The role of textbooks in Finnish upper secondary school mathematics: theory, examples and exercises

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Textbooks are written mainly for students, but they can also be seen as mediators between the intended curriculum and teachers' practices. The focus of the present study is on how students and teachers at Finnish upper secondary schools perceive and describe their use of the theory, examples, and exercises presented in their mathematics textbooks. Seventy-one students participated in a short survey, and six of them and three of their teachers were interviewed. The results indicate that the theory, examples, and exercises presented in their textbooks have a strong influence on teachers' work, but for the students the textbooks primarily represent a source of exercises.

A textbook is a very essential element in the teaching and learning of mathematics. It is an important tool for students, but it may also affect a teacher's choices in a number of different ways. It is generally known that teachers base a major part of their teaching on textbooks. As long ago as in the late 1970s it was found that in Germany and the USA roughly half of all teachers used no other materials besides their textbooks in their preparation of lessons (Haggarty & Pepin, 2002).

The role of textbooks has changed relatively little in the past few decades. In Sweden, for example, the inspectors conducting a school evaluation found that the teaching of mathematics at Swedish lower secondary schools relied on the use of textbooks more than was the case for any other subject (Johansson, 2006). Johansson (2006) found in her study that students were working individually with textbook tasks for more than half of each lesson, and the students' homework was also taken from the textbook.

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Viholainen, A., Partanen, M., Piiroinen, J., Asikainen, M. & Hirvonen, P. E. (2015). The role of textbooks in Finnish upper secondary school mathematics: theory, examples and exercises. *Nordic Studies in Mathematics Education*, 20 (3–4), 157–178.

Likewise, the examples and tasks presented by teachers were mainly from the textbooks. Johansson also found that mathematical definitions, conventions, rules, and procedures were generally the same and were presented in the teaching more or less as they appeared in the textbooks. Teachers also used the textbooks as a main source for background and motivational discussions. Holmlund (2013) examined the ways in which secondary school teachers in Sweden and Finland use textbooks in their planning of mathematics teaching. Holmlund found that textbooks are widely used in both countries, but Swedish teachers were more likely to limit their personal reliance on them.

The TIMSS group of researchers (Valverde et al., 2002) have discussed the relationship between textbooks and curricula. They attempt to accommodate the role of the textbook within a tripartite curriculum model consisting of the intended, the implemented, and the attained. They concluded that textbooks function as mediators between the intentions of the curriculum and the teachers themselves. According to them, textbooks and other organized resource materials belong to a potentially implemented curriculum. This means that textbooks are based on the intended curriculum, but teachers may use them dynamically as templates in creating educational opportunities, and, as a result, there is considerable variation in the precise impact of textbooks on instruction.

In Finland a tradition has prevailed in which so-called literature-based teaching has been emphasized (Mikkilä-Erdman, Olkinuora & Mattila, 1999). Teachers have frequently regarded conscientious adherence to the textbook as the safest way to proceed, since textbooks were expected to be based on the national curriculum. It was thought that this would ensure that the same topics were taught to all students on an equal basis. Supplementary materials in a textbook series, such as teacher guides and ready-made examinations, underpinned the role of the textbook in teaching. This all agrees with Perkkilä's (2001) findings, which showed that textbooks and teacher guides play a very large role in teachers' work in Finnish primary-level mathematics. Teachers very often follow suggestions about what to teach and how to teach as presented in the teaching materials, and in general the materials also prescribe the order in which the content should be taught. Teachers in Finland select the textbooks to be used in their teaching together with their colleagues. In general, therefore, all of the study groups in the same school use the same textbook series.

Mathematics textbooks have existed since ancient times, but until the 1980s few studies of their actual content had been published. More recently, however, Fan, Zhu and Miao (2013) have analyzed 111 journal articles and other scientific publications concerned with mathematics textbooks which have been published in recent decades. Their sample is quite comprehensive, which suggests that this area had not been much

explored until recently. Fan et al. found that 25% of the empirical studies in their sample dealt with the use of textbooks in the teaching and learning of mathematics. The study presented in this paper belongs to this category. Earlier studies in this category have revealed, for example, consistency between mathematics teachers' teaching practices and the actual contents of textbooks used in the Netherlands (Krammer, 1985), and that textbooks constitute the main, but not sole, source material for mathematics teachers in Singapore and China (Fan et al., 2013). Teachers' practices in using textbooks may vary considerably, but Fan and his colleagues found no dependencies between teachers' practices with respect to textbooks and teachers' genders, their experience, or their schools and the regions in which they worked. On the other hand, Remillard and Bryans (2004) showed that teachers' orientations toward curriculum materials and their professional experience may influence their way to use textbooks. According to Fan et al. (2013), considerable progress in the conceptual and methodological aspects of research into the use of textbooks has been made in the last 15 years. However, Fan and his colleagues argue that more research work is needed on learners' use of textbooks, specifically studies on a larger scale with experimental design.

Research questions

In the present study, the role played by mathematics textbooks at upper secondary school level is analyzed with respect to three elements: theory, examples, and exercises. *Theory* refers to mathematical theory, consisting of definitions, theorems, formulas etc., as it is presented in the textbooks. The theory, however, is not necessarily presented formally, but different kinds of representation may be used in textbooks. *Examples* are assignments whose solutions are also presented, usually in their entirety. In contrast, ordinary *exercises* are presented in an unsolved form, although in some cases the correct answers will be presented at the end of the book. Both students' and teachers' viewpoints will be taken into account in this study.

The research questions dealt with in this study have been formulated in the following way:

1) In their learning of mathematics, how do upper secondary school students perceive and describe their use of the theory, examples, and exercises presented in their textbooks?

2) In designing and implementing their teaching, how do mathematics teachers at the upper secondary school level perceive and describe their use of the theory, examples and exercises presented in their textbooks?

In this article, we present the combined findings of two studies originally produced as Masters theses. Partanen (2013) studied upper secondary students' experiences and views on the use of mathematics textbooks, while Piiroinen (2013) examined how teachers use mathematics textbooks in their teaching. These two studies were conducted at two upper secondary schools in Finland, where the teachers who took part in Piiroinen's study were also teaching the students observed in Partanen's study.

Printed textbooks have continued to the present day to play a dominant role in teaching, but in recent years the number of electronic learning materials has strongly increased (Galligan, Loch, McDonald & Taylor, 2010). It is probable that printed textbooks will be at least partially replaced by electronic learning materials. In consequence, a discussion involving the role of textbooks and other learning materials is both current and important. Studies concerning teachers' and students' ways of using their textbooks are also valuable both in the development of new kinds of learning materials and in discussions of related pedagogical questions.

Methods

The data used in this study was collected partly with the aid of a short questionnaire designed for upper secondary students and also by interviewing students and their teachers. Three study groups working on the advanced mathematics syllabus at two Finnish upper secondary schools participated in the study.

In total, 71 students (33 female and 38 male) answered the questionnaire. The subjects were in their second, third, or fourth year at upper secondary school (grades 11–13). The questionnaire consisted of ten statements concerning students' use of a mathematics textbook in learning mathematics, and the subjects were required to answer by using a four-step Likert scale. The statements were all concerned with the use of theory, examples, and exercises in the learning of mathematics. The statements are presented in table 1 in the Results section.

In addition, two students (one male and one female) were selected from each study group for a pair interview, and hence six students in total were interviewed. In the case of two of the study groups, a male and a female student who were the first to announce their interest in participating in the interview were selected, while in the case of the third study group the teacher helped in the selection of students. The most important criterion for the selection of the students was their willingness to describe their use of their textbooks for research purposes. The teachers of these three study groups were also interviewed. All of the interviews were theme-based (Kvale, 1996), where the focus was on the

use of textbooks in the learning of upper secondary mathematics. The interviews included discussion of the role played by mathematics textbooks with respect to the following themes:

Theme 1	The impact of textbooks on the course structure
Theme 2	The methods employed by teachers in guiding students in the use of textbooks
Theme 3	The role of textbooks in the teaching and learning of mathematical theory in lessons
Theme 4	The role of textbooks in the study of theory at home
	Subtheme: Students' opinions concerning the presentation of theory in textbooks
Theme 5	The role played by textbooks in the study of examples
Theme 6	The role played by textbooks in teachers' and students' approaches to conducting exercises

The questions used in the interviews are presented in the Results section in the context of each theme. The interviews were semi-structured so that, in addition to these pre-designed questions, the interviewers were able to ask some additional questions in light of each interviewee's responses.

In the following, the three study groups are indicated with the letters A, B, and C. The students interviewed are referred to as "student A1/A2/B1/B2/C1/C2" and the teachers interviewed as "teacher A/B/C", where the letter A, B or C refers to the study group for which s/he was responsible. Students A1, B1 and C1 were female and students A2, B2 and C2 male. Students A1 and A2 were in their second year of studies (11th grade), students B1, B2 and C1 were in their third year of study (12th grade), and student C2 was in his fourth year (13th grade) at upper secondary school. Teachers A and B were male, while teacher C was female. Teacher A had taught mathematics for 12 years, and teacher C for 13 years, while teacher B had also taught for several years, but the exact number of teaching years could not be attained. All of the teachers had taught mainly at upper secondary level. Teacher C had also experience of teaching physics and chemistry. Study groups A and B were from the same school and used the same textbook series, whereas Study group C was from another school where another textbook series was in use. The students in study group C had used the same textbook series throughout their studies at the upper secondary school, whereas the students in study groups A and B had earlier used a different textbook series. The interviewees and their mutual connections are illustrated in figure 1.

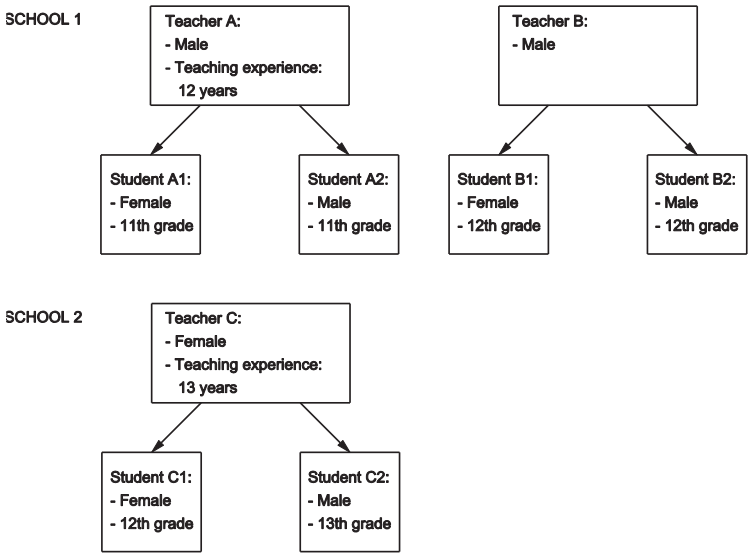

Figure 1. *Interviewed teachers and their students*

The data collected via the questionnaire was analyzed by calculating the means and standard deviations. These were intended to provide a rough overview of the students' use of textbooks in their studying. The interview data was transcribed and analyzed qualitatively with respect to each Theme. The purpose was to reveal the different practices and views that the students and teachers had with respect to each Theme. In Themes 1, 2, 3, 5, and 6 both the students' and the teachers' responses were included in the data. Both sets were first analyzed separately and then combined to check whether they complemented each other significantly. Common features used in their responses were also analyzed in order to reveal elements that might complement or explain each other. The data related to Theme 4 and its subtheme consisted solely of students' responses.

Results

The results are presented in several parts. First, the results based on the questionnaire are described in order to supply a general picture of the students' use of a textbook in mathematics. Subsequently, the results of the teachers' and students' interviews are discussed in terms of Themes in order to reveal significant aspects of the use of textbooks from these two perspectives.

General picture of students' use of textbooks

The results based on the questionnaire offer a very general overview of the students' use of their textbooks. The statements collected via the questionnaire and the means and standard deviations of the responses are presented in table 1. Statement-specific proportional response distributions for the statements are presented in figure 2.

Table 1. *Students' experiences while using mathematics textbooks as part of the advanced mathematics syllabus at three Finnish upper secondary schools (n=71)*

Statement*	Mean	St. dev.
1. How much do you use the mathematics textbooks in order to study the theory?	2,7	0,66
2. How much do you use the mathematics textbooks in order to do exercises?	3,4	0,67
3. How much do you use the examples presented in the mathematics textbooks?	2,9	0,66
4. How often do you seek help in solving exercises from the examples presented in the textbook?	3,1	0,61
5. How often do you seek help in solving exercises from other sources than the textbook?	2,0	0,81
6. How often do you seek help in understanding the theory from other sources than the textbook?	1,9	0,85
7. How much do you use mathematics textbooks at school?	3,1	0,70
8. How much do you use mathematics textbooks outside school?	2,5	0,71
9. Do teachers urge the use of a textbook?	3,1	0,69
10. Have you observed differences between teachers in terms of the ways in which they use the textbooks in the course of their teaching?	2,9	0,73

Note. * The students responded to the statements by using a four-step scale, in which 1 = not at all/never, 2 = occasionally/rarely, 3 = quite a lot/quite often and 4 = a lot/frequently.

On the basis of the results presented in table 1 and in figure 2, it seems that students use their mathematics textbooks mainly for doing exercises (statement 2). Most of the students also quite often or frequently seek help from the examples presented in the textbooks in support of solving exercises (statement 4). However, only a few students frequently study the theory presented in their textbooks (statement 1). In addition, it is uncommon for the students to turn to other sources than their textbooks while working on exercises (statement 5), nor studying the theory (statement 6). It also seems that students use textbooks more at school than

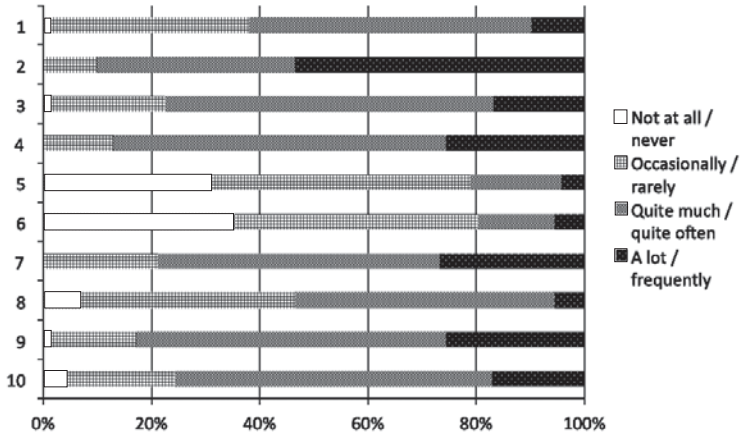

Figure 2. *Proportional response distributions for the statements in the questionnaire*

outside it (statements 7 and 8). According to the students' experiences, teachers tend to urge students to use a textbook at least occasionally (statement 9), but the students also found differences between teachers in terms of the ways in which they, too, used textbooks (statement 10).

There were no large differences in the standard deviations between the statements: all of them were between 0.61 and 0.85. Figure 2 reveals that all the response distributions were single-peaked.

Theme 1. The effect of textbooks on the course structure

Question to students:

Do the courses follow the order in which the topics appear in the textbooks?

Question to teachers:

How much do you use a textbook when designing your teaching? Do you follow the order in which the topics appear in the textbook?

According to the experiences of the students interviewed, their teachers frequently followed the sequence of topics as presented in their textbooks. Students C1 and C2 reported that, as a result of time-pressures, some topics had sometimes been skipped, and student A1 mentioned a single case where at the start of the course a teacher had diverged from the topic sequence used in the textbook. No other exceptions came to mind.

According to Mikkilä-Erdmann et al. (1999), the teaching materials have frequently played a more significant role in the teaching than the curriculum per se. The comments made by teacher C and teacher B about the planning of their teaching confirm this hypothesis.

> Yes, it (the textbook) is very much the basis for my course. We have about 17–18 45-minute lessons per course. I browse through the textbook and check to see if it will be possible to study one chapter per lesson. Then, when I am planning the lesson, I go through the chapter and select in advance which exercises will be done during the lesson and which task will be set as homework. In general, the teaching will proceed according to the style and structure of the textbook. (teacher C)

> Yes, it (the textbook) determines the order of topics completely. It would be pointless to jump back and forth rather than following the textbook. (teacher B)

Thus, the structure of the courses taught by these mathematics teachers are very strongly dependent on the textbook. In contrast, however, teacher A connects his teaching with the textbook more lightly:

> I need to check the curriculum, but mainly I try to use the textbook more or less by following its sequence of topics. It's easier for the students to follow if the topics do not jump. In some courses, however, I may consciously take another topic from later in the textbook at an earlier stage. Then I try to arrange the topics in a hierarchical order. As a consequence, I don't use the textbook in my course planning, but I do try to make sure that the course will roughly follow its sequence. (teacher A)

Teacher A assumes that it is easier for students to follow the course if the course structure at least approximately follows the order of the textbook. Teacher C argues that following the order used in the textbook helps in solving practical problems whenever students happen to miss lessons:

> I proceed according to the topic sequence (of the textbook). [...] If students miss lessons, it is quite easy if you can say that last time we studied this chapter and now you have to read and study it yourself. (teacher C)

Theme 2. Teachers guiding of students in the use of textbooks

Question to the students:
How does a teacher guide you in the use of textbook? Does s/he recommend reading theory, doing exercises, etc?

Question to the teachers:
How do you guide your students in their use of the textbook?

The students also had a variety of experiences concerned with how their teachers guided them in the use of their textbooks. According to student B1, some teachers even provided detailed instructions concerning which sections of the textbook should be read, whereas some teachers gave no instructions at all. Students B1 and C2 reported that their teachers usually provided no guidance in their use of the textbook, nor did they encourage their students to read it. According to student C1, a teacher might, for example, skip a proof presented in the textbook and instead tell them to read it for themselves at home. According to this student, teachers may also recommend students to study the examples in the textbook whenever they felt that something remained unclear to the students. Student A2 suggested that teachers tended to advise students to seek help from the textbook if they encountered problems:

> I think they all set exercises from the textbook and say 'please read this and that, look at the examples, and look for help in the textbook if you encounter problems'. (student A2)

Teachers' responses confirm differences with respect to guidance in the use of textbooks. Teachers A and C reported that they tended to present the content and structure of the textbook to the students and also at the start of the courses to provide some advice about the techniques of study related to use of the textbook. In contrast, however, teacher B leaves the responsibility for discovering how to use the textbook almost entirely in the hands of the students themselves.

> The student has the responsibility. I don't want to dictate any manual for that. (Teacher B)

Theme 3. The role of textbooks in studying theory during lessons

Question to the students:

How is theory taught in class? Does the teacher use a document camera to display the theory contained in the textbook or does s/he provide notes that s/he has devised on her own, etc?

Question to the teachers:

How do you use a textbook in relation to teaching theory in class?

According to the students interviewed, teachers' methods of teaching mathematical theory vary. However, it is common that the teacher will write on the blackboard/whiteboard and the students will take notes. For example, student C1 commented in the following way:

> I think most of the teachers write on the paper [on the document camera table] and we copy it (student C1)

Students B1 and B2 remembered that their teachers used a document camera to show the students the theory and examples in the textbook. However, student C1 did not regard this as a common method. Students A1 and A2 had noted differences between various teachers' teaching styles. They reported an example involving the teaching of formulas concerning vectors: one teacher wrote the formulas on the blackboard, whereas another teacher taught the same topic by showing it to the students from the textbook using a document camera. Student C2 expressed the criticism that theory shown from the textbook was not understandable:

> If the theory is shown directly from the book, it doesn't make sense, at least for me. (student C2)

Teacher B reported that it depended on the way the theory in the textbook was represented that decided whether he would simply use the book or present the theory in his own way:

> [...] how the theory is presented in the textbook. If it is presented well, I will show it to the students, but if I can invent something better, I will use that. (teacher B)

Teacher C refers to the relevant pages in the textbook when she teaches theory:

> I tend to start introducing the theory by typing the heading and after that the textbook pages I've told the students that if I make the notes and present the theory, they can follow from the textbook, and they don't have to take notes. (teacher C)

This comment reveals that teacher C teaches the theory in a way that is similar to its presentation in the textbook. In contrast, teacher A uses the textbook very little when teaching theory:

> In practice, when we start to deal with the theory, I use the textbook only very little. I prefer to take my own material, and I avoid examples that are presented in the textbook. My examples will generally be somewhat different. I hope that students will then study the examples in the textbook by themselves. (teacher A)

These comments confirm students' perceptions of teachers' different ways of teaching mathematical theory.

Theme 4. The role of textbooks in studying theory at home

Question for the students:

Do you study the theory in the textbook outside the classroom? Do you find the theory dealt with in the lessons useful?

When students were asked about the use of textbooks when studying mathematics theory at home, many students responded that instead of familiarizing themselves with the theory sections, they usually focus on doing the exercises. Student C2 reported that he did not study theory at home:

> I don't look at the theory at all, but I only work on the calculations. I am not interested in theories, and because of that, I am probably unable to solve the most difficult exercises because I tend to be lazy when it comes to studying the actual theories. (student C2)

When the interviewer asked about the benefits of studying theory, student C2 was critical:

> I have managed without reading any theories. However, I can see that if you want to get the highest grade in the matriculation examination, or if you want to learn how to prove the issues and learn why things are as they are, and not simply that that's how it is ... then it makes sense to also present the arguments. But to save time, I've not studied [theories]. (student C2)

On the other hand, student C2 seemed to connect studying theory with a deep understanding of mathematics, although he did not regard it as important for average students who do not aim to get the highest grades. Student B2 also reported that he only rarely used the textbook theory sections at home.

Students A1 and C1 said that they read the theory in the textbook when preparing for examinations:

> I revise the theory sections, too, when I am preparing for an exam. (student C1)

> Before an exam I always check the theory, but otherwise ... (student A1)

Student A2 also recognized that it was useful to read the textbook theory sections when studying for an examination. Student C1 analyzed the usefulness of the theory sections by mentioning that they made it easy to check formulas and that when a student was unable to attend a class, the theory sections facilitated self-study.

Student B1 was the only one to report that she used theory in order to understand the issues dealt with in the lesson. Student B1 emphasized that the textbook theory was useful only if it was properly understood.

It can be concluded that students A1, A2, and C1 emphasized the technical advantages of the theory sections. In contrast, students B1 and C2 saw the importance of the theory for gaining a proper understanding, even though student C2 questioned the usefulness of understanding itself.

Subtheme. Students' opinions concerning the presentation of theory

No distinct question about the presentation of theory in textbooks was asked in the interviews, but the discussion about the use of theory and the usefulness of theory led to this subtheme. In addition, discussions about the qualities of a good textbook have been taken into account in this analysis.

All the students interviewed criticized the way in which mathematics theory is presented in the textbooks. They considered it too difficult to understand. Students B1 and C1 complained about complicated terminology:

> Too difficult terminology ... I don't think I'm stupid, but I need to give quite a lot of thought to what is said here. (student B1)

> At the beginning it may be said that something is $f(x)$ and something else is $g(x)$ ... and some other terms, which make you totally confused about what is said in the text. You can't grasp the meaning of the symbols properly. (student C1)

Students reported that it may take a lot of time to understand the content of the theory sections. Student A2 thought that the mode of presentation was better in the formula book:

> They (the theory sections in the textbooks) are sometimes quite confusing. I think MAOL (a formula book) is a really comprehensive and compact package. It presents everything very clearly and compactly. (student A2)

Students B1 and B2 thought that it was often expected in the textbooks that the topics that had been studied earlier were in consequence well understood. They omitted any revision in the theory sections of topics discussed at earlier stages of education.

> I think it seems to be expected in the books that all of the earlier courses have been completed with the highest possible grade. (student B2)

Theme 5. The role of textbooks in studying examples

This was introduced with a discussion about whether the examples studied in class came from a textbook or not.

Question to the students:
Do the examples presented by teachers in lessons come from a textbook or have they been developed by teachers themselves?

Question to the teachers:
Do you invent examples yourself, or do you take them straight from the textbook?

According to the students' experience, classroom examples were almost always dealt with by the teacher presenting a ready-made example and the students copying it down in their notebooks. However, the sources of the actual examples may differ. Sometimes the teacher will take ready-made examples from the textbook or use exercises from the textbook, but on other occasions the teacher may also develop her/his own examples. Practices differ from one teacher to another. Student C1 reported that a teacher may present a ready-made example from the textbook by also presenting details which are not explicitly presented in the textbook. However, according to students A1 and C2, it was more common that the examples presented in the classroom were not directly from the textbook.

> They usually come from elsewhere, but sometimes there may be an exercise from the book. (student A1)

Teacher B described how he selected both examples and exercises from the textbook used in the course:

> It is pointless to strain your own head, because the textbook is made for this purpose. If there are exercises comprehensively related to the theory, why take them from elsewhere? (teacher B)

Teacher C sometimes takes word problems, in particular, from other textbooks:

> I prefer to take the word problems from a parallel textbook series. [...] So, in practice, I use examples from the textbook and also from elsewhere. (teacher C)

None of the teachers interviewed mentioned inventing their own examples.

In the student interviews discussion also focused on the use of examples. The discussion dealt with both the examples in the textbooks and also examples studied in class.

Questions to the students:
Do you use examples when you try to solve exercises, study theory etc? Have you found examples useful?

All of the students interviewed regarded the examples in the textbooks as useful. When students are working on exercises, they may try to imitate corresponding ready-made examples. This is common especially when students encounter problems while doing the exercises.

> They [examples] are useful, at least when there is a similar type of task to the one that you're attempting to solve. Then, when you see the example, you will know how you are expected to solve the task. (student C1)

According to the students interviewed, the examples and exercises are often similar. This makes learning based on the imitation possible. In addition, the examples presented in the lessons may steer students towards this kind of learning. Student C1 saw that the examples presented in the classroom might replace the examples in the textbook:

Int: Do you seek help from the examples for the homework?

Stud C1: Sometimes. But, usually, if a lot of similar tasks have been calculated during the lessons, then you will have already mastered the topic so that you don't need to look at the examples at home.

Student B2 prefers using examples based on his own notes before looking for hints in the textbook:

> If I cannot solve a task at home, I will look for help in my notes. If I don't manage to find a proper example there, then I will look at the textbook. (student B2)

Student C2 reported that when he is studying a new topic, he first tries to solve the exercises in the textbook by himself. If he still encounters problems, he will look at the ready-made examples.

> Even if you don't succeed, you will sketch where you have experienced the problem when you look at the example and find out what has gone wrong. (student C2)

Students also criticized the way in which textbook examples often skip too many steps:

> The examples are usually shortened in the textbooks so that some of the steps are not presented. The example will still work well if it is worked through together with the teacher. Otherwise you have to think for a very long time in order to see what has to be done before the answer can be found. (student B1)

It seems that student B felt it difficult to read examples, because filling in any missing steps was necessary to properly understand them.

Theme 6. The role of textbooks with respect to exercises

Question to the students:
Are the exercises (lesson exercises and homework) always taken from the textbook?

Question to the teachers:
Do classroom exercises come from the textbook? Do you set homework from the textbook?

Almost all the exercises that students had worked on during mathematics lessons or at home were from the mathematics textbooks. According to the students interviewed, such things as additional exercise-handouts were not used in the mathematics teaching. The teachers' responses also confirmed this observation.

> I don't give homework based on my own material. It takes a lot of time if you want to use your own material. Why would we even need to have textbooks in that case? (teacher A)

> I take them [exercises] from the textbook to avoid the trouble of writing my own. I don't take them from the Internet, though. (teacher B)

The only exception may be the revision course, where tasks from the previous matriculation examinations are sometimes used.

Nowadays mathematics textbooks in Finland include a lot of different exercises, which means that students usually cannot do them all, so there is a need for proper selection. According to the students, there were differences between the ways in which different teachers selected exercises. Quite often the teacher will select the exercises that are to be done, but some teachers may leave the responsibility for the selection to the students themselves.

Question to the students:
Do you have any opportunity to choose exercises yourself, or are they decided by the teacher?

> At least Mr H. works on the principle that the exercises are given and they are expected to be done. However, in the last period, there were always a given number of exercises and you had to do, for example, seven of them. (student A1)

It is even possible that exercises are not selected at all, but this practice was criticized by the students:

Stud B1: Nowadays, Mr. J is in the habit of giving us a whole page and asking us to do them one by one. I think it's quite stupid.

Stud B2: Other teachers select the exercises so that ...

Stud B1: ... so that they take one example of each exercise-type. They select them so that we will be exposed to a wide range of exercises.

Int: Do you feel that if a whole page of tasks is set, the students won't be able to select the most essential ones from them?

Stud B2: Yes.

Stud B1: Mr J. expects all the tasks to be done. I think this is stupid.

A belief that selecting exercises is difficult for students emerges from this extract. Student B1 believes that the exercises selected by the teacher will form a multifaceted set. It seems that the interviewer initially misunderstands that Mr J. expects the students to select exercises, but according to student B1 this is not the case: the teacher expects them to do all of the exercises. An obvious criticism of this practice is that it will entail the completion of an excessive number of exercises.
Teacher C uses a large range of exercises in her use of differentiating instruction. In her teaching, students who learn and proceed faster may solve more challenging exercises than those tackled by the others.

It also emerged that there were noticeable differences between teachers in terms of the amount of homework given to students:

> Mrs. K always gives most; she sets a huge number of tasks. But Mr P. sets only a few and Mr R. is similar. (student B1)

Students also use textbooks when they are preparing for examinations. The textbooks often contain a series of revision exercises designed for this purpose. The repetition series may also be used in class.

Summary of results

On the basis of the students' and teachers' interviews the following conclusions can be drawn:

– Mathematics textbooks seemed to play a strong role in teachers' planning of their teaching with respect to the contents and the sequencing of topics. In practice, the textbook often replaces the curriculum, or at least teachers trust that textbooks follow the curriculum.

- Mathematics theory is usually presented in a teacher-led style, but the extent to which the presentation is similar to the textbook varies somewhat from one teacher to another.
- Some mathematics teachers give instructions about how the textbook should be used, while others may leave this responsibility for the students to take on.
- Students' use of the theory sections in the textbooks is minor and often restricted to their preparation for examinations. Students reported problems in understanding the presentation format of the theory in the textbooks.
- Lesson examples are studied in a teacher-led style, and teachers often take their examples either from the textbook in use or from another, similar textbook.
- Students mainly use classroom-devised examples and examples presented in the textbook when they are trying to solve exercises. They frequently try to find examples that are similar to the exercises in order to imitate them.
- Mathematics textbooks are very important sources of such exercises. Almost all of the exercises studied during in class or at home come from the textbook. However, there are differences in practice with respect to the selection of exercises.

Discussion

The results of this study indicate that textbooks work quite broadly as mediators of the curriculum in upper secondary mathematics. The three teachers involved in this study tended to use the textbook as the main source in planning their teaching. It seems that teachers were strongly confident about the pedagogical solutions found in the textbooks, especially in terms of their selection of content, the order of topics, and exercises. For the students involved in this study the textbook was mainly a source of exercises.

This study was a synthesis of two inter-related sub-studies (Partanen, 2013; Piiroinen, 2013) in which teachers' and students' experiences were combined. No explicit discrepancies were found between the perceptions of the teachers and their students, and the results of these two studies seemed to be coherent.

According to Haggarty and Pepin (2002), it is frequently the case that each country will have its own cultural traditions concerning the way in

which textbooks are used in teaching. Hence, it is interesting to notice that the results of this study are in many senses similar to those published in Sweden by Johansson (2006). Both of these studies were limited to a couple of classes, and hence it is not possible to draw any generalized conclusions about the situations in Finland or Sweden. However, it is possible that mathematics teachers in Finland and Sweden use textbooks in rather similar ways, which could be explained by reference to their common cultural traditions related to practices in schools. Nevertheless, there may be rather large differences between individual teachers within each country.

In this study it was found that there was a tendency for students' to avoid reading mathematical theory and to concentrate almost solely on solving exercises. This indicates that in the teaching that they receive, the learning of problem-solving methods receives a greater emphasis than the acquisition of a thorough understanding of mathematics. This is the case especially if theoretical reflections are not required in the exercises. In addition, some students' resorting to searching for examples that are similar to the exercises and then imitating them strongly resembles *imitative reasoning*, which Lithner (2008) regards as the opposite of *creative reasoning*. It is also analogous to *scheme-oriented learning*, where mathematics is seen as a collection of calculation rules and methods, and the achievement of a proficient use of them is seen as an essential learning goal (Viholainen, Asikainen & Hirvonen, 2014). The teacher-led teaching style in which theory and examples are given and presented by the teacher may also underpin this kind of scheme-oriented learning. In contrast, *process-oriented learning* means that mathematics is seen as an active construction process, where the acquisition of reasoning skills and the construction of new things are seen as crucial learning goals (Viholainen et al., 2014). No clear references to this kind of learning view emerged in this study. In fact, it seems that teachers' practices do not support process-oriented learning.

The students found that the presentation of theory is often difficult to understand. They experienced problems with the language and notation. This may be a significant reason for their avoidance of reading theory. Österholm (2006) also found that the use of mathematical symbols may affect students' reading comprehension of mathematical texts. In Finnish mathematics textbooks the theory was often based on the earlier topics, which students had possibly forgotten. Students also questioned the importance of the theory. Hence it is probable that the structural features of mathematics are not much learnt at upper secondary level, and students' structural overview of mathematics remains weak. This may cause problems for students especially when they move to the tertiary level and start their mathematics studies there (Moore, 1994; Gueudet, 2008).

The development of electronic learning materials will pose teachers with a new kind of challenge. They will be forced to change a number of outdated practices in their teaching and think about a new kind of pedagogical questions concerning the effective use of learning materials that may include, for example, dynamic and interactive elements. In a broader sense, developments in the nature of learning materials may provide an opportunity for teachers to reflect critically on the prevailing culture of textbooks and their use.

Awareness of current practices and views related to the use of textbooks is important both for teacher educators and also for designers of learning materials. On the basis of the results of this study it may be concluded that the designers of learning materials should think especially about the ways in which they could present the theory so that it would become more comprehensible and accessible for students. In addition, students may need more guidance from their teachers in the ways in which they might use their learning materials more effectively. Further studies of the impacts of different learning materials on both learning outcomes and also the general view of mathematics would undoubtedly be needed for this multifaceted phenomenon to be properly understood. In particular, it is important to study and discuss the ways in which deep understanding and creative reasoning could be better emphasized, rather than imitative reasoning and the inculcation of routines.

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