Using textbooks in the mathematics classroom – the teachers' view

MADIS LEPIK, BARBRO GREVHOLM AND ANTTI VIHOLAINEN

Mathematics teachers' self-reported practices of textbook use were investigated by a survey of more than 400 teachers in Estonia, Finland and Norway. Do they have different approaches in their use of textbooks and to what extent do they rely on textbooks in planning and preparing their lessons? What kinds of patterns characterize teachers' practice when using textbooks in mathematics lessons? The answers to these questions indicate that in Estonia and Finland teachers have similar attitudes towards textbooks. They are responsible for the choice of book and the textbook has a strong effect on their didactical choices. In Norway the teachers are less dependent on the textbook. In Finland the textbook is the crucial resource for exercises while in Estonia and Norway teachers use other resources more often. The study reveals the limited use of the full potential of the textbook. Almost 45 % of the teachers use the textbook simply as an exercise book. As a result the pupils do not get the opportunity to fully exploit the book as a multifaceted learning resource.

Textbooks are considered to be among the most powerful influences on school mathematics (Mullis, Martin & Foy, 2008; Valverde, Bianchi, Wolfe, Schmidt & Houang, 2002). Textbooks are equally important resources for both groups – for pupils to learn mathematics and for teachers to plan and teach their mathematics lessons. Mathematics classroom instruction is, in many cases, generally organized around and delivered through the mathematical tasks and activities found in textbooks. Therefore, textbooks are probably among the most immediate determinants of educational practice (Amit & Fried, 2002; Chval, Heck, Weiss & Ziebarth, 2012; Li, Zhang & Ma, 2009; Silver, 2009; Törnroos, 2005).

Madis Lepik, *Tallinn University*
Barbro Grevholm, *University of Agder and Kristianstad University*
Antti Viholainen, *University of Eastern Finland*

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Much has been written in the literature about the work of teaching, but surprisingly little effort has been devoted to examining and conceptualizing teachers' approaches to textbook use. Also, there are relatively few studies dealing with teachers' implementation of textbooks in mathematics lessons, at least in the Nordic and Baltic countries (see Fan, Zhu & Miao, 2013). At the same time however, it is argued that a marked dependence on textbooks is "perhaps more characteristic of the teaching of mathematics than of any other subject" (Robitaille & Travers, 1992, p. 706).

Learning mathematics with a textbook comprises activities such as reading explanatory texts and acquiring new content, looking through worked examples, solving tasks, etc. It is the teacher who orchestrates the students' use of textbook materials during the lesson. So, the same textbook as an instructional tool could be used differently in different mathematics classrooms. Teachers may or may not use the textbook in the lessons; they may simply use it as a source of exercises or they may utilize the full potential of the materials presented in the textbook.

In this paper the authors analyse teachers' approaches to the use of textbooks in three countries: Estonia, Finland and Norway. Textbook use in mathematics classrooms in the three countries has not been studied systematically, and it appears that the different ways in which teachers in these countries employ textbooks in their lessons are relatively unknown.

Typically, the use of textbooks by teachers is studied by using teacher interviews in combination with classroom observations (Fan et al., 2013; Haggarty & Pepin, 2002). Both methods provide a deep understanding of concrete cases of classroom practices. Such results are rich and illuminating but mostly do not allow generalisations because of the limited number of teachers and classes involved. The purpose of this study is to provide a broader and more general picture of teachers' approaches in textbook use in their classrooms. The authors' investigation is based on teachers' reflections on their textbook-related practices. Teachers' reflections provide a useful perspective on teachers' use of textbooks in their instruction (Nie et al., 2013). The perspective in this study is that the teacher is seen as the mediator between students and textbook; therefore the teachers' view is considered very important and valuable (Love & Pimm, 1996; Pepin & Haggarty, 2001; Rezat, 2009; Sosniak & Perlman, 1990).

The study is based on a survey of approximately four hundred teachers from three countries, which overcomes the above-mentioned limitations. At the same time however, surveys always allow responses from a larger group of interviewees, and may therefore lead to only statistical and numerical results. Such results demand appropriate discussion and constructive argument in order for the interpretations to be convincing in

answering the research questions. The authors offer such arguments and interpretations below. Firstly, the study describes and compares mathematics teachers' approaches to textbook use in the three countries. The aim is to understand the general trends and the range of approaches in which textbooks are used in different mathematics classrooms. Secondly, the research explores the different strategies of textbook use more closely – the general patterns of textbook use, common to teachers from all three countries, are described and discussed.

The research questions to be discussed in this paper are as follows:

1. What are the teachers' self-reported practices of textbook use in the three countries?

    1.1. To what extent do teachers rely on textbooks while planning and preparing their lessons?

    1.2. What approaches do teachers take when using textbooks in the lessons?

2. What kind of general patterns characterize teachers' practices when using textbooks in mathematics lessons?

Earlier studies on teachers' use of textbooks

Textbooks are an important resource for teaching mathematics. Very often the only resource that all pupils will have access to during the lesson is the textbook.

> The textbook should arouse students' interest in learning mathematics, help students to study mathematics actively, develop students' potential in creativity through the process of learning basic knowledge, improve students' mathematical thinking when trying to understand the essence of mathematics knowledge, and raise students awareness to apply mathematics knowledge in everyday lives .
>
> (Li et al., 2009, p. 743)

This very important role of textbooks is also reflected in the remarkable body of literature devoted to the analysis and understanding of the potential effect of the different features of the textbook on mathematical learning (Fan et al., 2013; Mikk, 2000; Pepin & Haggarty, 2001; Remillard, 2005; Valverde et al., 2002). These analyses reveal nuanced insights into variations in what is made available to students in textbooks and how it is made available. Such analyses certainly serve as an effective basis for the on-going development of new and better textbook versions.

Another important direction in textbook research draws attention to the role of the textbook in the teachers' planning and preparing for lessons. Textbooks are also an integral part of a teachers' daily work and are intimately connected to the enactment of instruction. The framework for the Third International Mathematics and Science Study (TIMSS) differentiates between the *intended* curriculum, the *implemented* curriculum and the *attained* curriculum. Since textbooks usually reflect national goals set down in the (intended) curriculum and at the same time they strongly shape the instruction given in classrooms (implemented curriculum), evidence suggests that it would make sense to expand the three-level curriculum model by adding a fourth level, namely the *potentially implemented* curriculum. This level encompasses textbooks and other curricular materials used in classrooms (Valverde et al., 2002).

Textbooks are often perceived to reflect the officially intended curriculum by the teachers. Research has documented a strong influence of textbooks on mathematics content that is taught and learned. Thus, textbooks control material selection and sequencing (for overview see Pepin & Haggarty, 2001). The topics presented in the textbook are also very likely to be introduced in the classroom; on the other hand however, topics not included in the textbook are most likely to not be presented by the teacher (Johansson, 2006).

Textbooks, in many cases, are also a primary information source for teachers in deciding how to present the content. There is a large body of literature that points to textbooks as driving teachers' pedagogical behaviour. In many mathematics classes, student assignments, the questions the teacher asks, the ways in which students are grouped, the forms of assessment, and much more, originate in curriculum materials. Thus pedagogical approaches reflected in the textbook in all probability will be translated into practice in the classroom, and textbooks in many ways serve as the models of instruction (Bush, 1986; Johansson, 2006; Lloyd, 2002; Tyson-Bernstein & Woodward, 1991). For example, Haggarty and Pepin (2002) showed that half of the teachers interviewed in England, France and Germany didn't use any material other than the textbook for their lesson preparation.

Research literature is also clear about the fact that textbooks are used extensively in mathematics classrooms (see Fan et al., 2013; Pepin, Gueudet & Trouche, 2013; Reys, Reys, Lapan, Holliday & Wasman, 2003). However, the question of *how* textbooks are employed remains relatively unanswered. There is insufficient information available regarding the nature and quality of textbook implementation – the way in which textbooks are actually used in mathematics classrooms.

Two classes making use of the same textbook may differ markedly in how they use the textbook. One teacher, for example, may build up

the whole learning process based on students' individual work with the textbook while another teacher may tend to use the textbook simply as a source of exercises and homework tasks. For example, in their study based on teachers' logs, Freeman and Porter (1989) analysed the way in which four fourth grade mathematics teachers in the US were using the same textbook. They detected striking differences in the practices of these teachers in the use of textbooks. Thompson and Senk (2014) studied how 12 teachers from nine different states in the US applied the same mathematics textbook in their teaching of congruence. They found differences between the teachers both in the selection of topics and in instructional strategies. Teachers made the selection of topics on the basis of their own experience and teaching philosophy, the students' abilities and state-specific curricula. Teachers' instructional strategies also varied as well – for example, with respect to homework and whole-class instruction. Therefore, the use of similar textbooks does not guarantee equal learning opportunities for all students, because the teacher has a key role in affecting how the resources provided in the textbook are utilized in practice. Remillard (1999) also describes the same phenomenon. However, one may also find contradicting results from the literature. For example, Fan et al. (2004) did not find significant differences in the teachers' use of textbooks in their study of 36 secondary mathematics teachers in China.

Haggarty and Pepin (Haggarty & Pepin, 2002; Pepin & Haggarty, 2001) describe different traditions of textbook use in lower-secondary mathematics lessons in England, France and Germany. Semi-structured interviews with 10 teachers from each country were used to explore teachers' views that underpinned their use of textbooks in classroom. Teachers in all the countries analysed emphasised the use of textbooks for exercises. At the same time however, French teachers used the books for explanations, but insisted on providing the rules and the essence of the lesson without the textbook and in a different way to it. German teachers used worked examples that were different to those provided in the textbooks, in order to initiate class discussion about the problems that might be encountered. English teachers mostly introduced and explained a concept or skill to their students and then gave examples on the board and expected pupils to practice on their own. The research indicates that in France and Germany the textbook is regarded as the key element of teaching and learning, whereas in England textbooks are viewed as one of many resources that teachers use in their classrooms.

Johansson (2006) provides another study that gives an insight into how the textbook is used in the classroom. Three lower-secondary teachers in Sweden were observed and interviewed. In all three classrooms textbooks were in direct use about 60 % of the time and the dominant activity was pupils' individual work with tasks in the textbook. Also

teachers' introduction of the new content proved to be directly or indirectly based on the textbook.

Existing research reveals the critical role that teachers play in the use of textbooks in classrooms. Teachers act as mediators of the content of the textbook; they decide which sections of the textbook to use and the way in which to use it (Pepin & Haggarty, 2001). At the same time, the number of studies investigating how textbooks are used in classrooms is limited. We agree with Pepin and Haggarty (2001) in that "the ways in which the teacher mediates the textbook are largely unknown" (p. 166). It is also clear that existing studies have been carried out on a small scale and do not allow generalisations. Thus, very little is known about the role that the textbook plays in teaching and learning of mathematics and teachers' styles of textbook implementation in particular. The work reported in this paper aims to partly fill this gap and attempts to shed light on the extent to which textbooks are used in mathematics classrooms in Estonia, Finland and Norway. It also searches for general patterns in the teachers' approaches to textbook use in these countries.

The use of textbooks in Estonia, Finland and Norway

To set this study in context, the following discussion presents an overview of the main or dominant practices. Reference is made mainly to the situation in lower secondary schools. The background information regarding textbook use is presented according to a structure inspired by the work by Pepin and Haggarty (2001), and also reflects the authors' own experiences and focus.

As mentioned above, in all three countries *textbooks are used for mathematics teaching and learning*. The TIMSS textbook study even highlights the fact that in the Nordic and Baltic countries teachers still tend to use textbooks more than in other parts of the world (Valverde et al., 2002). In all three countries analysed here, pupils are expected to follow the common curriculum in grades 1–9 and for mathematics, there are several alternative textbook series on the market. In Norway, approximately seven different series can be found; there are three different series in Estonia and seven in Finland.

What is the relationship between textbooks and the curriculum?

Until 1992 in Finland, until 2000 in Norway and until 2003 in Estonia, all textbooks were pre-inspected by the National Boards of Education, which ensured that textbooks satisfied the curriculum criteria. For example, in Finland learning goals, content, teaching arrangements, etc. were all

checked, and in addition, textbooks were supposed to be understandable and objective (Partanen, 2013). This may have given teachers a sense of security when using the textbook. However, later it became apparent that, due to the development of technology and media for example, the concept of *learning material* could not just be limited to textbooks. This was one important reason why the decision was made to terminate the pre-inspection of teaching materials (Laaksola, 2007). Nowadays therefore, it is the responsibility of the publishing house to ensure that there is concordance between the textbook and the curriculum and teachers still expect that the contents of the textbook follow the official curriculum (a fact that may not always be the case). It is claimed that both students and teachers in Scandinavia see the textbook as the mathematics to be taught in schools (Skolverket, 2003). However, Finnish mathematics teachers see the textbook as a concretized curriculum that is not only the source of information, but also prescribes the content, as well as the goals and didactical solutions for teaching (Mikkilä-Erdman, Olkinuora & Mattila, 2009). Also, according to the study by Perkkilä (2002), Finnish primary school teachers believe that widespread use of textbooks in teaching ensures that they will be following the national curriculum. Love & Pimm claim: "There is a good deal of evidence that many teachers like the security and freedom of responsibility that a text series provides. In theory when using a text series, teachers need not involve themselves in ordering the topics, in ensuring that notation is consistent nor in concerning themselves whether a student will have met the necessary pre-requisites for a new topic" (Love & Pimm, 1996, p. 384).

Who are the authors and producers of textbooks?

In all three countries, the groups of authors include both teachers and teacher educators (see e.g. Kongelf, 2011). Commercial publishers produce the textbooks series that are used in schools and in principle the publishers decide what the authors can write.

Who decides which textbook to use?

Textbooks are free for pupils studying at compulsory school levels in Estonia, Finland and Norway and schools supply all pupils with the necessary textbooks. Normally, teachers make a joint decision at the school level regarding which textbook series will be used. It is common practice in all three countries that schools prescribe that textbooks are to be circulated, i.e. they are used for two or three consecutive years. This means that at the end of the school year pupils are expected to return

their textbooks. It also means that a quick change of a textbook to be used is not possible. One can also ask on what basis teachers choose a textbook.

There are no clear criteria for making this decision and in many cases it is not possible to discover the features of a textbook just by browsing through it. It needs a deeper and more holistic analysis to explore a book (Grevholm, 2014).

What is the structure of the textbooks?

Typically the materials presented in Estonian textbooks are structured according to the topics, which could include material for several sequential lessons. Every topic starts with the presentation of the new content and often there are worked examples followed by a set of tasks and activities. In order to meet the different needs of pupils, the tasks in the textbooks are organised in modules based on their complexity. Thus, in addition there are always more challenging tasks available for more able pupils. A similar structure is used in Norway (Kongelf, 2011). Textbooks in Finland are laid out in such a way that each page-spread in the textbook is designed to be dealt with during one lesson. Ready-made exams are also published. Therefore, the teacher becomes an implementer of a learning process that was designed and regulated by textbook authors (Mikkilä-Erdman, Olkinuora & Mattila, 2009).

How are textbooks used, and who makes the decisions?

Textbook use in the three countries has not been studied systematically. In all three countries it is a prerequisite for pupils to have the textbook for every lesson. So, textbooks serve as common instructional tools but there is not much concrete information on the way in which they are employed during the lesson. In Norway the teacher often devotes the first half of the lesson or about half of the time during the lesson to the introduction of new material using whole class discussion and then pupils are asked to work with the tasks and activities from the textbook (Kleve, 2007; Hundeland, 2011). Perkkilä (2002) studied Finnish primary school teachers' mathematics-related beliefs and the role of textbooks in the teaching of mathematics. Perkkilä found that teachers' attempts to cover the whole textbook in a limited time frame caused a feeling of urgency and pressure during lessons. Similar results are shown in Norway (Hundeland, 2011).

Very often the textbook is more important than the curriculum for teachers when they plan their teaching. In Finland especially, the textbook has a very strong effect on the content and order of topics in teaching. Almost all the examples presented by teachers and the exercises that

the students work with, either during lessons or at home, come from textbooks as well (Viholainen et al., 2015). Extra resources are rarely used except for the calculator, computer or maybe the interactive board. When discussing the implications from the TIMSS-study, researchers claim that Norwegian teachers talk too little about mathematics with their pupils. There is too much time spent in the classroom performing quiet, individual calculations (Grønmo, 2013) and she claims that this leads to an overemphasis on just one method of working.

Methods and methodology

The investigation reported here is part of a larger study (the NorBa study) incorporating a survey of mathematics teachers in the Nordic and Baltic countries. The objective of this study was to explore mathematics teachers' belief structures in Baltic and Nordic countries and compare possible cross-cultural differences (Hannula, Pipere, Lepik & Kislenko, 2013; Lepik & Pipere, 2011; Lepik, Pipere & Hannula, 2012). The data was collected from mathematics teachers of grades 7 to 9 in Estonia in the autumn of 2011, in Finland in the spring of 2012 and in Norway in the autumn of 2012. The overall sample size was 402 teachers; the country sub-samples are presented in table 1. Data was collected differently in each country because of the need to choose the best way to increase the response rate.

In Estonia, the sample consists of 241 lower-secondary mathematics teachers from 144 schools in 15 different administrative regions of the country. Informative e-mails were sent to all lower-secondary schools in the selected regions inviting teachers to participate in the survey. Head-teachers of the schools who accepted the invitation received the paper-based surveys with the necessary instructions and distributed them among the teachers. Teachers completed the surveys and sent them back. The respondents' identity and records were kept confidential. The response rate was about 85 %.

In Finland, the teachers were approached through the principals of a representative sample of Finnish speaking schools ($n = 114$). A list of all schools in Finland that teach the lower-secondary level was used for sampling purposes. The number of students attending each school was available and this information was used to select schools randomly and weighted by their size. In this way oversampling of small schools was avoided. Finally, a manual check was carried out and some selected schools were changed in order to ensure that the sample of schools was geographically representative. Surveys to be completed were sent to the schools together with a response envelope. In this way data was collected from 94 teachers throughout the country. The estimated response rate was approximately 30 %.

In Norway, a statistically representative, random selection of 136 schools was invited to respond (selection used by the TIMSS-researchers). Replies were received from 67 teachers in 38 of these schools spread geographically throughout Norway. The teachers, who accepted the invitation, received the surveys with the necessary instructions, completed them anonymously and returned them to the designated e-mail address.

Table 1. *Sample of the survey*

Country	Number of teachers
Estonia	241
Finland	94
Norway	67
Total	402

It should be noted that countries' subsamples are somewhat unbalanced. In order to minimise the influence of this phenomenon on the results, only relative frequency tables were used while comparing the countries during the analyses.

Fan, Zhu and Miao (2013) concluded in the summary of their review of research on textbook use in teaching and learning that "researchers have paid increasing attention to the use of mathematics textbooks. On the other hand, it is clear that most of the studies were carried out on a small scale. [...] Further research on a large scale [...] is much needed" (p. 642).

Proceeding from this advice, this study analyses over four hundred teachers' responses to the questionnaire module concerning their use of textbooks while planning and teaching a lesson. Based on these responses, a broader picture of teachers' approaches was developed and the general patterns of textbook implementation in mathematics classrooms were examined. There is reason to believe that the self-reported practices of teachers in most cases truthfully reflect the reality of their classrooms. Responses were given anonymously and other survey studies (e.g. TIMSS) indicate that teachers most often do not try to present themselves in a different way from how they act in the classroom. The authors acknowledge that in spite of all efforts made, the representativity of the Norwegian and Finnish subsamples may cause some problems. We note that in Norway and Finland it is quite hard to convince teachers to respond to questionnaires without special arrangements. Thus, the generalizations made should be treated with a degree of caution. A repeat of the study with larger samples would be recommended.

Another limitation could be the questions posed. Because module F of the NorBa study was only one of many parts of the study, the number of questions needed to be limited. The authors chose to formulate what they had found to be the most urgent questions based on their background information and insights developed in earlier research. Teachers reacted positively to the questions and were able to provide answers.

Questionnaire module (Module F) consists of two parts. In the first part teachers were presented the following 5 items.

F1 I have chosen the textbooks I use for teaching.

F2 The textbook is the primary tool to plan and prepare my lessons.

F3 The pedagogical strategies I use are often influenced by the instructional approach of the textbook.

F4 The tasks in the textbook are well adapted to fit both weak and strong students.

F5 Overall, I am very satisfied with the textbooks I use.

Teachers were required to respond using a 5-point scale:

1 (Fully disagree) ... 2 ... 3 ... 4 ... 5 (Fully agree)

The second part of the module concerned the usage of textbooks in the lessons and consisted of the following 5 items.

How often do students in your class use textbooks for the following activities?

F6 To study new concepts individually.

F7 As the only source for exercises.

F8 As a source for group work tasks.

F9 To find additional material related to the content covered by the teacher during the lesson.

F10 As the only source for homework tasks.

Teachers were requested to respond using a 4-point scale:

(1) Never (2) Some lessons (3) About half the lessons (4) (Almost) every lesson

The results are presented by summarising the teachers' replies in the form of frequency tables for each country and item-by-item. To find out if differences between countries are significant, we used the *F-test*. The

ANOVA *F*-test is used to determine whether there are any significant differences between the means of three (or more) independent groups. The ANOVA cannot tell you which specific groups were significantly different from each other, only that at least two groups were. To determine which specific pairs of countries had significant differences, we used a *post hoc* test (Field, 2005, p. 311, 325, 339).

To discover the most common patterns of employing textbooks in mathematics classrooms, the *cluster analyses* were carried out based on teachers' responses to items about their textbook usage (items F6... F10). *Clustering* is the task of grouping a set of objects in such a way that objects in the same group (called a *cluster*) are more similar (in some sense or another) to each other than to those in other groups (clusters) (Aldenderfer & Blashfield, 1984). In this study the clusters grouped teachers together who used textbooks in a similar way; this makes it possible to interpret developed clusters as reflections of common patterns of employing textbooks in mathematics lessons.

Analysis and results

Teachers' approaches to textbook use in the three countries

This section is based on teachers' responses and provides an analysis and comparison of teachers' practices of using textbooks in Estonia, Finland and Norway. The purpose of this analysis is to shed light on the extent to which teachers rely on textbooks in preparing and planning their lessons and the ways in which they use the textbooks in their classrooms. It is also interesting to find out if there are differences between the countries in the strategies used by teachers in utilizing textbooks.

Distributions of the teachers' responses to statements F1 to F5 are presented in table 2. Differences between the countries were analysed using the ANOVA *F*-test. *P*-values calculated as the result of comparisons of the teachers' responses in the three countries are presented in the last column of table 2 and those representing statistically significant differences ($p < 0.05$) are shown in **bold** type. The pairs of countries which teachers' responses proved to be significantly different ($p < 0.05$) are also listed in the last column of table 2.

Choice of textbook and level of satisfaction

The use of the textbook begins with a choice between the alternatives available in the textbook series. If the textbook is considered as the major curriculum resource then it would be recommendable for the teacher to have the freedom to make his/her own choice between the available

Table 2. *Distribution of teachers' responses about textbook use*

		Estonia (%)	Finland (%)	Norway (%)	*F*-test *p*
F1 I have chosen the textbook I use for teaching	1- fully disagree	14	34	43	**0.000** E-F E-N F-N
	2	9	13	15	
	3	12	3	13	
	4	26	21	12	
	5- fully agree	39	29	16	
F2 The textbook is the primary tool to plan and prepare my lessons	1- fully disagree	3	1	6	0.083
	2	6	16	10	
	3	29	19	34	
	4	46	50	39	
	5- fully agree	16	14	10	
F3 The pedagogical strategies I use are often influenced by the instructional approach of the textbook	1- fully disagree	3	5	3	0.364
	2	16	16	16	
	3	32	26	43	
	4	38	49	33	
	5- fully agree	11	4	5	
F4 The tasks in the textbook are well adapted to fit both weak and strong students	1- fully disagree	3	7	2	**0.028** F-N
	2	16	22	15	
	3	31	25	19	
	4	39	35	48	
	5- fully agree	11	11	16	
F5 Overall, I am satisfied with the textbooks I use	1- fully disagree	1	3	6	0.629
	2	14	19	13	
	3	33	22	28	
	4	42	46	43	
	5- fully agree	11	10	9	

alternatives and to use the version which best suits the teacher's preferred instructional style. Only then will the teacher make the most effective use of the different materials and activities presented in the textbook. Responses of teachers to the statement F1 – *I have chosen the textbook I use for teaching* – proved to be significantly different between all three countries. A total of 65 % of Estonian teachers, 50 % of Finnish teachers and only 28 % of Norwegian teachers agree with this statement. It seems as if the policies for choosing the textbook are different in different

countries. Estonian teachers tend to be freer to make their choice. In Norway 58 % of teachers disagreed with this statement, so they were not the individuals making the decision about which textbook to use. Since the common practice in all three countries is that schools are required to circulate textbooks, then quick changes of the textbook to be used are not possible and teachers are then forced to adapt to textbooks that may not represent their first choices.

Such situations could be one explanation for the fact that only half of the teachers in all three countries are satisfied with the textbook they use. As can be seen from the responses to the statement F5 – *overall, I am satisfied with the textbooks I use* – between 52 and 56 % of all teachers agree with this statement. At the same time teachers are not very critical towards the textbook they use – only about 15 to 21 % of the teachers disagree with the statement. The widest spread in this respect proved to be in the Finnish teachers' responses.

Several studies have shown that in the classroom, teachers rely on textbooks mostly for the exercises (i.e. Pepin et al., 2013). Thus, the composition of tasks presented in the textbook may be one important feature of the textbook for teachers. It is very important to provide both able and less able pupils with suitable tasks. In this respect it is important that textbooks include a large variety of tasks with different levels of difficulty, which enables teachers to easily differentiate the learning process. Based on teachers' responses to item F4 – *the tasks in the textbook are well adapted to fit both weak and strong students* – it seems that textbooks in the three countries are quite different in this respect. A total of 64 % of Norwegian, 50% of Estonian and 46 % of Finnish teachers agreed with this statement. Thus, Norwegian teachers seem to be the most satisfied with the composition of tasks in their textbooks and Finnish teachers tend to be the most critical; about one third of them disagreed with this statement.

Using the textbook as a tool to plan and prepare lessons

Several empirical studies have shown that "textbooks appear to play a role in teachers' pedagogy by conveying pedagogical messages and providing an encouraging or discouraging curricular environment for them to employ different teaching strategies" (Fan, Zh. & Miao, 2013, p. 636). From previous research we find that the textbook influences the teacher's decisions regarding the content to be covered as well as the instructional approaches used in the classroom (Johansson, 2006).

Based on teachers' responses to the statement F2 – *the textbook is the primary tool to plan and prepare my lessons* – it appears that in Estonia and in Finland teachers rely heavily on the textbook while planning and preparing their lessons. A total of 62 % and 64 % respectively of teachers

agreed with this statement. In Norway the situation tends to be different where less than half of the teachers (49%) claimed that the textbook is the primary tool to plan their lessons with one third of the teachers remaining neutral. The percentage of teachers who disagreed with this statement in Estonia was half of the figure in Finland and Norway. It appears therefore, that in Estonia the majority of teachers "teach by the book". One explanation for that may be the fact that in Estonia there are no teachers' guides in use as is the case in Finland and Norway. It should also be remembered that in the survey sample, Estonian teachers claimed to have the most freedom in choosing which the textbook to use. The results of the survey showing Norwegian teachers' having a relatively greater independence from the textbook is in accordance with results from some earlier investigations. Pepin et al. (2013) described Norwegian lower-secondary mathematics teachers' practices in lesson planning as follows: "For the lesson preparation, teachers of the same grade worked together [...] Activities were chosen from a range of textbooks, and the web" (p. 693). At the same time, however, the progression of topics still tends to reflect those in the textbook.

With respect to the statement F3 – *the pedagogical strategies I use are often influenced by the instructional approach of the textbook* – the teachers from these countries responded quite similarly. Only about 20 % disagree, others agree (49 % in Estonia, 53 % in Finland, 37 % in Norway) or remain neutral. Therefore, in all three countries, only one teacher out of five opposed the claim. Again, the level of agreement is lowest among Norwegian teachers – a greater part of them (43 %) preferred to stay neutral. If the teachers' responses to both this and the previous statement are combined, it becomes obvious that a majority of the teachers clearly rely on the textbook while planning the content and pedagogy of their lessons. The number of teachers relatively independent of textbook influence (disagreed with both statements F2 and F3) was only 5 % in Estonia, 10 % in Finland and 8 % in Norway.

It is therefore obvious that a majority of the teachers in the survey build on the textbook they use in terms of teaching content and also instructional strategies. Thus, the results of this study support the understanding of the textbook as being *potentially implemented curriculum* (Valverde et al., 2002), which in turn strongly shapes the instruction given in mathematics classrooms.

The second part of the questionnaire module concerned textbook usage in the lessons. Teachers were asked (to describe) how often students in their classrooms have been involved with different textbook-based activities. The distribution of teachers' responses is presented in table 3. Differences between countries were analysed using the *F*-test. The

p-values calculated as the result of comparison of the teachers' responses in the three countries are presented in the last column and those representing statistically significant differences ($p < 0.05$) are shown in **bold** type. The pairs of countries which teachers' responses proved to be significantly different ($p < 0.05$) are also listed in the last column of table 3.

Table 3. *Distribution of teachers' responses concerning usage of textbooks in the lessons*

		Estonia (%)	Finland (%)	Norway (%)	*F*-test p
F6 To study new concepts individually	never	6	30	24	**0.000** E-F F-N
	some lessons	62	62	43	
	about half the lessons	25	6	20	
	(almost) every lesson	7	2	13	
F7 As the only source for exercises	never	3	1	15	**0.000** E-F E-N F-N
	some lessons	5	2	16	
	about half the lessons	30	7	51	
	(almost) every lesson	62	90	18	
F8 As a source for group work tasks	never	25	28	9	**0.030** F-N
	some lessons	54	63	64	
	about half the lessons	13	5	27	
	(almost) every lesson	7	4	0	
F9 To find additional material about the content covered by teacher during the lesson	never	12	11	13	0.440
	some lessons	40	45	27	
	about half the lessons	26	32	45	
	(almost) every lesson	23	13	15	
F10 As the only source for homework tasks	never	1	2	2	**0.017** E-N F-N
	some lessons	7	10	10	
	about half the lessons	27	12	43	
	(almost) every lesson	65	76	45	

Use of textbooks as the main source for tasks

Analysis reveals a strong reliance on tasks from textbook by teachers in the survey. In most lessons *the textbook serves as the only source for exercises* (item F7) for almost all teachers in Estonia and Finland. A total of 90 % of Finnish teachers claim to employ such practice in almost every lesson. Thus, it appears that this is one of the basic ways of using textbooks in Finland. The textbook appears to be the only source for exercises in every

lesson for 62 % and in every second lesson for another 30 % of Estonian teachers. A total of 51 % of Norwegian teachers also claim that this is the case in about every second lesson and an additional 18 % say it is the case in almost every lesson. In contrast to the other countries, about one third of Norwegian teachers seem to use additional sources for exercises on a regular basis. A case study of Norwegian lower-secondary mathematics teachers' curricular practice (Pepin et al., 2013, p. 693) provides one possible interpretation of this result:

> in class, the teacher mostly relied on the textbook or worksheets from other books, or the web [...] Whilst textbooks were not often seen to be directly used in the classroom, exercises and activities were often simply downloaded from textbooks to worksheets [...].

Exercises presented in the textbook also serve as the main source for pupils' homework. In all three countries, about 90 % of teachers claim to *use the textbook as the only source for homework tasks for their pupils* in either every lesson or every second lesson. Therefore in summary, it could be said that exercises are the component of the textbook most heavily used by the teachers and thus, also by their pupils. Several other studies have noted the same observation – for example, in the study of textbook use by English, French and German mathematics teachers (Haggarty & Pepin, 2002).

Teachers were also asked *how often they use the textbook as the source for group work tasks in their classes* (F8). The responses suggest that this is not a common activity for the teachers; 73–91 % of teachers never practice it or use it only in some lessons. The popularity of the textbook for this kind of activity still seems to be somewhat higher in Norway, while in Estonia and Finland about 25 % of the teachers claim they never use textbooks as a source for group work tasks and only 10–20 % use them regularly. In Norway, only 9 % of teachers claim they never use the textbook as a source of tasks and about 27 % say they practice it in every second lesson.

Use of textual materials of the textbook

Traditionally teachers are the mediators of new knowledge in mathematics lessons. At the same time, all textbooks also include texts introducing new concepts and providing additional material about the content covered. Thus, one of our interests was, how common it is among the teachers to use texts from the textbook for pupils' individual acquisition of knowledge. According to teachers' responses to item F6 – *How often do students in your class use textbooks to study new concepts individually?* – a majority of them never let students study new concepts individually from the textbook or do so only in some lessons. In the Estonian sample,

the proportion of such teachers is 68 %, in Norway 67 % and in Finland as high as 91 %. In this respect Finnish teachers are significantly different from others, and it seems that pupils' self-study of mathematical concepts based on the textbook is not the tradition here. This is regular practice for approximately one third of teachers in Estonia and Norway. With respect to the item F9 – *How often do students in your class use textbooks to find additional material about the content covered by the teacher during the lesson?* – the approach of teachers in the three countries appears to be quite similar: about half of the teachers say that students do so either every lesson or every second lesson (45 % in Finland, 49 % in Estonia and 60 % in Norway) and another half never do so or use this approach only in some lessons. Therefore, we can agree with the conclusion made by Pepin and Haggarty (2001): "Although the textbook seems to influence the lesson to a large extent, it appears that it is rarely used as a *pupil book*" (p. 164).

Reading mathematical texts in the textbook could be an important part of a pupil's learning process. At the same time, our analysis reveals limited use of texts in learning situations; reading of mathematical texts in the textbook is not seen as an opportunity for learning by about two thirds of the teachers. If the textbook is mainly used as a source of exercises and tasks to solve, pupils' reading is limited to the reading of the text of a given task. Österholm (2005) presents the notion of *content literacy*, which refers to "the ability to read, understand and learn from text from a specific subject area" and stresses the need to develop this ability by regular practice of specific reading activities in mathematics. Mathematical literacy should be recognized as an integral part of mathematical competences (Niss & Højgaard Jensen, 2002). "Thereby, reading and reading comprehension could be more explicitly included in mathematics education, in teaching as well as examinations, and some agree it *should* be included" (Österholm, 2005, p. 326). In the study carried out by Haggarty and Pepin (2001), which was based on textbook use in England, France and Germany, the researchers came to the same conclusion that pupils in their study had almost no opportunity to develop their reading and comprehension skills in mathematics.

Common patterns in teachers' practices of using textbooks

The previous section analysed and compared teacher practices in the use of textbooks in three countries. One possible conclusion is that beside the differences, there is much in common in the way teachers claim that they utilize textbooks in their lessons and it would be possible to find teachers with similar practices in the use of textbooks in all three countries.

What are the most common patterns of utilizing textbooks in mathematics classrooms? To answer that question the cluster analysis was carried out based on teachers' responses to items about their textbook usage in the lessons (items F6 to F10). Formed in this way, the clusters unite teachers whose practices in using textbooks are similar and it is possible to interpret developed clusters as reflections of common patterns of utilizing textbooks in mathematics lessons. It should be noted that the developed cluster model is valid for our teachers' sample and its generalisation should be treated with care.

In the process of analysis, models with different numbers of clusters (2, 3, 4, 5 and 6) were derived. The choice between models is based foremost on how interpretable and meaningful the derived cluster sets were. This way the four-cluster model was accepted by the authors after discussion about possible interpretations and considered to be the most informative. To assess the solution's validity the means for each cluster on each dimension (F6 to F10) using ANOVA was examined. All possible pairs of clusters in the 4-cluster model proved to be significantly different (in terms of at least 4 dimensions out of 5). So, the clusters proved to be conceptually distinguishable. The four-cluster model that was developed is represented in figure 1 and interpreted in the following.

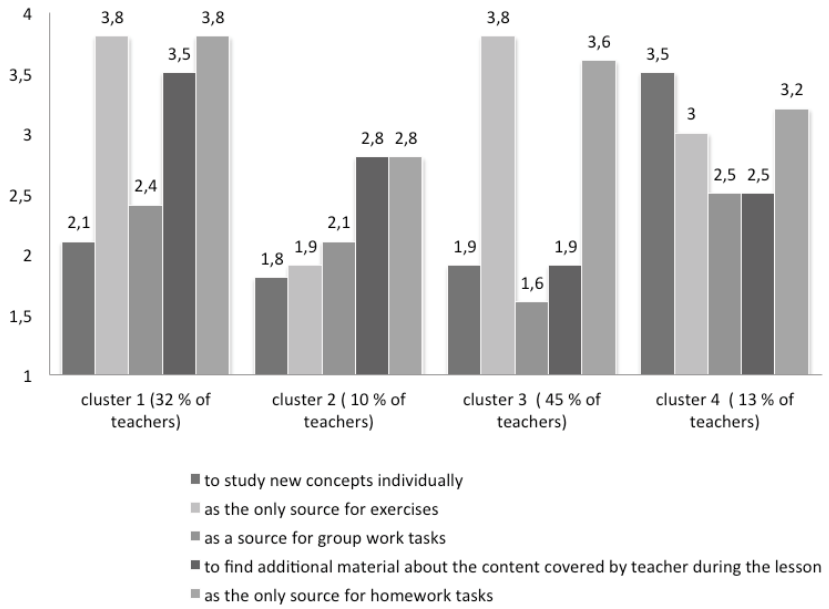

Figure 1. *Patterns of teachers' use of the textbook*

Cluster 1 includes 32 % of all teachers. It represents teachers who in almost every lesson emphasize the use of textbooks for exercises (the textbook as the only source for exercises) and for homework (the textbook as the only source for homework tasks). But in addition, it also represents those teachers who tend to frequently initiate tasks in which pupils need to search through the textbook for additional information. Thus, they also use the textbook as an important supplementary resource for the pupils to find additional material about the content covered by the teacher during the lesson.

Cluster 2 includes 10 % of all teachers in the survey. It represents teachers who do not consider the textbook to be the primary tool to use in the lessons. Obviously they use different sources to find tasks and also they seldom ask students to study new concepts individually from the book. Even for homework tasks they seem to use additional sources in parallel to the textbook.

Cluster 3 includes 45 % of all the teachers in the survey. Teachers belonging to this cluster emphasize the use of textbooks for exercises – whether to be solved in the lesson or as homework. Other elements in the textbook are used only in a few lessons. Probably these teachers consider pupils unable to study new concepts or methods independently from the textbook. Thus, for them, the theory in textbooks is written for teachers and the textbooks are just exercise books for pupils.

Cluster 4 includes 13 % of all the teachers. These teachers differ from all the others in their approach by also using the textbook as a text book for pupils – they often let pupils study new concepts from the textbook individually. Of course like all other teachers, they also emphasize the use of textbooks for exercises and for homework.

Table 4. *Distribution of teachers from each country between clusters*

Cluster	Estonia (%)	Finland (%)	Norway (%)
1	32	34	25
2	7	3	34
3	46	61	18
4	15	2	23

Clusters were developed based on a sample of teachers from all three countries. Consequently, each cluster may include teachers from each country. At the same time however, it could be assumed that there may not be an even distribution of teachers from the different countries within each cluster and certain patterns could be more common for

certain countries. As can be seen from table 4, the popularity of clusters is quite different in each of the countries. The majority of Estonian and Finnish teachers tend to represent the pattern reflected by Cluster 3 – the use of the textbook mainly as the source of tasks and exercises. Cluster 1 reflects the second most popular pattern for these teachers where in addition to using the textbook for exercises, some additional material is also used from the textbooks. In the Finnish case, these two patterns involve more than 95% of the teachers. In Estonia, 15% of teachers also tend to practice the pattern reflected by Cluster 4 – in addition to being a source of exercises they use textbook for pupils to study new concepts individually. Among Norwegian teachers there are no dominant patterns of textbook use. All four patterns are employed relatively equally. The most popular proves to be Cluster 2 – reflecting less emphasis on the textbook and instead, using different alternative tools, which tend to be among the less used patterns in Estonia and Finland.

Discussion of presented results

The results of this study indicate that teachers in Finland and Estonia share a similar relationship with the textbook. In these countries teachers are often responsible for the selection of textbooks and the textbook has a strong effect on teachers' pedagogical choices. However, on the basis of this study, it appears that in Norway teachers are not as dependent on the textbook as their colleagues in Estonia and Finland. It could be the case that teachers in Norway consider it important for them as professionals, not to be dependent on textbooks. If the massive use of textbooks is criticized in society in general and by researchers in particular, teachers may take action accordingly (Holmlund, 2011). The fact that Norwegian teachers are mainly neutral to the question whether the pedagogic strategy of the textbook influences them, might indicate that they are not in favour of any specific strategy. It is noteworthy that almost 2/3 of the Norwegian teachers agree with the fact that the textbook includes tasks for able and less able pupils. The regulations in Norway require teachers to provide "tilpasset oppläring" (adapted teaching) for each pupil.

In Finland the textbook is also a crucial source of exercises. In Estonia and Norway, other sources are used more often. It also seems, that individual study of new concepts, and group work with the help of the textbook, are not as common in Finland as they are in the two other countries. This raises a question whether the study methods in Finnish mathematics classes are in general less multifaceted than they are in the two other countries. To some extent, the study of Viholainen et al. (2015) answers this question. It indicates that in Finland, mainly teachers utilize the

theory section of the textbooks whereas students use textbooks mostly as a source of exercises. If students encounter problems with the exercises, they may search for help among the examples presented in the textbook, but they tend to only read the theory sections when preparing for exams. It also became evident in the study by Viholainen et al. (2015) that in lessons, both theory and examples are usually studied with a teacher-led style and that exercises done by students come almost completely from the textbook. These findings are all in compliance with the results presented in this paper.

Johansson (2003, 2006) has documented findings from Sweden, which are in many ways similar to the results of this study. According to her studies, textbooks also have a very central role in Swedish mathematics classes. She analysed three mathematics classes and found that in lessons, students work individually with the textbook tasks more than half of the time. In addition, the examples presented by teachers usually come from the textbook and textbooks also affect the way in which a teacher presents mathematics in a lesson.

Differences between the countries concerning the role of the textbooks may at least partially be due to the effect of traditions. However, it is notable that in all the countries there are a significant number of teachers whose responses do not match the overall trends of their country. Therefore, it is probable that within each country there are large differences between individual teachers with respect to their use of textbooks.

During recent decades a considerable amount of energy has been put into developing new and better mathematics textbooks. At the same time, teachers' approaches to textbook use have seemingly remained unchanged. Our study shows only a limited use of the full potential of textbooks by many teachers – almost 45 % of all teachers in the study tend to use the textbook only as an exercise book. Consequently, their pupils are not given the opportunity to learn mathematics from the textbook without the teacher's mediation. Pupils are obviously perceived as not being able to cope independently with the texts in the textbook. Only 13 % of the teachers tend to make full use of different materials included in the textbook. It appears that too often pupils have limited access to the textbook and in this way textbooks are not fully used as multifaceted resources for pupils' learning.

Sosniak and Perlman (1990) draw the conclusion that the way in which the teacher mediates the use of textbooks leads to a limited view of the opportunities provided by the textbooks. They write: "Mathematics textbooks turn out to be useful only for the practice opportunities they provide, not for the knowledge they might make available" (p. 434). One might ask why the tradition of using the textbook mainly as a source for

exercises is so strong. Could it be there is a connection to how mathematics as a science is perceived? If proficiency in mathematics is seen simply as procedural skills and contains little conceptual thinking, then practice following given examples is perhaps enough. The study seems to support the view that the teacher is the mediator of the textbook to students. If this is the case, we should strive for teachers who try to mediate another view of the textbook, which would give the textbook the chance to fill an even more important role in students' learning.

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Madis Lepik

Madis Lepik is Associate Professor of mathematics education at the Department of Mathematics, Tallinn University, Estonia. His research interests include teachers' beliefs and professional development, textbook studies, and proof and proving.

mlepik@tlu.ee

Barbro Grevholm

Barbro Grevholm is Professor Emerita in didactics of mathematics at University of Agder in Norway and at Kristianstad University. She has been active in creating and developing the doctoral programmes in both universities and at Luleå University of Technology. She currently supervises doctoral students and has published in books and journals. From 2006 she was the leader of the network for research on mathematics textbooks in the Nordic and Baltic countries (supported by NordForsk).

barbro.grevholm@uia.no

Antti Viholainen

Antti Viholainen (Ph. D) is a Senior Lecturer at the Department of Physics and Mathematics, University of Eastern Finland. He has taught several courses in the mathematics teacher education program. His main research interests are mathematical orientations, mathematical reasoning and use of learning materials.

antti.viholainen@uef.fi