for systematic analysis of texts (for a non-comprehensive list see Lamnek and Krell, 2010, pp. 434–497). Within content analysis, one basic distinction must be mentioned: A content analysis may be quantitative, i.e. based on numerical counting of pre-defined instances, followed by a statistical analysis of the data (e.g. Valverde et al., 2002). Even more numerous are qualitative content analysis studies, defined "as an approach of empirical, methodological controlled analysis of texts within their context of communication, following content analytical rules and step by step models, without rash quantification (for this definition see Mayring, 2000, p. 5)[3]. A study of this type is Howson (1995). Within qualitative content analysis, we find two ways to generate the categories: Somehow following a more grounded approach, we can see an "inductive" model of category development, whereas there is also a "deductive" step model (for a graphical representation see figure 2, from Mayring, 2000, p. 11 / 14; for grounded theory Strauss & Corbin, 1990, or Glaser & Holton, 2004).

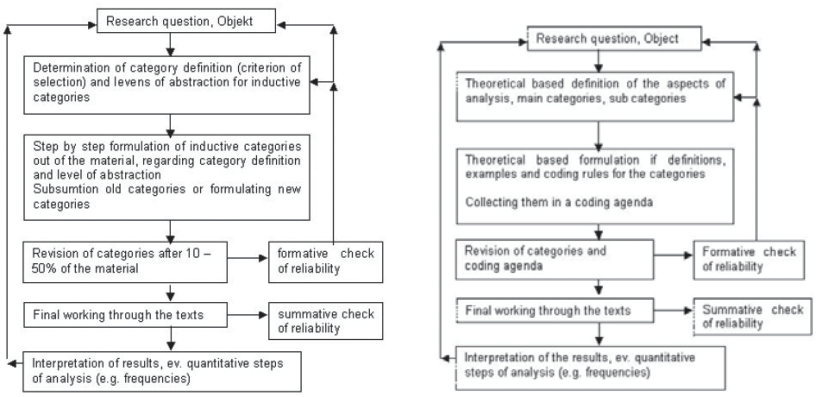

Figure 2. *Inductive (left) vs. deductive (right) category generation (from Mayring, 2000)*

Content analysis of mathematics textbooks can have a whole variety of research topics. Howson (2013, p. 653f) can be read as a list "of possible attributes that a reviewer of a textbook [...] might consider:

- Mathematical coherence.
- Clarity and accuracy of explanations.

- Clarity in the presentation of kernels.
- The range, quantity and quality of the exercises.
- The connections with real-life and with other curricular subjects displayed in the explanations and the exercises.
- Gender, racial and other social balance.
- The use of appropriate language and the development of language reading skills.
- Evidence that research results and accrued professional experience have been taken into account.
- Provision for the differing abilities of the students who will use the text.
- The physical attractiveness of the texts: format, type, colour, illustrations.
- Some signs of originality in material, examples or form.
- The provision of teachers' guides that go beyond answer books and balance the twin demands of developing the teachers' mathematical understanding and assisting the management of lessons".

While all these different aspects require different theoretical frameworks in order to specify and operationalize these phenomena under study the actual investigation of these phenomena will all be based on the same method, namely content analysis.

In the same paper, Howson identifies one of the most important research topics of content analysis, namely the inspection and classification of exercises (Howson, 2013, p. 656):

> A comparative study of texts should, therefore, place considerable emphasis on them and observe to what extent they go beyond the routine and well-signposted. More attention should be paid to how they encourage thought, analysis and synthesis, and act to stimulate both an interest in the subject and to demonstrate its usefulness both within and outside mathematics.

If we reconsider Nordic research on mathematics textbooks cited above, it appears that both approaches (inductive as well as deductive) of category generation have been used. The researchers have taken into account quite a few of the possible attributes to be analysed in research on the textbook itself. However, whatever approach to content analysis is used, whatever phenomenon the research is focusing on and whatever

theoretical framework is used, it is important to keep in mind that content analysis only reveals opportunities to learn. No inferences about the actual impact of textbooks on instruction or competencies of students may be drawn. The use and impact of textbooks need additional methods, which will be presented in the following sections.

Research on the use of textbooks and its impacts

Looking into research on the use of textbooks has to take into account one major caveat: "use of textbooks" has multiple meanings, even if reduced to the range of textbook used by teachers. Lennerstad et al. (2005) seem to be a special case, analyzing the evolution of a textbook for beginning students of mathematics at university. Students were able to discuss about the textbook and change the content using an online tool. As for the use of textbooks in school, and regarding "textbook" and "curriculum" as synonyms, Remillard (2005) distinguishes four notions of "use":

- curriculum use as following or subverting the text,
- curriculum use as drawing on the text,
- curriculum use as interpretation of text, and
- curriculum use as participation with the text.

The mere enumeration of these different notions of use point to difficulties when analysing the use of textbooks. Methodological approaches are depending on the notion of curriculum materials (or textbooks) and their "use". Here, a sociocultural perspective distinguishes between the artefact textbook, the user (be it teachers or students) and the object, in our community mathematics. In order to grasp the teaching-learning-situation as a whole, Rezat and Sträßer (2012) have condensed this into the "didactical tetrahedron of textbook use in the narrow sense", which is restricted to the upper four vertices of the the socio-didactical tetrahedron in figure 1, incorporating the two basic actors in the didactical situation, namely teachers and students.

Doing research on persons implies certain methodological difficulties[4], which must be taken into account by not forgetting about the difference between what is in the data and interpretation and what should be taken as a description of the persons' competence.

As for methodological approaches, there is a plethora of possibilities. Surveys can be done as questionnaires or interviews (e.g. Bromme & Hömberg, 1981, on the role of mathematics textbooks in teachers' lesson preparation work or Zimmermann, 1992, on the use of textbooks by students). Surveys imply certain validity problems directly linked to the

object under study, e.g. the problem of recalling the use of textbooks by those who are questioned or interviewed. There is also a certain likelihood of the answers being biased because of the social desirability created by these methodological approaches.

Recently, we find more case studies, which may be observations, experiments or mixed methods case studies. In particular the latter offer one way out of the problems identified with the different methods, because they combine advantages of different methods and reduce disadvantages, thus attempting to overcome validity problems by means of triangulation. This is the case in Randahl's (2012) study of first-year engineering students' use of calculus textbook. In order to identify students' approaches to mathematics textbooks and opportunities and constraints that influence them she combines data from a questionnaire, an interview with the teacher, observations of lectures and task-solving sessions, interviews of students, and some informal conversations with students. The choice of methods is well reflected and driven by the aim to reduce the problems with each single method in order to increase the objectivity and validity of the data.

Normally, it is not the use itself that research in mathematics education is interested in, but the impact of the use on the user, thus, answering the question what was the effect of the use of textbooks. We use the term "impact" instead of "effect" in this paper to distinguish it from the technical term "effect size", which is a common measure of social studies (especially meta-analyses) nowadays.

The field of studies on the impact of textbooks can be divided according to the domain on which impacts are considered. Three major domains can be distinguished:

- The impact on mathematics lessons in terms of contents and of teachers' teaching strategies.
- The impact on students' achievement.
- The impact on students' cognitive processes.

Impact of textbooks on mathematics lessons

From all different domains on which impacts have been considered, the impacts of textbooks on the content of mathematics lessons and on teachers' teaching strategies received utmost attention. Johansson (2006) analysed the impact of textbooks on the content of teaching in Swedish mathematics lessons of three different teachers, using a mixed methods approach combing classroom observation with interviews and questionnaires. The impact of textbooks was measured according to three

categories: "textbook direct", "textbook indirect" and "textbook absence". The video recorded lessons where split into sequences according to these three categories. "Textbook direct" refers to video sequences of lessons where the textbook is physically apparent and used in the classroom. "Textbook indirect" refers to sequences in which the teacher makes verbal or written analogous statements as can be found in the book without referring explicitly to the book. "Textbook absence" means that no analogy to the book can be identified. Johansson finds out that despite differing use of textbooks in each lesson of each teacher there are only few occasions of textbook absence and the textbook dominates the mathematical content of the lessons. Although Johansson analyses video recordings and no written communication her method is comparable to content analytic methods, because the contents of the lessons are analysed according to three predefined categories in order to make inferences from the content of the lessons to the relevance of the textbook. All studies that aim at measuring the impact of textbooks on the content of mathematics lessons and on teaching strategies have to code data on content of mathematics lessons or teaching strategies according to a category system related to textbook content or teaching strategies. The sources of these data differ among the studies. Some studies use observations or video recordings of mathematics lessons (Johansson, 2006), other studies use teacher logs and interviews (Freeman and Porter, 1989; Floden et al., 1981) – showing that the methodological toolkit has to be enlarged when leaving the narrow field of textbook analysis or use of textbooks.

The impact of textbooks on students' achievement

A very important question related to impact of textbooks is the question about the impact of textbooks on students' achievement in mathematics. Törnroos (2005) tackles this question asking "What kinds of correlations can be found between opportunity-to-learn data of different kinds and student achievement?" This question even reveals a methodological focus of the study. Törnroos approaches opportunities-to-learn in three different ways: 1) A first measure of opportunities-to-learn was provided by the proportion in which each TIMSS-topic was treated in each textbook. 2) The second approach was based on a teacher-questionnaire, in which teachers were asked to provide information if and when they had taught the TIMSS-topics. 3) The third measure of opportunities-to-learn was provided by an item-based analysis of textbooks[5]. In the latter approach different textbooks where analysed according to the degree of preparation they provide for students to answer the TIMSS 1999 items. Törnroos carries out a quantitative correlation analysis with these three

opportunity-to-learn data and data on students' achievement from TIMSS. He concludes "an item-based analysis of [...] textbooks yielded relatively high correlations between opportunity to learn and student achievement".

Studies investigating the impact of textbooks have to tackle the methodological challenge to gather valid data on students' achievement and on opportunities to learn. Törnroos' study shows that it matters how opportunities to learn are measured. The same accounts for the students' achievement measure. While Törnroos relies on the TIMSS results Senk et al. (2014) use a pretest-posttest-design. Data on students' achievement was gathered using two different pre-test and three different post-tests. Compared to Törnroos (2005) the opportunity to learn data was refined, because Senk et al. collect data on opportunities to learn from teachers in terms of lessons taught and questions assigned throughout a whole school year. Data was analysed using multiple regression analysis to test if the used textbooks and the used opportunity to learn measures would predict achievement in the post-tests. From all five analysed variables the used textbook was the strongest predictor of achievement on the functions subset of one of the used post-tests. This study actually approaches impact in the sense of "effect size". It is prototypical for the quantitative statistical approach, which tries to control as many variables as possible and then looks into effects that have to be identified in quantitative statistical analyses.

The impact of textbooks on students' cognitive processes

A different approach than focussing on the relation between textbooks and students' achievement could be to investigate the impacts of the use of textbooks on students' learning processes or strategies. An experimental study in this context was carried out by Lithner (2003), who observed students' mathematical reasoning when solving exercises in university textbooks. The students worked in the presence of a video camera, they were asked to "think aloud", but they should try to act as closely as possible to their usual way of conducting homework and to use their normal aids. This already reveals that a major issue of experimental studies is the problem of ecological validity. The participants act in an artificial situation, which might bias the results of these studies. Like most observation studies in an experimental situation, the study by Lithner is limited to particular situations, its ecological validity at least can be questioned. This limitation was overcome in a later study (Sidenvall et al., 2014), where data was gathered in class using video recording and fieldnotes. The methodological difference between these two studies also shows a possible path of methodological development within textbook research.

After identifying and defining the relevant concepts in experimental situations the concepts might be applied and evaluated in ecological valid research contexts in the field.

Another issue related to studies investigating the impact of textbooks is, that they are limited to what can be observed. They cannot approach the phenomenon under study directly, because it is part of the inner world of the participants. Thus, they have to infer the impacts of textbooks use from observable facts. Lithner (2003, p. 32) himself draws attention to this issue and comments: "A sequence of reasoning analysed in this paper is always the part of a person's reasoning that is accessible as a set of empirical data that can be represented in a documentary form (text, symbols, figures, pictures, video recordings, etc.), and not the actual reasoning that took place in the person's mind", reminding us of the caveat on what can be observed.

Österholm (2006, also Österholm & Bergqvist, 2013) also carries out an experimental study, where students' mathematical reading ability was investigated.

Both studies seem to be appropriate for the text type under scrutiny and the respective research questions under study, but share the problem of ecological validity with the majority of experimental studies. [6]

Aiming at "valuable contextual knowledge" (Beach & Player-Koro, 2012, p. 117) and thus facing the problem of ecological validity, Beach and Player-Koro (2012) investigate the question how subject textbooks were used in a teacher education context and what meanings and understandings were generated by this use (Beach & Player-Koro, 2012, p. 118) based on two ethnographic studies. They argue that their ethnographical approach "produces knowledge about the basic conditions of education systems and practices and the perspectives of the participants involved in them, while minimalizing the influence of purely personal ideas" (Beach & Player-Koro, 2012, p. 117). Compared to the experimental design their approach is likely to produce ecological valid knowledge, because data has been gathered in a natural context. The bias of the researcher as a person acting in the field and thus also affecting the field can be minimalized. However, this method strongly relies on field notes. Thus the data itself is not objective, but always an interpretation of the reality by the researcher.

Textbooks – linking the artefact, its use and its impacts

As we have seen in the previous section, research on the artefact textbook and its use can be easily captured by means of a tetrahedron (see upper part of figure 1, the didactical tetrahedron in the narrow sense). Studying the impacts of the textbook on lessons, students' achievement

and students' cognitive processes adds a perspective, an aim to this tetrahedron (or triangles as part of it), trying to find out what comes out of the triangle/tetrahedron. This can be the quality of mathematics taught or learned, the type of interaction promoted by a (set of) textbook(s) or the subject matter learned measured by curricular or other societal standards.

From a methodological point of view, we basically found three common methods of textbook research: content analysis, surveys and case studies. Whereas content analysis is capable to reveal opportunities to learn, the two latter research approaches in a way have complementary potential and problems. At a first glance, surveys seem to be easier to set up and run through, but are likely to have validity problems. They should be evaluated in terms of representativity of the sample and the accepted quality criteria of reliability, validity and objectivity. In contrast to this, a case study approach normally does not have problems with its validity, but is often difficult to start because of difficulties to define and get data on the case under study. In addition to this, reliability and objectivity are not as obvious as they may seem with surveys. Case studies often use a "grounded theory approach" (for characteristics Glaser & Holton, 2004), relying on "theoretical sampling" and "theoretical saturation".

If we widen the lens coverage from the Nordic countries to a more global one including textbook research in general, not restricted to research on mathematics textbooks, additional research methods come into sight. In order to cope with the necessities from the objects under study, triangulation and typologies are used as complementary methods in order to increase the validity of the results and to allow for generalizability (for an example of this approach see Rezat, 2009, for typologies especially chapter 7, pp. 283–309, or Rezat, 2013). These methods can also be used to cope with inconsistent results from case studies. In general and not only related to case studies, the issue of generalization can be tackled, if not overcome by the construction of typologies in the sense of "types". Kluge (2000) distinguishes "normal types", "empirical types" and "prototypes". She states (loc. cit.):

> Every typology is the result of a grouping process: An object field is divided into some groups or types with the help of one or more attributes. The elements within a type have to be as similar as possible (internal heterogeneity on the "level of the type") and the differences between the types have to be as strong as possible (external heterogeneity on the "level of the typology"). The constructed subgroups with common attributes that can be described and featured by a particular constellation of these properties are defined with the term type.

Conclusions

We provided an overview of methods used in textbook research according to three areas of research on mathematics textbooks:

1. Research on the influences on textbooks
2. Research on the textbook itself
3. Research on the impacts of textbooks

From this overview it appears that most research on mathematics textbooks focuses on areas where standard methods are available. The prevalent method in studies on mathematics textbooks is qualitative or quantitative content analysis, which is an established method from the social sciences. Research based on content analysis is able to answer questions about the contents of textbooks and of the relation between the contents of textbooks and the contexts in which they are used. In this context, contents is not restricted to mathematical contents, but also comprises didactical aspects of the mathematical contents, such as cognitive demand of tasks, teaching strategies and other aspects of teaching and learning mathematics. Since methodology is always connected to the phenomena under study it can be argued that the methodological focus on content analysis is equivalent to a restricted domain of research issues related to textbook research.

On a more general basis, Fan (2013) calls for a paradigm shift about methods in textbook research:

> it can be argued that there is a strong need for a paradigm shift, using Kuhn's term, about the methods of textbook research, for two main reasons. The first reason is that research methods are directly related to research issues, and they are designed and employed to serve the purpose of addressing these issues. Therefore, to advance research on mathematics textbooks from mainly addressing descriptive issues to focusing more on correlational issues and causal issues, as discussed earlier, calls for a range of new research methods.
>
> (Fan, 2013, p. 774)

Even if Fan's understanding of decent research seems not to comprise descriptive studies and case studies (unnecessarily narrowing down the toolkit of research into textbooks), it also becomes clear from our overview that research, which focuses on correlational and causal issues and aims at an understanding of underlying mechanisms (as also signposted by Fan), is confronted with methodological challenges. From our point of view, these relate especially to three questions:

1. How to approach the cognitive dimension of textbook use?
2. How to take into account the social dimension of textbook use?
3. How to restrict the focus on the variable "textbook"?

Research on mathematics textbooks has to tackle questions of design and evaluation of opportunities to learn and how these relate to other variables in education like curriculum design, instruction, teachers' and students' behaviour. This list of issues is not intended to be exhaustive. However, it already makes clear that research on mathematics textbooks is almost a micro-cosmos of research in mathematics education.

A future challenge of research on mathematics textbooks will be that textbooks might appear in a different modality:

> Indeed, despite the attractions of readily available information on computers it would appear that textbooks, or more modern IT equivalents, will always be needed—even if in years to come they are no longer printed on paper. Already, South Korea has decided to move to e-textbooks in all its schools in 2015 and experiments are being carried out in France, the USA, Japan and Singapore (for the pros and cons of e-textbooks cf. Miller et al., 2012 and Philip & Moon, 2013). The possible savings that e-books could bring along with the perceived ease with which they might be updated cannot be ignored, and the financial constraints currently in force in so many economically developed countries could accelerate a move to them. (Howson, 2013, p. 647)

As a consequence questions of use and impacts of textbooks might even be brought to the forefront, because of increased interactivity and availability, which calls for new methodological approaches.

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Notes

1 "Under de observerade matematiklektionerna i år 7–9 arbetar eleverna individuellt med bokens uppgifter och läraren ägnar tiden till att ge individuell hjälp. Gemensamma genomgångar sker sällan eller aldrig."

2 This paragraph refers to previous published material (cf. Rezat & Straesser, 2013)

3 Fan (2013) in his overview seems to not include these qualitative studies in research on textbooks. We obviously start from a different point of view.

4 From Rezat & Sträßer, 2012, p. 473: [T]here is a fundamental difficulty with researching persons: There is no direct access to the person's inner world, if the researcher does not reduce a person to its observable activities. As long as a person is not reduced to her/his performance, as long as Didactics is interested in competence – and the present competency based approaches

somehow suggest to do so – there is a gap between what is to be observed and what is assumed to be the reality of a person's competence. The difference between "competencies" and "competence" in the English language is a very pale indication of this gap, which has consequences for a research methodology in Didactics of Mathematics: It has to take into account the difference between performance and competence, because observables are merely indicators of the competence of the proband under study.

5 Opportunities to learn according to no.1 is an analysis on a rather global, conceptual level, whereas no.3 looks into the concepts to be used in individual tasks.

6 The problem of ecological validity is in fact not taken into account by Fan (2013).

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