Teachers' use of resources in and for mathematics teaching

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Over the past decades the amount of available resources for mathematics teaching has vastly increased, in particular the availability of resources on the web. However, we know very little about how teachers select and use the available resources. In this paper we investigate how four primary school teachers used curriculum resources in and for their mathematics teaching. Grounded in a case study approach, we have analysed lessons, teacher interviews, and documents they used for their lesson preparation and instruction. Subsequently, we identified five "usage categories": (1) resources to manage the teaching objectives; (2) resources to "inspire" teaching; (3) resources for student work; (4) resources to adapt the teaching to indivi-dual students' needs (differentiation); and (5) resources to organize the teaching. In this article we explain and discuss these five categories, and argue that the "lens of resources" offers an opportune window into teachers' work, in particular their work as mathematics teachers.

For some time mathematics curriculum materials, in particular textbooks, have been heralded as crucial resources for mathematics teachers' work (e.g. Cohen, Raudenbush & Ball, 2003; Pepin & Haggarty 2001). Undoubtedly due to the increase in digital/web resources, there is now an enormous amount of materials available, both traditional (e.g. textbooks) as well as digital resources (Gueudet, Pepin & Trouche, 2012). It is perhaps because of this immense availability of teaching resources, provided by publishers and also individuals (on the web), that research has focussed on the resources' quality and their use by teachers and educators (e.g. Fan, 2013). It is known that when teachers use resources they "[...] often feel compelled to change them, in order to be able to address pupil questions, difficulties, or misunderstandings" (Gueudet, Pepin & Trouche, 2013, p. 2). The literature (e.g. Gueudet & Trouche, 2012; Brown, 2009; Pepin, Gueudet & Trouche, 2013a) claims that there is an

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interactive relationship between the teacher and the resource/s. In this paper we investigate particular aspects of these interactions between teacher/s and resource/s.

In Norway, where this study was conducted, the national curriculum (LK06) has had a strong influence on textbook resources and on teachers' practices (Pepin, Gueudet & Trouche, 2013b). The Norwegian national curriculum (LK06) consists of teaching objectives that should be achieved after the 2nd, 4th, 7th and 10th grades in schools, and it does not include concrete guidelines for teaching, how to teach particular topic areas. This puts a large responsibility on the teacher to develop their lessons in order to achieve the prescribed objectives, and teachers often feel at a loss to do this in appropriate ways. At the same time there has been a free textbook market since 2000 in Norway, which means that textbooks can be published without an approval from the ministry of education. Juuhl, Hontvedt and Skjelbred (2010) show that in many cases, teachers lean on (or download) commercially produced curriculum materials, which in turn gives power to those commercial publishing companies. In this study[1] we examine what kinds of materials selected mathematics teachers used most, and how they used them.

Theoretical background

Over past decades there has been a considerable amount of research on curriculum materials and resources in mathematics teaching (Remillard, 2005). Different researchers have used different expressions for these materials and resources, e.g. c*urriculum resources*, *curriculum materials*, *teaching tools* etc. Adler (2000) coined the term "re-source", and she explained that "it is possible to think about a resource as the verb 're-source', to source again or differently. This term is provocative. The purpose is to draw attention to resources and their use, to question taken-for-granted meanings" (p. 207). In our study we define resources in line with Pepin and Gueudet (2014):

> We define mathematics curriculum resources as all the resources that are developed and used by teachers and pupils in their interaction with mathematics in/for teaching and learning, inside and outside the classroom. Curriculum resources would thus include the following:
>
> 1. Text resources, such as textbooks, teacher curricular guidelines, websites, student sheets, and syllabi
>
> 2. Other material resources, such as manipulatives and calculators
>
> 3. ICT-based resources, such as computer software. (p. 132)

The research on teachers' use of resources can be divided into two groups. One group mainly includes text resources and curriculum materials (e.g. Brown, 2009; Remillard, 1999, 2012; Sherin & Drake, 2009). The other group has a wider perspective on resources and includes both ICT and human resources (Adler, 2000, 2012; Gueudet et al., 2013; Gueudet & Trouche, 2009, 2012). In our study we acknowledge the interactive relationship between the teacher (individual or in a collective) and the resource. The following theoretical frameworks use this idea (for more details see Pepin, Gueudet & Trouche, 2013).

The first theoretical framework is the d*ocumentational approach* (DA), as presented by Gueudet and Trouche (2009, 2012). In this approach the resources are viewed in connection with how they are used, the *scheme of utilization*. When resources are used, they develop into *documents*.

Document = Resources + Scheme of utilization

(Gueudet & Trouche, 2012, p. 25)

This is a dynamic process that takes time, and the influence goes both ways: as the teacher is influenced by the resource (*instrumentation*), the resources are influenced by the teacher (*instrumentalization*). The DA "[...] aims at presenting a more holistic view of the teachers' activity" (Gueudet & Trouche, 2012, p. 38).

While the DA uses a wide definition of *resource*, our second theoretical framework, the *Design Capacity for Enactment* (DCE) approach, is developed to look at text resources, in particular *curriculum materials*. Curriculum materials are also included in the first element of the definition of resources by Pepin and Gueudet (2014). In the DA, teaching is seen as a design activity, where the teacher designs lessons when using resources to achieve an objective. The DCE approach identifies three different ways teachers work with curriculum materials: *offloading*, *adapting* and *improvising*. When offloading the teacher predominantly leaves the agency to the resource/s, when *adapting* the agency is more equally divided, whilst when *improvising* the agency rests with the teacher (Brown, 2009). In the DCE approach the teachers' ability to "use" resources is called *pedagogical design capacity* (Brown, 2009).

Both the DA and the DCE framework talk about the *interaction* between the teacher and the resources. The development of teacher knowledge is important in both frameworks, but the teaching context is more prevalent in DA than in DCE. Because the two frameworks focus on different aspects of teachers' use of resources, they complement each other. In this study these frameworks were used to analyse our data.

Remillard (1999) provides a third frame, when she talks about how teachers use resources in different arenas. She identified three different *arenas* in which teachers use resources. In the *design arena* teachers choose

and design tasks for their students, in the *construction arena* teachers enact their choices in the classroom and respond to their students, and in the *curriculum mapping arena* teachers organize the curriculum. These three arenas are not time-set arenas, and teachers use resources in different ways within them.

The fourth frame is provided by Sherin and Drake (2009), who offer another perspective in terms of teachers' adaptations of resources. They found that the ways teachers adapt resources could be set along a continuum. At one end of the continuum teachers *create* new elements for their teaching, at the other end teachers simply *omit* elements from the resources without replacing anything or re-designing, whilst in the middle teachers may be seen to *replace* elements from the resources. These three adaptions have similarities with Brown's (2009) *offloading*, *adapting* and *creating*. Whilst the adaptions in DCE focus on how agency is divided between teacher and resource, Sherin and Drake (2009) found that the ways teachers adapt resources was influenced by when and how they evaluated the resources.

When interacting with resources, the literature contends that teachers *read* the resources in different ways. According to Sherin and Drake (ibid.) teachers read either for details, for an overview, or for a combination of these two. On a similar line, Remillard (2012) also claims that teachers "read" resources in different ways; she calls these different *modes of engagement*. Remillard has identified four different ways teachers read resources: "what she reads for; which parts she reads; when she reads; and who she is as a reader" (p. 115). These different ways to read the resources have similarities with the categories from Sherin and Drake (2009). Remillard (2012) also shows examples of how teachers either read for *big ideas*, for *tasks*, or for a *script*.

These perspectives on teachers' use of resources in mathematics teaching lead us to our research question:

> How do primary school mathematics teachers use resources in their lesson planning and their mathematics teaching?

Using the definition of resources by Pepin and Gueudet (2014), we examine four teachers' use of resources in both their planning of teaching, and their mathematics instruction in the classroom.

Research design

Using a case study approach (with the teacher being the unit of analysis), we sampled four primary level mathematics teachers; hence we had four cases. The four teachers were chosen, because they taught at similar grade levels (grades 5–7) in primary school and because of their involvement

in the mathematics teaching at their school. All four were in charge of mathematics teaching at their grade level at their schools, i.e. they were responsible for making lesson plans in mathematics for all classes at their grade level. In this paper they will be referred to as Cathrine, Lillian, Sandra and Torgeir. They worked at three different schools in Sør-Trøndelag, Norway, and they had different educations and seniority in their schools.

Cathrine was the youngest teacher amongst the four. She was a qualified teacher (under the Norwegian system). She had taken some additional courses in mathematics, and she had a Master degree in science. She had three years of teaching experience, and she now taught the 7th grade. Her school was a medium sized school with 350 students from a working class area of the city. In addition to the students living in the catchment area, hence "naturally" belonging to the school, the school had special classes for immigrant children (who also went to special Norwegian as a foreign language classes). These students were present in some of the lessons Cathrine taught.

The second teacher, Lillian, was also a qualified teacher, but with only one course in mathematics. She has been teaching for 13 years and her first job was at a "demonstration school" (a school that invited other teachers to observe the mathematics teaching). This period had apparently been important for her professional development. She had recently started at her present school, a small and "traditionally" run school. The school had less than 300 students from lower socio-economic backgrounds with several social service supported children in each class. Lillian now taught the 6th grade.

Our third teacher, Sandra, also had qualified teacher status, had taken additional courses in mathematics, and she had a degree in special educational needs. She has been a teacher for 11 years and has taught at all grade levels. At her present school she taught all seventh graders in mathematics. Her school was a relatively large school with almost 400 students, and these students came from different socio-economic backgrounds.

The fourth teacher, Torgeir, was the only case teacher without a Norwegian teacher education degree. He had studied French, social science and science, in addition to pedagogy for mother tongue. He had no formal education, above high school level, in mathematics, but had volunteered to lead the mathematics teaching at the grade level he taught, 5th grade. Torgeir taught at the same school as Cathrine.

The data collection strategies in this study were the following:

– Classroom observations of teachers' practices: 2–4 sessions per teacher were observed. The observations were conducted as "participant observer" (Cohen, Morrison & Mannion, 2007) with a

particular focus on the *use* of resources. The observations provided information of the teachers' use of resources *in* their teaching.

- Interviews with teachers (after observations): about their use of resources in their lesson planning and instruction in class. The interviews where semi-structured, with an interview schedule to secure that the same topics were covered in all four cases. In addition, during interview the interviewer also alluded to lesson situations previously observed.
- Document analysis: teacher documents relating to their lesson planning and teaching. Documents consisted of half-year plans, weekly plans, lesson plans, tasks provided for students, copies from textbooks and teacher guides, etc.
- Teachers' drawings of their *Schematic representation of the resource system* (SRRS) (Gueudet et al., 2013). The SRRS is a "mind map" of the teacher's resources, as perceived by the teacher. The four teachers drew four very different SRRSs, which gave us important additional information on their use of particular resources. For example, in the SRRSs teachers listed all the resources they used, and several had not been mentioned in the interviews. As another example, the SRRSs also gave us information about how teachers saw the connections between the different resources. One teacher even ranked the resources in her SRRS according to their importance in her work. Together with the interviews and the documents, the SRRSs gave us information about how the resources were used *in* and *for* teaching.

In terms of analysis, we used the constant comparative method as outlined by Cohen et al. (2007). We conducted the analysis in two steps. First we did a within-case analysis of each teacher, followed by a cross-case analysis (Creswell, 1998). The within-case analysis provided the first level of analysis where we analysed each case teacher's data firstly in terms of "open coding" (Cohen et al., 2007), and subsequently compared the emerging codes with themes from the literature (e.g. types of resources, interaction between teacher and resource/s, agency between teacher and resource, teachers' readings of the resources, etc.). The within-case analysis resulted in a case-description for each teacher's use of resources in and for mathematics teaching.

These case descriptions were sent back to the teachers for respondent validation, one of our attempts to counter threats to validity. Two of the teachers simply consented with the descriptions, whilst the two others

provided complementary information. The four amended and validated cases were then used for cross-case analysis.

In the cross-case analysis we compared the four cases and identified particular themes that were similar or different across the four cases. In this work we used the different types of coding from grounded theory, *open coding* and *axial coding* (Cohen et al., 2007). From the coding we first identified 9–15 usage categories for each teacher. After further analysis we managed to make broader categories, approximately 4–6 for each teacher. We used the constant comparative method to provide evidence (or not) for these categories. Subsequently, we used the literature to describe and explain these broad categories, but stayed closely to our data following the constant comparative method and making sure we did not "invent" the categories from the literature, but that they emerged from the data. This analysis resulted in five "usage categories", which are described and explained in the findings section.

Findings

Through our analysis we identified the following five "usage categories":

1. Resources to manage the teaching objectives.
2. Resources to "inspire" teaching.
3. Resources for student work.
4. Resources to adapt the teaching to individual students' needs (differentiation).
5. Resources to organize the teaching.

These were *usage* categories (and not resource categories). In the following explanation of the categories, some resources will occur in different categories, which indicates that the same resource was used in different ways. The usage categories described many aspects of mathematic teachers' work related to lesson planning and classroom instruction. Whilst the five categories covered different aspects of teachers' work, they were also closely connected to each other. Figure 1 below shows how the five categories were linked and related to each other.

Whilst the five usage categories were identifiable within all four teachers' work, there were differences in the ways they presented themselves in each individual teacher case. In the following we describe each category and the supporting evidence from the four teacher cases. Figure 1 will be explained in each category.

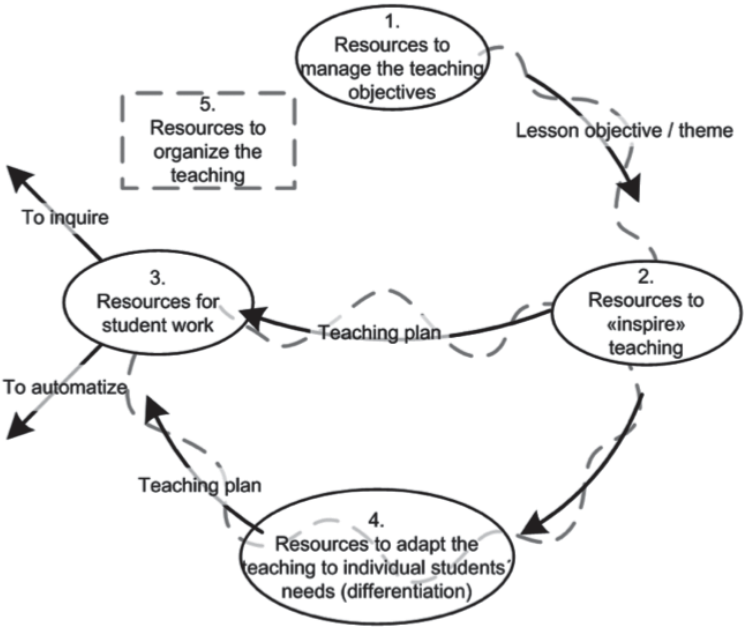

Figure 1. *The five usage categories*

Resources to manage the teaching objectives

All four teachers in our study used resources to manage their teaching objectives. The ways they did this could be divided into two subcategories: they used resources to manage (1) long-term objectives, for a longer period, up to a year; and (2) short-term objectives, for single lessons or several weeks. In both these subcategories there were two crucial resources: the national curriculum; and the textbook with an accompanying teacher guide. We will now describe the practice of two teachers, Cathrine and Lillian, who had quite contrasting practices in this usage category.

In Cathrine's practice, the textbook was the most important resource to manage both her long-term and short-term teaching objectives. At first sight, when studying her half-year plan, where she listed her long term teaching objectives, it appeared as if she had prepared it according to the national curriculum. However, through more thorough observations and from the interview, we understood that the half-year plan was only a formal document, which was not used. What directed her teaching was the textbook. She said about her planning that "we just continue where we stopped last lesson".

In Lillian's practice, the national curriculum was her most important document for lesson planning. Lillian had recently changed school. In her new school they used a textbook she did not like. This resulted in her using the national curriculum to organize her teaching objectives, both short-term and long-term, to the detriment of the textbook. When studying her half-year plans and weekly plans, we found close alignment between her teaching objectives and the objectives from the national curriculum. Her teaching objective for the period we observed was: "To be able to multiply multi-digit numbers and feel that you master one calculation method". This objective closely resembled two objectives in the national curriculum:

- Describe and use the place value system for decimal numerals, reckon with positive and negative whole numbers, decimals, fractions and percentages, and place these along a number line.

- Develop and use methods for counting in his or her head, make estimates and written calculations, and use a calculator for these methods. (Directorate for Education and Training, 2013)

To understand these two teachers' work in terms of this usage category, we leaned on Remillard's (1999) different "arenas". When the teachers were working with the management of their long-term objectives, it can be argued that they worked in the *curriculum mapping arena*. In this arena the textbook would take an important role, as we saw with Cathrine. About the role of textbooks in this arena Remillard writes:

> Textbooks offer a curriculum map that organizes mathematical topics into sections, each including specific concepts or skills. Teachers map the curriculum when they decide how or whether to use these structures. (Remillard, 1999, p. 334)

Cathrine decided to use the structures from the textbook, whilst Lillian decided not to. This resulted in two different practices in this usage category. Using Brown's (2009) three different adaptions in DCE, Cathrine *offloaded* "a large degree of agency for guiding instructional activity onto the materials" (Brown, 2009, p. 24). This is in contrast to Lillian, who did not simply offload, but aligned her teaching. Both her weekly plans and half-year plans were structured in accordance with the national curriculum. Taking this as her starting point, she adjusted the teaching to achieve her teaching objectives.

When the teachers used resources to manage their short-term teaching objectives, we contend that they worked at the borderline between *design arena* and *curriculum mapping arena*. Whilst the curriculum had

already been organised in half-year plans, at the same time the teachers organised the sequencing of teaching for shorter periods, e.g. a week, and designed the detailed aspects of teaching. We assert that when teachers organised and *designed* their teaching with short-term objectives, they moved from the *curriculum mapping arena* to the *design arena* (Remillard, 1999).

This usage category, *resources to manage the teaching objectives*, was a prominent category in all four teachers' cases, for at least two reasons. The first reason concerns the form of the Norwegian national curriculum. As we described earlier, at the time of our study (and this is still the case) the national curriculum consisted of teaching objectives that should be achieved after the 2nd, 4th, 7th and 10th grades, and it was the teachers' responsibility (in each school) to organise the teaching in ways that made this possible. This demanded that teachers organised and managed the detailed teaching objectives themselves. The second reason for the prominence of this category is likely to be linked to the free textbook marked in Norway (i.e. that mathematics textbooks can be published without authorization from the government) (Juuhl et al., 2010). While following the national curriculum was statutory, following a textbook was optional, and those teachers who decided to follow the textbook had no guarantee that the textbook was in accordance with the national curriculum.

In figure 1 this is the first category. The outcome of this category was "lesson objectives/themes", and the four teachers' lesson planning typically started with this category. Once they had managed lesson objectives and themes, they moved to the next category, *resources to "inspire" teaching*, whilst still referring to the lesson objective/s.

Resources to "inspire" teaching

The second usage category, *resources to "inspire" teaching*, was directly linked to the teaching objectives in the first category. The teachers talked about how they used resources for inspiration to plan their lessons in order to achieve the teaching objectives. Our study showed that there were two main resources the teachers used for inspiration, namely the textbooks and the Internet.

All four teachers worked in schools, where the school management had decided which textbook the whole school should use. But how they decided to use the textbooks, varied amongst the four teachers. Sandra and Lillian used the same textbook, but worked in different schools. Neither of them was pleased with the textbooks chosen by their school, but they used it "for inspiration" in two different ways. Lillian found "themes" in the textbook, and for the rest of her lesson planning she used the national curriculum

and other resources, such as her own old lesson plans, other textbooks, the Internet etc. She only "used" the textbook for the students' homework.

Sandra, in contrast, continued using the textbook as her main resource for inspiration, but supplemented it with other resources. One of the resources Sandra used as a supplement to the textbook was the Internet. She used the Internet both to inspire herself, and her students. In terms of using the internet for her own inspiration, Sandra selected YouTube videos made by other teachers, lesson plans of other teachers and such like. In terms of inspiration for her students, for example, she found problem-solving tasks and games for her students on the Internet, both on the publishing firms' home pages and other preferred websites. She claimed that her students were more motivated when doing tasks on the Internet than tasks from the textbook, and it was her belief that students needed motivation and good mathematics experiences to learn mathematics. In fact, all four teachers used the Internet mainly to inspire their students.

In addition to the Internet and the textbook, the teachers used previous lesson plans, colleagues' ideas, other textbooks and concrete materials for inspiration. Lillian, who did not want to use the textbook, used old lesson plans very frequently. Leaning on the *documentational approach* (Gueudet & Trouche, 2009), her old lesson plans functioned as resources for her. However, with different "schemes of utilization", her usage schemes in different classes, they developed into *documents*. We observed one example of this, when she taught multi-digit multiplication with the array model. This model was not used in her present textbook, but had been important in her former textbook. After one of her lessons she talked about the amendments that she had to do concerning her lesson, in order to familiarise her students with this method, and she showed us how her documents were constantly being developed and changed to adjust to this particular method.

The ways these four teachers used resources for inspiration in their planning phase can be placed in the *design arena* (Remillard, 1999). The *design arena* is about choosing and creating tasks for the students. This was something both Lillian and Sandra did with the textbook, the Internet and other resources, but they did it in quite different ways (as described). Remillard (1999) claims that the decisions in the *design arena* are directed by the teacher's beliefs. The way Sandra used the Internet to give inspiration and motivation to her students was an example of this.

From figure 1, one can get the impression that *Resources to "inspire" the teaching* was about getting inspiration for the teaching, but as we have described, it also involved to inspire students. There was a close connection between the first and the second usage category. This second usage category led directly to the teaching plan, or to *Resources to adapt the teaching for individual students' needs*, as shown in figure 1.

Resources for students' work

In this usage category, the students mainly used the resources, but the teachers initiated their use: typically tasks and concrete materials in the lessons, or textbooks, Internet etc. at home. Our study revealed that the teachers initiated two usage ways for their students: either to automatize skills, or for inquiry.

In Lillian's practice the differences between developing automatization skills and developing inquiry-based practices were evident. She wanted her students to work in an inquiry-based way in her lessons, together with peers, and to develop automatization skills at home where pupils worked on their own.

In Cathrine's practice the differences between these two were not as clear as in Lillian's practice. In one of the lessons we observed she taught calculation with brackets. Her textbook presented two different ways to solve the task "Ulrik has 20 NOK. He buys a chocolate prize 11 NOK and a box of candies prized 7 NOK. How much does he have left?" (our translation, see figure 2). The teacher guide recommended that the students should get the chance to investigate both methods (on particular tasks) and that the teacher should help them to see the connection between the two methods (Alseth, Nordberg & Pedersen, 2008a, p. 14–16) as a preparation for calculating with brackets.

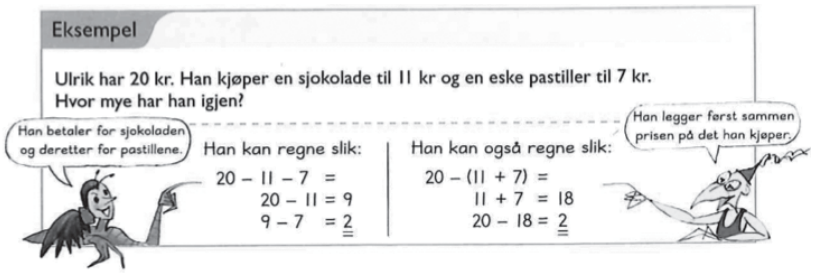

Figure 2. *Task from Alseth et al. (2008b, p.14)*

In her lesson Cathrine briefly presented the two methods by asking a student to read the example aloud (figure 2). After the reading she gave the students a new example and instructed them to always calculate the brackets first, method nr 2 in figure 2. After this instruction she set them to work with the subsequent tasks in their textbook, but only by calculating according to method 2. In the interview following this lesson observation, Cathrine talked about her beliefs concerning learning mathematics:

Cathrine: I believe that in mathematics you need a big amount of concentration, and you need to know what to do and when to do it. You need everything to be separated into smaller parts.

This description from Cathrine's lesson can be understood in terms of DCE. In DCE the resources interact with the teacher, and vice versa, and the result of this interaction leads to the *instructional outcomes* (Brown, 2009). Evidence from Catherine's lessons showed that the resource/textbook clearly recommended inquiry, but Cathrine's beliefs as a mathematics teacher would contradict and override this. The outcome of this interaction can be termed *adaptation* (Brown, 2009), which is a different interaction than what we previously saw in Cathrine's practice.

In one of Sandra's lessons we saw an example of resources being used by students for inquiry. The teaching objective for this lesson was "To be able to do addition and subtraction of fractions with different denominators". Her textbook (Pedersen et al., 2006) recommended that the students should use fraction strips[2] to solve the tasks by investigation. The textbook also asked the students to explain to a peer how each task was solved, in order to provide opportunities for discussion and reasoning on their answers. This was in accordance with Sandra's beliefs and intentions. She said in the interview that she wanted her students to use fraction strips to see connections between different denominators and between addition and subtraction of fractions, and that she wanted them to cooperate in order to help each other and to communicate about their findings to secure deep understanding.

In her lesson she gave an introduction to fractions and fraction strips. Following that, the students were supposed to cut their own fraction strips and solve the tasks from the book, together with a peer. As the cutting took a long time and some students were doing other things, Sandra decided that no one could take their break before they had finished all the tasks. This made the students work quicker, but at the same time prevented them from discussing with peers. They mostly worked individually, and mastered the addition tasks with the fraction strips. However, when working with the subtraction tasks, most of the students had problems and asked Sandra to explain. It is likely that the students considered addition with fraction strips as a new algorithm, and did not consider the work as a way to discover connections between denominators. In that case they would have needed a new algorithm for subtraction. Van Galen et al., (2008) say the following about students' use of fraction strips:

> Working with fraction strips can only contribute to the intended development if the teacher makes sure that the students test their solutions by explaining the relationships between the measurements they used. Good mathematics education involves more than the quality of the concrete materials; it also involves the quality of the students' discussions of the reasoning behind their solutions. (Van Galen et al., 2008, p. 66)

When Sandra limited the time for students' work, they refrained and did not take the time to discuss with peers, and hence did not utilize the full affordances of the fraction strips. The lesson evolved from (intended) inquiry to almost automatization. In terms of Brown's (2009) DCE, Sandra's interaction with her resources can be termed *adaptation*. Leaning on the *documentational approach*, where a *document* is the combination of *resources* and *scheme/s of utilization* (Gueudet & Trouche, 2012), the teaching situation gets a prominent position. In Sandra's case both her intentions/beliefs and the resources indicated that the students should investigate fractions whilst using the fraction strips. But her choices in the lesson, in the teaching situation led to the unintended automatization. This shows how the teaching situation, the *scheme of utilization* in this specific lesson, influenced the *document* and overrode Sandra's intention for the lesson. In the interview after the lesson Sandra said:

> Sandra: Yes, it is my idea that they (the students) should do it by seeing, by understanding. But actually that is not correct. Very often the students become more confused by it.

In the described lesson, she experienced that the students got more confused, possibly because they treated the fraction strips just as a new algorithm, or because they were stressed and did not take the time to discuss and reflect with peers. The Norwegian curriculum emphasised inquiry-based teaching at all levels. This finding showed how important the usage of the resources was for the enactment of inquiry-based teaching.

To sum up the category *resources for students' work*, one can say that within this category teachers worked in the *construction arena* (Remillard, 1999). The teachers enacted their intended lesson plans in class. But work in the construction arena also involved decisions made in the moment (Remillard, 1999). This was illustrated by Sandra's lesson using the fraction strips.

It was also interesting to see how prominent the textbook has been in these three "usage categories", even though it was used in different ways. This finding is in accordance with research on textbooks (e.g. Valverde et al., 2001), which claims that the textbook is one of the most crucial resources in mathematics classrooms.

Figure 1 shows how this usage category, *resources for students' works* came as a consequence after the teachers had made their lesson plans. The examples supported this: in the teachers' lesson plans they had decided *how* students would use the resources, whether they should use them for automatization or for inquiry, but the examples have also shown how the teaching situation influenced the in-the-moment decisions.

Resources to adapt the teaching to individual students' needs

Norwegian education has traditionally had a strong focus on equality (Kunnskapsdepartementet, 1998) claiming to provide equal learning opportunities for all students, and hence "setting" and/or "streaming" practices are not encouraged (Kunnskapsdepartementet, 1998). In order to attend to the learning needs of individual pupils in their class, Norwegian teachers are advised (and expected) to differentiate their teaching. In this section we describe how the four teachers used resources to carry out their differentiation strategies. We have identified three different ways the four teachers differentiated: 1) they differentiated via the *arbeidsplan*; 2) they used concrete materials with low achieving students; and 3) they used what they called "alternative working methods". We will now describe these methods briefly.

In Norway there is a prevailing use of the *arbeidsplan*. This is a document, normally written/developed by the teachers teaching the same grade for their students. The arbeidsplan typically lists the learning objectives for the period in the main subjects, and consists of information about what the students are supposed to do in the main subject areas, both in school and at home for a set time period (e.g. a week or two). Typically all students in the same grade level get the same arbeidsplan, but it might list different tracks/or achievement levels for students´ work at school and at home.

The use of the arbeidsplan originates from teachers' work with differentiation (Klette, 2007). The teachers in our study used the arbeidsplan to differentiate between high achieving and low achieving students. One way was to differentiate between different achievement levels. Lillian differentiated between three levels, whilst Sandra differentiated between two, but in addition provided an extra level called "challenges", which was for everyone. For both teachers, the levels were flexible and they decided which level each student should follow, together with each student and their parents. Both Sandra and Lillian used textbooks from the same publishing firm. The textbooks offered differentiation in three levels, red, yellow and blue, where red was the easiest and blue is the most demanding. Whilst Sandra simply used the differentiation provided by the textbook, Lillian had clear intentions with respect to the different levels:

Lillian:	I differentiate, so that on the first level, there is no text-tasks.
Int.:	Mmm.
Lillian:	There it might be students with dyslexia and other learning difficulties that might give them a low self-efficacy in mathematics.
Int.:	Mmm.
Lillian:	So I have decided that they should master mathematics even though they have these difficulties, so they only get numbers, no text. At the

> middle level there are some tasks with only numbers and some text-tasks. At the highest level I put some tasks to check if the students have the proficiency needed, but mostly they work with problem based tasks.

With these criteria for differentiation she sometimes used the suggested differentiation in the textbook, whilst at other times she had to mix the textbook levels, or create her own tasks.

Leaning on Sherin and Drake (2009), Sandra's and Lillian's organisation of differentiation (in the arbeidsplan) could be understood by placing their practices on a continuum: Sandra at one end – following the textbook and simply *omitting* some parts, whilst Lillian may be placed at the other end of the continuum – she *created* her own differentiation scheme, with her own defined criteria. We contend that in Sandra's case the arbeidsplan was only an organizing resource, since she followed the differentiation from the textbook, whilst in Lillian's case it was clear that the arbeidsplan was a resource used for differentiation.

Cathrine used the arbeidsplan to differentiate in a different way. She differentiated between tasks the students *had* to do and tasks they *could* do. This was a type of differentiation of the *amount* (Klette, 2007) of work students were expected to do. But Cathrine had not decided for the students, who should do both categories, and this resulted in only one student doing the *can*-do tasks.

Lillian was the only teacher organizing the differentiation (in the arbeidsplan) after mapping tests. She used the tests in the textbook, and decided then which level her students should follow, together with the students and their parents.

As we have shown, the teachers used the arbeidsplan to differentiate students' work in different ways. The work they did with the arbeidsplan to differentiate can be understood in terms of Remillard's (1999) *design arena*, since the teachers chose particular tasks for their students to work on.

In our study we also found that the teachers used concrete materials for differentiation. In Lillian's practice this was most prevalent. She said in the interview that she used concrete materials especially for her low-achieving students, but she also used concrete materials to introduce new themes in mathematics, also on higher-grade levels. Sandra said that she used more concrete materials when teaching younger students, and that she reduced the amount of concrete materials as the students grew older since they no longer appreciated it. This could be seen in connection with the mathematics becoming more abstract in higher grades. Both Sandra and Lillian used concrete materials for differentiation *in* their lessons. These ways of using resources can be understood in terms of the

construction arena, as the teachers used concrete materials to differentiate for their students in the lessons, in contrast to how they differentiated pupil work in the arbeidsplan *before* the teaching (Remillard, 1999).

The third way to use resources for differentiation, "alternative working methods" was especially evident in Lillian's practice: Lillian talked about how her students worked with "practical mathematics outside" and how she used *tasks to reset students*. By this she meant tasks where every student was able to start and do something, and at the same time these tasks could be extended, as there were possibilities for more thorough investigations for the high achieving students. These tasks had resemblances with the tasks mentioned by Ollerton (2001), tasks which included all pupils in the same mathematics course without the need for streaming or setting (Ollerton, 2001). We found that, as this had not been the focus of our study, this category was difficult to substantiate, and Lillian was very ambiguous about the resources she used for these "alternative working methods". Some of the tasks she had made herself, others were from older textbooks, or from the Internet.

This fourth usage category, *resources to adapt the teaching to individual students' needs* (*differentiation*), is a difficult category to delineate, because it had close resemblances to many of the other categories presented. When teachers used the arbeidsplan to differentiate, this could be called an organizing resource and placed in the fifth category (see below). The use of concrete materials had close resemblances to resources used for inspiration, and "alternative working methods" had close resemblances to resources used for inquiry (category three). Still, we chose to present it as a separate category because of the strong focus on differentiation in Norway.

The three different ways the teachers used resources to differentiate can be understood when considering the three different time periods Sherin and Drake (2009) present: when teachers differentiated in the arbeidsplan they used resources for lesson preparation (that is *before* the lesson) and when they used concrete materials or "alternative working methods", they used resources to differentiate in the lesson. Unfortunately, we had no data supporting ways of how teachers differentiated *after* the lesson.

Figure 1 shows two different ways and links from *resources to "inspire" teaching* to *resources for students' work*. One of these ways was through *resources to adapt the teaching to individual students' needs* (*differentiation*). We have shown these two ways because the differentiation aspect was less evident in Cathrine's and Torgeir's practice, and clearly evident in Lillian's and Sandra's.

Resources to organize the teaching

The fifth usage category, *resources to organize the teaching*, was mostly concerned with teachers' organisation of their teaching, and to a lesser extent with the mathematical content of their teaching. We found that the *year plan/half-year plan* and the *arbeidsplan* were used by all four teachers. In addition they used other resources, like digital learning platforms, the blackboard or interactive boards.

Concerning the year plan/half-year plan, we found that some of the teachers organized it according to the order of chapters in the textbook (e.g. Torgeir), whilst others organized it differently to the textbook (e.g. Lillian). When they made the year plan/half-year plan, they worked in the *curriculum mapping arena* (Remillard, 2009). This was covered in usage category one.

Even though all four teachers had some kind of year/half-year plan, it varied among them how important this was as an *organizing resource*. In Cathrine's practice it was simply a formal document she had to provide (for her pupils and their parents). She followed the textbook, and continued where she had left off the previous lesson, regardless of what the plan said. Sandra's practice was in clear contrast: in her year plan she had made learning objectives for each period, and she followed this plan closely in her planning, teaching and evaluation. Hence, we claim that for Sandra the year plan functioned as an important resource for organization. In the interview she talked about how important it was to work thoroughly with the year plan when starting a new school-year, and that she used it as a support in her planning throughout the year.

Whilst the arbeidsplan was used by all four teachers, it was used in different ways. Cathrine's, Sandra's and Torgeir's arbeidsplan*s* provided a list of the tasks the students should do, both in school and at home for homework, in addition to the learning objective for the week. In their schools they had designated lessons for students working with their arbeidsplan*s*, called *working lessons*. This meant that Cathrine, Torgeir and Sandra had to choose tasks considering the time students might take to work with them, and it was stipulated that the students should spend approximately the same amount of time on mathematics each week. When the arbeidsplan said what students should do both at home and in school, it functioned as an organizing resource for students. But we also claim that it organized teachers' work, since creating the arbeidsplan with all these elements was a huge part of teachers' planning and lesson preparation in the *design arena* (Remillard, 2009).

In Lillian's school they had recently changed their practice concerning the arbeidsplan. As the arbeidsplan (now) simply stated the learning objective/s for the week, and the homework, we claim that it functioned

more as an organizing resource for the students, but less as a work plan for Lillian as a teacher.

All four teachers had weekly learning objectives on their arbeidsplan, and we claim that the learning objective/s could also have an organizing function. Sandra, Lillian and Torgeir all started their lessons with the learning objective/s, and Lillian even ended the lesson with the lesson objective. In figure 1, a dashed line represents this usage category. The dashed line goes through all the other usage categories, to show how the organization affected all the other usage categories.

Conclusions

From our investigation of the four mathematics teachers' work, we make claims in terms of three dimensions: (1) the dominant resources teachers used, (2) how they used them, which includes the usage categories, (3) methodological theoretical insights.

In our study of the four teachers we have shown that some resources were more important (for our teachers) than others, both for their lesson preparation and for their instruction in class. One of the most used resources was clearly the textbook; and it was also the most influential in terms of teachers' practices. In our discussion of the five usage categories, the textbook was evident in all categories, and especially prevalent in the three first categories. It was not surprising that the textbook had this dominant role, a finding in most countries according to international comparative studies (e.g. Valverde et al., 2001). Since in Norway mathematics textbooks no longer need authorization from the ministry of education, textbook authors take on a huge responsibility. Moreover, it becomes especially relevant to investigate how teachers use the textbook, as we have done in this study.

In addition to the textbook, we found that the national curriculum and the Internet were also very important resources for our teachers. However, the agency the teachers attributed to each of them differed among the teachers. We also found how the arbeidsplan, concrete materials, half-year/year plans, "old" lesson plans, and human resources (such as colleagues) influenced teachers' practices in different ways.

At the same time, our study shows how teachers used textbooks in different ways, and differently according to the different teaching situations. Cathrine's practice was a good example of this. In the first category, *resources to manage the teaching objectives*, Cathrine followed the textbook closely, *offloading* a lot of agency to the textbook, but in the third category, *resources used by the students*, Cathrine made *adaptations* because of her own beliefs about mathematics learning. This was an important

finding: the same resource inhabited different roles depending on the usage category. This finding would be particularly relevant, if one was to evaluate resources (e.g. textbooks).

According to our four teachers' use of resources in and for their teaching, we identified five "usage categories". In the *resources used to manage teaching objectives* category, the resources were used to guide the teaching, both long-term and short-term. The textbook and the national curriculum were the main "inhabitants" of this category. In the *resources to "inspire" teaching category*, the resources were used to plan lessons. Among the four teachers, different resources could be identified in this usage category, e.g. "old" lesson plans and the Internet, as well as the textbook, were the prominent ones. In the category *resources used for students' work*, the teachers initiated how students should use the resources, either for *inquiry* or for *automatization*, and *concrete materials* played an important role, in addition to the textbook and the Internet. In the *resources to adapt the teaching to individual students' needs* (*differentiation*) category, the teachers designed and appropriated resources, with the objective of adapting the teaching to the needs of individual students, as they were required to do (by law). In this usage category the textbook and the arbeidsplan were the most influential resources, but also concrete materials played a role. The fifth category, *resources to organize teaching*, was concerned with resources that helped teachers in the organization of their teaching. In this category the teachers mainly used half-year/year plans and the arbeidsplan.

As these five categories show, resources influenced many aspects of teachers' work in and for mathematics teaching. Resources influenced teachers' lesson planning, their work with the national curriculum, their enactment of the curriculum in the classroom, their decisions in terms of which tasks to provide for their students' work with mathematics, and their organization of the teaching and students' work, to name the "beneficiaries" most affected. Figure 1 illustrates how these categories were interlinked. From teachers' planning and *curriculum mapping*, over instructional decisions, to assessment, tests and students' homework, resources played an important role. Even though we studied only four case teachers, these four worked with and used resources in significantly different ways. Moreover, our study showed that resources influenced their practice: when designing their lessons (Brown, 2009), for example, or when making in-the-moment decisions in class, teachers were active users of resources (Gueudet et al., 2012).

In terms of research tools, we claim that "resources" are a useful instrument with which one can study mathematics teachers' work. Through the lens of "resources" we could identify and explain five usage categories,

which are arguably some of the most important aspects of mathematics teachers' work. In addition, we could identify the most crucial resources for lesson planning; for organising the teaching objectives; and for organising and enacting inquiry-based activities in class. Teachers' use of and interaction with resources can reveal important characteristics of teachers' practices of their mathematics teaching and beliefs about mathematics learning. Cathrine was an example of this: although she had recently undergone "thorough" and "fresh" teacher education courses, with several of them in mathematics, she followed the textbook closely. However, she had firm beliefs about mathematics learning that led to an *adaption* of the textbook (Brown, 2009).

As our sample of cases was relatively small, we do not claim generalizability of our results. However, one of the strengths of the case study approach has been that it has provided opportunities to study teachers' usages of resources in some depth, obtaining detailed case descriptions (Cresswell, 1998). Hence, we contend that our results detail teachers' work, and in particular their work with mathematics resources. In other words, we claim to have added to knowledge, albeit in a small way, in terms of deepening the phenomenon of "mathematics teacher resource use" by distinguishing selected categories of mathematics teachers' use of selected resources. This is particularly relevant, as there seem to be an abundance of resources available on the web, and at the same time in-service as well as pre-service mathematics teachers often find it difficult to use them meaningfully in and for their teaching. We believe that it is important to draw attention to the "re-sourcing of teachers' work" (Adler 2000).

Whilst our study may answer some questions, it also raises new questions. As we have shown in usage category three, there is still a great need to find out more about how teachers use resources to differentiate, and attend to individual pupils' needs. As this study was only small scale, we did not have the opportunity to follow the teachers' practices over time. It would be relevant to examine the usage categories, and the links between them, over a longer period of time, and also with more refined research instruments.

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Notes

1 The paper is based on the findings in Ingvild Grave's master thesis, submitted at Høgskolen i Sør-Trøndelag in May, 2013.

2 Fraction strips are equally long strips of paper. The printing on the strips represent different fractions, e.g. the printing on the third fraction strip divides it in three equally large areas with the printing 1/3 on each. Putting fraction strips beside each other allows us to compare different denominators.

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