Past and current approaches to decimal numbers in Dutch primary school mathematics textbooks

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In the Netherlands, most contemporary textbook series for primary school mathematics education are influenced by the so-called *Realistic mathematics education* (RME) reform. This reform dates back to the 1970s. In the study described in this paper we investigated what this reform means for the approach to decimal numbers. We analyzed how this content domain is treated in a contemporary RME-oriented textbook series and two pre-RME textbook series. Our study revealed that most, although not all, of the RME characteristics included in our analysis framework were found to be present in the researched contemporary RME-oriented textbook *The world in numbers* (2009). Furthermore, it was found that onsets of several RME characteristics were already present in the two older textbooks, *New arithmetic* (1958) and *Functional arithmetic,* (1969) that date from before the RME reform started.

Approaches to mathematics education change and evolve over time (e.g. Howson, 1982; Walmsley, 2007). This means that current ideas about the learning and teaching of mathematics inevitably trace back to the past. Therefore, knowledge of mathematics education in earlier times may contribute to a better understanding of today's approaches. One way – and maybe the only one – to reveal this knowledge from the past and get to know what mathematics was taught previously and how, is studying the textbooks that were used in those days. Mathematics textbooks can be seen as the potentially implemented curriculum (Valverde et al., 2002). They reflect ideas on mathematics education and translations of these ideas into actual educational approaches. Of course, textbooks are not equivalent to the enacted curriculum, but nevertheless they do bring into view both content and methods of teaching from the period in which

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they are, or were, in use (e.g. Bjarnadóttir, Christiansen & Lepik, 2013). Although from the past, no data are available about how strict teachers followed their textbooks, presently in the Netherlands research has shown that for the vast majority of primary school teachers the mathematics textbook is a main source for their teaching (Meelissen et al., 2012). Also it is found that over three quarters of the Dutch primary school teachers are following more than 90 % of the textbook content (Hop, 2012).

In the study described in this paper we looked back at primary school mathematics education of half a century ago with the aim of getting a historically-informed view on the contemporary teaching of mathematics in the Netherlands. The underlying assumption of this study is that reviewing old teaching approaches with the eyes of today and vice versa, may provide indications for improving current mathematics education.

The context of our study is the reform movement in mathematics education that became known under the name *Realistic mathematics education* (RME) (see e.g. Van den Heuvel-Panhuizen, 2001, 2010; Van den Heuvel-Panhuizen & Drijvers, 2014). The inception of this reform movement was the start, in 1968, of the Wiskobas project with Treffers as one of its leading persons. Wiskobas is an acronym for "Wiskunde op de basisschool", meaning "mathematics in primary school". In 1971 this project became further institutionalized with the establishment of the IOWO (Institute for research of mathematics education) of which Freudenthal was the first director. The main goal of this institute, which in 1991 was renamed as the *Freudenthal institute*, was to think of an alternative for the then prevailing mechanistic approach to mathematics education. Characteristic of this mechanistic approach (see, e.g. De Jong, 1986) is its focus on learning to calculate with bare numbers and the small amount of attention that is paid to solving real world problems. Students only have to apply mathematics after they have learned and practiced the calculation procedures with bare numbers. Another feature of the mechanistic approach is that mathematics is taught in an atomized way. Students learn the procedures in a step-by-step manner with the teacher demonstrating to them how they have to solve a problem. Contrary to the mechanistic approach, in RME "realistic" situations are given a major place in the learning process. These so-called contexts are used in two ways. First, they serve as sources for building up mathematical concepts, tools and procedures. Progressively, this knowledge becomes more general and less context-specific. Second, the learned mathematical knowledge can be applied in other problem situations. In RME to support the shift from context-based strategies to more formal ways of working, didactical models such as the number line play a crucial role. This emphasis on models makes it clear that the differences between the mechanistic approach and the RME approach do not only lie in the procedural aspects of learning mathematics (e.g. learning how to calculate), but that they are even more pronounced with respect to the conceptual aspects of learning mathematics (e.g. learning the decimal structure of the number system as units of units and learning how operations with numbers are related to each other) (see e.g. Treffers, 1991).

Although after forty-five years of reform much has changed in Dutch mathematics education (Van den Heuvel-Panhuizen, 2010), the current situation is not that every class is precisely taught according to the principles of RME. Also, not every RME-oriented contemporary textbook is exactly designed according to all RME principles. Yet, since the beginning of the development of RME, the nature of textbooks has changed dramatically.[1] Until the 1970s, mathematics textbooks in the Netherlands generally had a mechanistic approach to the teaching of mathematics. In the early 1980s, the market share of RME textbooks was only 5 %, whereas in 1987 already 15 % of the textbooks had a RME signature. In 1992 this market share had increased to almost 40 %, and in 1997 to 75 %. In 2004 RME textbooks reached a 100 % market share. Recently (see e.g. Van den Heuvel-Panhuizen, 2010), two new mechanistic textbooks have appeared on the market, but until now based on information we got from communication with publishers their market share has stayed below 5 %.

The purpose of the current study was to get a better understanding of what has changed in primary school mathematics textbooks in the Netherlands since the start of RME. To investigate this we carried out a comparative textbook analysis in which we took RME characteristics of teaching mathematics as a point of departure for examining how these are currently reflected in textbooks and which of these characteristics could also be traced in textbooks which date from before RME. The focus of our analysis was on the content domain of decimal numbers, since the teaching and learning of this content domain incorporates both conceptual aspects (e.g. understanding what a decimal number is and how it is related to other numbers) and procedural aspects (e.g. how to calculate with decimal numbers). More particularly our research questions were:

1. What RME characteristics of teaching decimal numbers can be identified in a typical contemporary RME-oriented textbook?

2. To what degree are onsets of RME characteristics of teaching decimal numbers already present in pre-RME textbooks and in what way does the approach to teaching decimal numbers in a contemporary RME textbook differ from the approach in pre-RME textbooks?

The RME approach to decimal numbers

Characteristic for the mechanistic approach to teaching decimal numbers is that from the very start the teaching takes place at a formal level. Decimal numbers are introduced by definition and presented as another way to write common fractions (Streefland, 1974). Furthermore, in the mechanistic approach there is a strong focus on teaching rules for placing the decimal point (Freudenthal, 1983). RME takes another course. Here the start of teaching decimal numbers is situated at the informal level of students' daily life knowledge. Based on the didactical phenomenology promoted by Freudenthal (1983), RME uses situations in which students can encounter particular phenomena in reality that may contribute to the development of particular mathematical concepts (Van den Heuvel-Panhuizen, 2014).

For decimal numbers this occurs in measurement activities. Through measuring, decimal numbers can emerge in a natural way (Streefland, 1991). Therefore, in RME the interpretation of decimal numbers as measurement numbers related to measuring length or distance forms the basis of introducing students to this content domain (Streefland, 1974; Treffers, Streefland & De Moor, 1996). This introduction is in agreement with the general RME idea of using context problems not only for the application of earlier learned subject matter, but also as a source for learning new mathematical content. For example, when measuring the length of the classroom with a particular unit of measurement, such as a meter, the result may be seven meters and a little bit more.

When we regard this approach in retrospect, this "little bit more" and especially how to express this extra length is crucial for developing understanding of decimal numbers. For sure this "little bit more" is smaller than 1 (meter), but what is it exactly? There are two ways to proceed: refinement of the unit of measurement (after meters continuing with decimeters and centimeters, and so on) or refinement of the number unit (keep the meter as the unit of measurement but continuing with tenths and hundredths of a meter, and so on). Students have to experience both refinements to understand how they are related. Connecting the concrete refinement (moving to a smaller unit of measurement) to the abstract refinement (moving from whole numbers to decimal numbers) can help students to get a better understanding of decimal numbers in several ways. To begin with, the tenths and hundredths can be interpreted as decimeters and centimeters respectively, which gives them a good basis for comparing and ordering decimal numbers grounded in understanding the place value of the digits in decimal numbers.

This connection of decimal numbers to measurement numbers can also prevent or dispel the fallacy that can result from the interference with students' whole numbers knowledge, that a number with more digits always represents a greater value than a number with fewer digits and that, for example, 5.68 is wrongly considered larger than 5.8. In the context of measurement it is clearer for students that this is not true: 5 meters and 8 decimeters is longer than 5 meters and 68 centimeters and therefore, 5.8 is larger than 5.68 (Treffers, Streefland & De Moor, 1996). To a certain degree this support can also be found within the context of monetary values notation. There, it is also clear that 5.8 is more than 5.68. However, using money to explain decimal numbers is not so suitable for getting understanding of the continuous character of decimal numbers because banknotes and coins factually represent discrete quantities instead of continuous quantities. With money the refinement factually ends after cents, whereas in the measurement context refinement can go on infinitely. When working with measurement numbers students can experience that between 3.4 meter and 3.5 meter there lie other numbers such as 3.41 meter and 3.412 meter and so on. Also, by writing down the result of a measurement like "7.632 ... meter", the dots represent the theoretically possible infinite ongoing refinement in tens (Streefland, 1974).

Contrary to the mechanistic approach, in RME the main focus is not on written calculation of decimal number, or, more precisely, on digit-based algorithmic processing of these numbers. Consequently, the focus in RME is also not on accompanying rules for how the decimal point should be put in the right position in each algorithm, which in RME is considered as blocking students' insight (Freudenthal, 1983; Treffers & De Moor, 1984). In RME, the focus on written calculation (either whole-number-based or digit-based algorithmic processing) of decimal numbers is reduced and postponed to later stages of learning decimal numbers. Characteristic for RME is that greater emphasis is put on estimation and mental calculation with decimal numbers. Moreover, within RME attention is also paid to calculating with decimal numbers using a calculator. However, the reason for this is not just to provide students with a device to find an answer. The calculator is also used as a didactical means to investigate the decimal number system and a means to check found answers (Treffers & De Moor, 1984). Also, estimation is not only a goal in itself, but is employed to support students' understanding of where to place the decimal point. Estimating prevents students from giving a too large or too small number that does not make sense as an answer (Treffers & De Moor, 1984; Treffers, Streefland & De Moor, 1996). In agreement with the RME characteristic of intertwined learning strands, the four different ways of calculating with decimal numbers are highly integrated, and related to each other.

Another characteristic of RME is the use of models. For learning decimal numbers and dealing with them in the context of measuring length and distance, the model that is pre-eminently suitable is the number line. According to Freudenthal (1983), the power of the number line is that it visualizes both natural numbers and measurement numbers. Consequently, through these measurement numbers the number line can be opened up to decimal numbers and whole number knowledge can be extended to knowledge about decimal numbers. Students can use the number line to compare and order decimal numbers. Additionally, the number line is rather appropriate to visualize the continuous character of decimal numbers. By zooming in on a particular section, enlarging it and adding finer units, more and more digits can be added behind the decimal point. Another model that in RME is used for teaching decimal numbers is the place-value chart (Treffers, Streefland & De Moor, 1996). In addition, also the abacus with a decimal point (De Jong, 1986) and the so-called "ladder of refinement" (..., 1000, 100, 10, 1, 0.1, 0.01, 0.001, ... presented vertically; see Freudenthal, 1983) are mentioned, but not frequently. These models are not primarily meant for carrying out calculations with decimal numbers, but for understanding the place value of the digits in decimal numbers (Treffers, Streefland & De Moor, 1996). In the case of multiplications with decimal numbers the area (or rectangle) model is used as an alternative for using rules to understand the place of the decimal point. Whereas rules may block students' insight, because the rule for multiplication competes with the one for adding decimal numbers (Freudenthal, 1983), the area model visualizes that when changing a measurement in centimeters into a measurement in millimeters, a refinement of ten times a refinement of ten, results in a refinement of one hundred (Streefland, 1974).

Finally, in line with the RME principle of having students actively involved in the learning process, in RME students are given much room to explore their own calculation methods, and to come up with self-constructed problems, called "own productions" (Treffers, 1987) or "free productions" (Streefland, 1990). Both the students' own calculation methods and their self-generated problems are discussed in class to evoke reflection to supports students' understanding of decimal numbers.

Method

Analysis framework

To carry out the textbook analysis we developed a framework in which we incorporated the main RME characteristics of teaching decimal numbers.

These characteristics were split into three perspectives (cf. Van Zanten & Van den Heuvel-Panhuizen, 2014): the taught mathematical content, the performance expectations (what performance are students expected to show regarding particular mathematical content), and the learning facilitators (see table 1).

With respect to the perspective of the content we focused on the types of decimal numbers that textbooks may deal with. For this we distinguished three sub-categories: bare decimal numbers, measurement decimal numbers, and monetary decimal numbers. The most significant type of decimal numbers, in the sense that this type is in line with how decimal numbers are conceptualized in RME is the sub-category of measurement decimal numbers. Alongside with these numbers we also included monetary decimal numbers in our framework. Although decimal numbers expressing monetary values can also be seen as measurement numbers, we considered them as a separate sub-category, because they are not so suitable for gaining an understanding of the continuous character of decimal numbers and consequently do not have a prominent place in the RME approach to decimal numbers.

Following Valverde et al. (2002), we did not regard performance expectations as part of the content perspective, but considered them a separate perspective, which refers to what type of calculations the students should be able to perform: mental calculation, estimation, written calculation (either whole-number-based or digit-based algorithmic processing), and calculation with a calculator. Here, the RME approach is best recognized by a not-so-prominent position of written calculation with decimal numbers to the advantage of a higher emphasis on estimation and mental arithmetic.

Table 1. *Analysis Framework*

Perspective	Category	Sub-category
Content	Types of decimal numbers	Bare decimal numbers Measurement decimal numbers Monetary decimal numbers
Performance expectations	Types of calculations with decimal numbers	Mental calculation Estimation Written calculation Calculation with calculator
Learning facilitators	Didactical support	Use of contexts Use of the number line Use of a place value chart Use of different calculation methods Use of own productions

For the perspective of the learning facilitators we included five types of didactical support promoted in RME: the use of contexts as a source for learning decimal numbers, the use of the number line as a model, the use of place value charts, the use of different calculation methods and the use of "own productions".

Textbooks and textbook materials included in the analysis

We included three textbook series in our analysis: *De wereld in getallen* [The world in numbers, WiN] (Erich et al., 2009), *Nieuw rekenen* [New arithmetic, NA] (Bruinsma et al., 1969), and *Functioneel rekenen* [Functional arithmetic, FA] (Reijners & Snijders, 1958). WiN dates from 2009 and was included because it is the most widely used contemporary RME-oriented textbook series. It has a market share of approximately 40 % and has a history of three previous editions, dating back to the early days of RME in the 1980s. Because we wanted to investigate to what degree onsets of RME characteristics of teaching decimal numbers were already present in pre-RME textbooks, we found NA and FA to be suitable textbooks. Both date from before the RME reform and despite the fact that in the study of De Jong (1986) they were classified as belonging to the mechanistic approach to mathematics education, De Jong's research team also identified several elements in these textbooks that were considered innovative for their time. NA dates from 1969 and was in use in more than 35 % of Dutch schools in the 1970s. It was still in use in the 1990s. FA dates from 1958 and was used in approximately 5 to 10 % of schools until the 1970s (see De Jong, 1986; Janssen et al., 1999; Wiskobas-team, 1979). Thus, our selection of textbooks covers over half a century.

1 Calculate.

a There are some lengths of artificial grass on sale. One is 12,5 m long, one is 3 m long, and one is 7,75 m long. How many meters are there in total?	**b** 0,5 + 0,44 = 5,03 + 99 = 0,06 + 60,1 = 7,5 + 3,75 =	**c** 2,55 + 35 + 102 = 60 + 4,89 + 3,01 = 5,49 + 3,21 + 130 = 12,4 + 0,45 + 3,05 =
d From a 12,5 m roll of fabric, 3,75 m is cut off. How many meters are left?	**e** 10,5 – 3,48 = 15,7 – 8,95 = 22,4 – 7,85 = 34,6 – 15,69 =	**f** 12,25 – 8,7 = 38,62 – 14,9 = 45,54 – 28,9 = 53,05 – 48,5 =

Figure 1. *A set of tasks (WiN, 2009) with nine addition and nine subtraction tasks (one word problem and eight bare number tasks each)*

Note. In the Netherlands, decimal numbers are written with a decimal comma instead of a decimal point.

In all three textbook series, decimal numbers are dealt with in grade 4, 5 and 6, so we limited our research to the materials for these grades. We analyzed all materials of the textbooks meant for all students and the accompanying teacher guides [2]. Optional materials meant for assessment, repetition or enrichment were left out of our analysis.

Unit of analysis

In all three textbook series, the content is organized in sets of tasks. With the term "task" we mean the smallest unit in the student books that requires an answer from a student. A "set of tasks" is always indicated by a number and mostly contains a series of tasks (see figure 1).

In our analysis we used the task as the unit of analysis, together with the directions and models belonging to this task as indicated in the set of tasks and the corresponding directions given in the teacher guides.

Tasks on decimal numbers included in the analysis

Although tasks in the domain of decimal numbers have an overlap with calculating with monetary values and with measurement tasks, we only included those tasks in the analysis that primarily focus on decimal numbers. For example, tasks in which certain measures have to be converted to another measurement unit (e.g. "2.5 m = ... cm") without problematizing the change from decimal numbers to whole numbers, were considered to belong predominantly to the domain of measurement. These tasks were left out of the analysis, whereas tasks that, for example, focus on place value (e.g. "3.42 m = ... m + ... dm + ... cm") were included, because here the students have to interpret the decimal digits.

Analysis procedure

First, all tasks on decimal numbers meant for all students were identified. Then, each task was coded according to the framework. For all the coded tasks it was checked whether the assigned codes were in agreement with the directions included in the teacher guide, because sometimes the tasks included the student books do not offer enough indications for how to code the tasks. To be sure that the coding was done correctly, the complete coding was done a second time. When differences between the two codes of a task occurred, the task was reviewed again and the codes were made the same, and if necessary codes of other tasks were also revised until all coding results were consistent.

Results

Attention paid to decimal numbers

As is shown in figure 2, all three textbook series introduce decimal numbers in grade 4 and pay most attention to it from the second half of grade 4 to the first half of grade 6. Over the three grades, the two older textbooks FA and NA offer more tasks on decimal numbers that are meant for all students (2333 and 3396 tasks respectively) than the contemporary textbook WiN (1997 tasks). Note, that these frequencies do not reflect the total amount of attention paid to decimal numbers, because additional optional tasks meant for differentiation were left out of our analysis. In WiN, the number of pages with additional tasks for differentiation is larger than the number of pages with tasks meant for all students. This is the other way around in NA, whereas in the oldest textbook, FA, only now and then an additional task meant for differentiation is given. So although it might seem, based on the number of tasks meant for all students, that the contemporary textbook in a quantitative way pays less attention to decimal numbers, this is not the case.

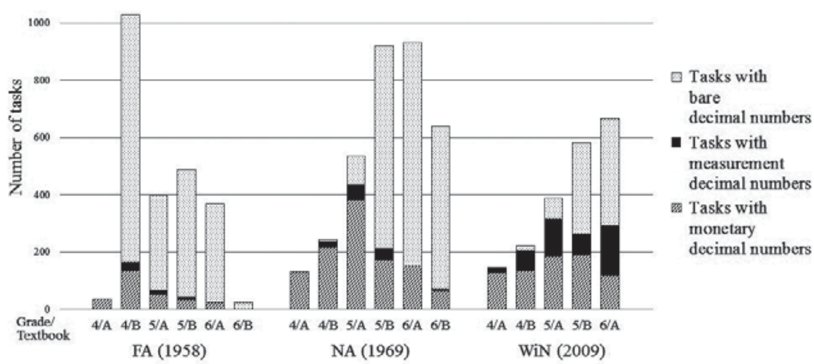


Figure 2. *Absolute and relative frequencies of tasks with a particular type of decimal numbers included in the textbooks FA, NA, and WiN; based on the tasks for all students; covering grade 4 to 6; specified for the respective textbooks for these grades*

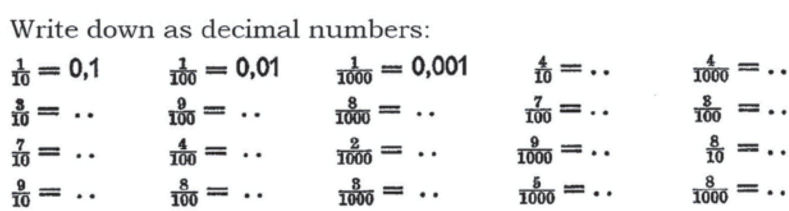
Types of decimal numbers included in tasks

All three textbook series offer tasks with bare decimal numbers, measurement decimal numbers and monetary decimal numbers, but the relative frequency of these tasks is quite different in the three textbooks series (see table 2). In FA, the vast majority of the tasks concern bare decimal numbers. Similarly, in NA most tasks are with bare decimal numbers, but NA also provides a substantial number of tasks with monetary decimal numbers. WiN provides about the same number of tasks with bare decimal numbers and tasks with monetary decimal numbers, and a substantial number of tasks with measurement decimal numbers. Compared to the older textbook series, WiN provides a considerable smaller proportion of tasks with bare decimal numbers and a considerable larger proportion of tasks with measurement decimal numbers. In the two older textbook series, measurement decimal numbers are present, but the proportion of tasks with this type of decimal numbers is very small. Regarding the number of tasks with monetary decimal numbers, no clear difference can be determined between WiN and NA. Both NA and WiN do provide substantially more tasks with monetary decimal numbers than the oldest textbook, FA.

Table 2. *Absolute and relative frequencies of tasks with a particular type of decimal numbers included in the textbooks FA, NA, and WiN; based on the tasks for all students; covering grade 4 to 6*

Type of decimal numbers	FA (1958)		NA (1969)		WiN (2009)	
Bare decimal numbers	2004	86%	2164	64%	783	39%
Measurement decimal numbers	57	2%	125	4%	469	23%
Monetary decimal numbers	272	12%	1107	33%	745	37%
Total number of tasks	2333	100%	3396	100%	1997	100%

All three textbook series offer tasks with bare decimal numbers for the first time in the second half of grade 4. The way in which the three textbook series introduce tasks with bare decimal numbers differs. In FA and NA bare decimal numbers are introduced as another way for writing fractions. In this sense FA and NA clearly reflect the approach to decimal numbers that suits their juncture in time. In FA, bare fractions are used (see figure 3). In NA, decimal numbers are introduced by making a reference to fractions expressed in monetary values (see figure 4). In WiN


Write down as decimal numbers:

$\frac{1}{10}$ = 0,1	$\frac{1}{100}$ = 0,01	$\frac{1}{1000}$ = 0,001	$\frac{4}{10}$ = ..	$\frac{4}{1000}$ = ..
$\frac{3}{10}$ = ..	$\frac{9}{100}$ = ..	$\frac{8}{1000}$ = ..	$\frac{7}{100}$ = ..	$\frac{8}{100}$ = ..
$\frac{7}{10}$ = ..	$\frac{4}{100}$ = ..	$\frac{2}{1000}$ = ..	$\frac{9}{1000}$ = ..	$\frac{8}{10}$ = ..
$\frac{9}{10}$ = ..	$\frac{8}{100}$ = ..	$\frac{3}{1000}$ = ..	$\frac{5}{1000}$ = ..	$\frac{8}{1000}$ = ..

Figure 3. *Introduction of bare decimal numbers in FA (1958), grade 4, student book B*

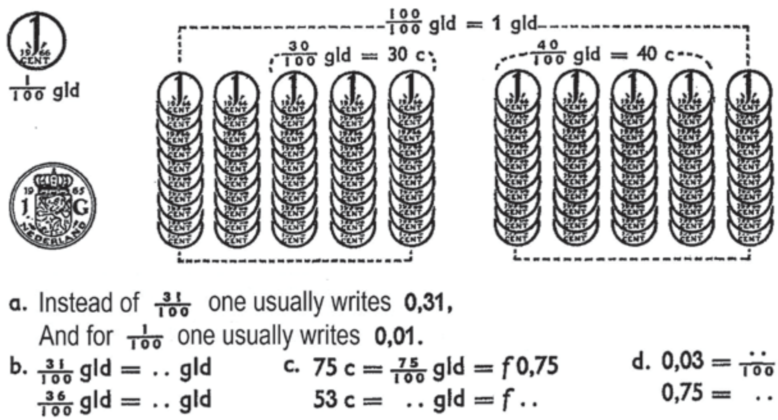


Figure 4. *Introduction of bare decimal numbers in NA (1969), grade 4, student book B*

bare decimal numbers are introduced through measurement numbers, without linking decimal numbers to common fractions. Instead, the context of hectometer signs along highways is used to support counting with tenths (see figure 5).

Tasks with measurement decimal numbers

Although the proportion of tasks with measurement decimal numbers is considerable smaller in the two older textbooks than in the contemporary textbook, the ways in which measurement decimal numbers are used to teach decimal numbers show certain similarities. In all three textbook series measurement decimal numbers are used to understand the place value of the digits behind the decimal point. Both FA, the oldest textbook, and WiN, the contemporary textbook, provide sets of tasks in which the relationship between the place value in monetary decimal numbers and measurement decimal numbers and the place value in bare decimal numbers is made explicit (see figures 6 and 7). In FA, tasks on place value with bare decimal numbers are provided before tasks with monetary decimal numbers and measurement decimal numbers. In contrast, in WiN this is done the other way around, which is consistent with the RME approach of using real-life situations as a source for learning.

In both NA and WiN measurement decimal numbers are used for comparison and ordering of decimals numbers which have a different number of digits or have one or more zeroes behind the decimal point. In contrast, FA only presents such tasks with bare decimal numbers.

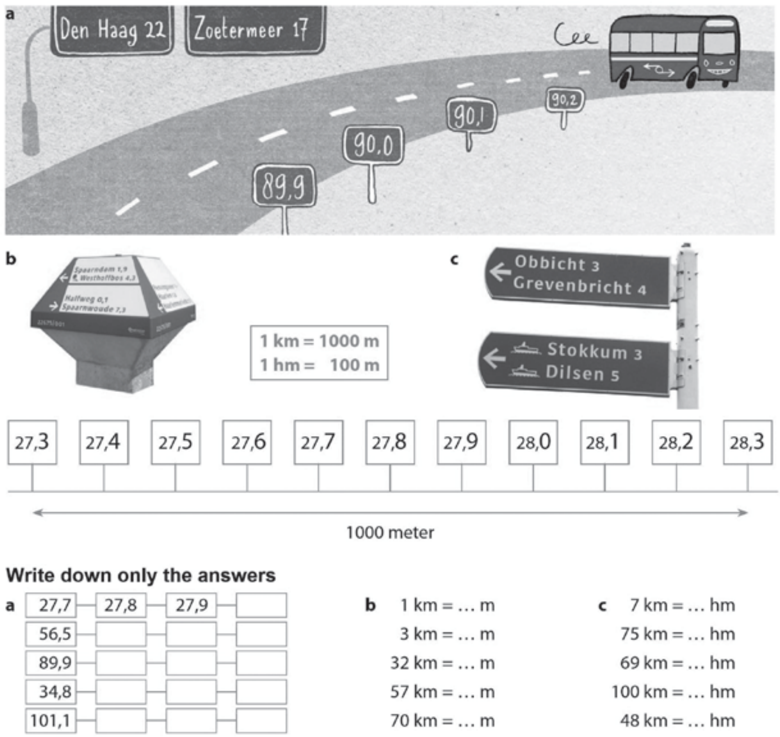


Figure 5. *Introduction of bare decimal numbers in WiN (2009), grade 4, student book B*

8,64 gld. The 8 = … 8,64 m. The 8 = …
8,64 l. The 8 = … 8,64 g. The 8 = …
35,89 m. This 35 + … *f* 35,89 This 35 = …
35,89 kg. This 35 = … 35,89 This 35 = …

When a decimal number has the name 'gld' behind it or '*f*' in front of it, the digits before the comma represent guilders. To the right of the comma, you first see the tenths of guilders (ten cent pieces), and then the hundredths of guilders (one cent pieces).
When a decimal number has the name 'kg' behind it, the digits before the comma represent … After the comma, you first see tenths of kg (…), and then hundredths of kg (…) and so on.
Try to tell this for a decimal number that has the name 'm' behind it. Also for 'km'. And what happens when no name is given?

Figure 6. *A set of tasks on place value in FA (1958), grade 4, student book B*

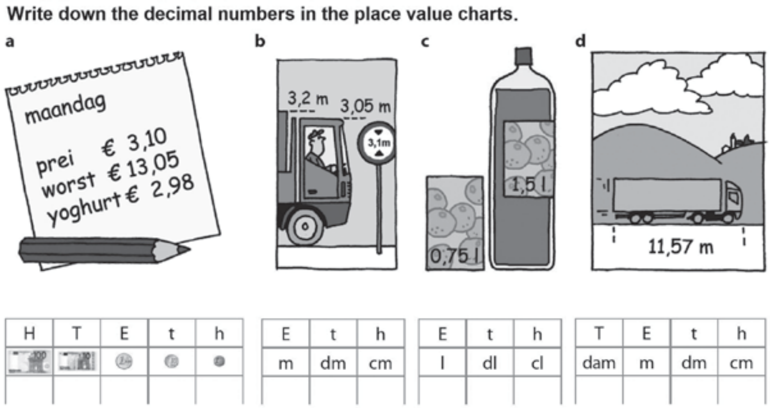


Figure 7. *A set of tasks on place value in WiN (2009), grade 5, student book A*

Note. The letters H, T, E, t, and h, respectively refer to the one hundreds, the tens, the units ('E' stands in Dutch for 'eenheden' which means 'units'), the tenths and the hundredths.

Another similarity between NA and WiN is that they both use measurement decimal numbers for tasks in which students have to round off decimal numbers, and for tasks in which fractions have to be written as decimal numbers. FA does not use measurement decimal numbers for such tasks. However, the three textbook series were again found to be similar with respect to using measurement decimal numbers for mental arithmetic in contexts.

There are also differences between the two older textbooks and the contemporary one. WiN provides tasks with measurement decimal numbers that are not present in FA nor in NA, including tasks in which students have to continue a counting sequence with decimal numbers (see for an example the tasks at the bottom on the left in figure 5) and tasks addressing the continuous character of decimal numbers, such as tasks in which students have to determine a decimal number in between two other decimal numbers (e.g. "Tooske puts her foot exactly between 5.2 meters and 5.3 meters. Where is that?").

Types of calculations with decimal numbers

In all three textbook series, a substantial part of the tasks with decimal numbers is spent on calculations with decimal numbers (see table 3). In FA, this concerns about 85% (1975 out of 2333 tasks), in NA this is about 80% (2706 out of 3396 tasks), and in WiN about 66% (1322 out of 1997 tasks). The overall trend based on these textbook series is that over the years the

Table 3. *Absolute and relative frequencies of tasks including a particular type of calculation with decimal numbers in the textbooks FA, NA, and WiN; based on the tasks for all students; covering grade 4 to 6*

Type of calculation	FA (1958)		NA (1969)		WiN (2009)	
Mental calculation	474	24%	346	13%	321	24%
Estimation	188	10%	472	17%	367	28%
Written calculation	671	34%	651	24%	238	18%
Calculation with a calculator	0	0%	0	0%	81	6%
Free choice by student	21	1%	67	2%	2	0%
Not indicated in textbook	621	31%	1170	43%	313	24%
Total number of calculation tasks	1975	100%	2706	100%	1322	100%

relative frequency of estimation tasks increased and that of written calculation tasks decreased. These changes are in line with the RME shift towards more emphasis on estimation, but the relative frequency of estimation tasks in NA makes it clear that the emphasis on estimation had increased already prior to the start of RME. Also the substantial attention to mental calculation has pre-RME roots. This attention was found in both FA and NA. Moreover, the relative frequency of mental calculation tasks is in FA about the same as in WiN. The only type of calculation with decimal numbers that was found in WiN and not in the two older textbook series was that of calculation with a calculator, which is obvious because calculators only became available in schools from the 1980s on. A type of task that is present in FA and NA but not in WiN, is that in which students themselves have to choose between using mental or written calculation. In WiN, a comparable type of task is present in which students have to choose between whether or not to use a calculator, but only very rarely (two tasks).

Estimation with decimal numbers

In all three textbook series students are expected to estimate the outcome of an operation before calculating the precise answer. This estimation is meant to support the correct placement of the decimal point in the answer. In FA this concerns 92 % (173 out of 188) of all estimation tasks with decimals, in NA 66 % (310 out of 472) and in WiN 31 % (112 out of 367). The relative low percentage in WiN is caused by the fact that in WiN most estimation tasks are not linked to a precise calculation, but estimation is a goal in itself. In all three textbook series most estimation tasks are offered on multiplication.

Regarding the strategy used for estimation, all three textbook series offer tasks in which students have to do the calculation with rounded-off numbers. Another strategy is that of clamping, which means that the students have to determine between which numbers an answer must lie. In the two older textbooks, the students have to apply this strategy with multiplications and divisions with decimal numbers, where they have to indicate between which whole numbers the answer lies (e.g. ”2.4 × 7.6 is bigger than ... and smaller than ...”). In the contemporary textbook WiN this always concerns division tasks where students are asked between which decimal numbers the answer lies (e.g. ”Between what decimal numbers does the answer lie? 205 ÷ 15. Choose: between 13.0 and 13.5 or between 13.5 and 14.0”).

WiN also offers estimation tasks in which students have to choose what answer can be correct (e.g. ”30 × 0.15 = 450 or 4.5 or 0.45”) and tasks that concern the recognition and correction of incorrectly placed decimal points (e.g. ”3.58 × 52.3 = 1872.34; correct the mistake”). The latter type of estimation task was also found in FA, but not in NA. Although these types of estimation tasks do not involve precise calculation, they nevertheless support the correct placement of the decimal point. The contemporary textbook WiN also offers tasks in which a calculator is used for this purpose. This applies to 73 % (59 out of 81) of the tasks on using a calculator. In these 59 tasks the calculator is used to check the estimation.

Use of contexts as a source for learning decimal numbers

In the contemporary textbook WiN contexts are clearly used as a source for learning decimal numbers. Figure 5, shown earlier, presents an example of this: the context of hectometer signs alongside highways is used to introduce decimal numbers. WiN also uses contexts for dealing with the

Figure 8. *A context on measuring distances with different precision in WiN (2009), grade 5, student book A*

Figure 9. *Positioning decimal numbers on a number line in WiN (2009), grade 5, student book B*

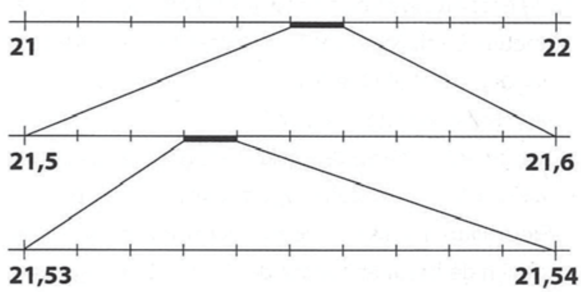


Figure 10. O*ngoing refinement of decimal numbers visualized on a number line in WiN (2009), grade 5, teacher guide*

specific difficulty of ordering and comparing decimal numbers that have a different number of digits behind the decimal point. For this, the context of measuring distances with different precision is used (see figure 8).

The two older textbook series also offer context problems for dealing with specific difficulties such as comparing decimal numbers with a different number of digits or with zeroes behind the decimal point. However, here contexts are not used as a source for learning. Instead, they only serve for applying what students have been taught earlier. The contexts are offered after the students had to compare the decimal numbers in bare number tasks. Though, if one would consider the coins shown in NA when the bare decimal numbers are introduced (see figure 4) to be a context, then this set of tasks in NA would be an exception to the previous conclusion that in the two older textbooks contexts are only offered after the students had to solve bare number tasks.

Use of the number line as model for dealing with decimal numbers

In WiN, the number line as a model for dealing with decimal numbers is used to visualize the partitioning of the units into smaller and smaller units, to relate decimals numbers to fractions, and to position and order decimal

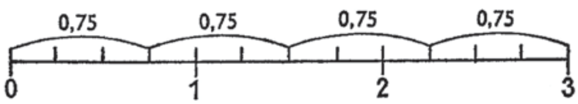


Figure 11. *Visualization of a multiplication with decimal numbers (4 × 0.75) on a number line in NA (1969), grade 5, student book A*

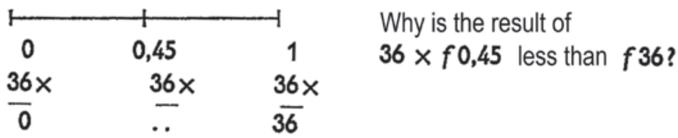


Figure 12. *Visualization of multiplying with a number smaller than 1 on a number line in NA (1969), grade 5, student book A.*

numbers (see figure 9). For this, 169 tasks (8 % of all tasks) with bare decimal numbers and with measurement decimal numbers are accompanied by a number line. In addition, the teacher guide explains how a number line can be used in instruction, for example, to contribute to students' understanding of the infinitely ongoing refinement of decimal numbers (see figure 10). Using the number line for visualizing an addition with decimal numbers was only found once in a WiN student book.

In contrast with WiN, in FA the number line is not present. In NA, however, the number line was found accompanying 20 tasks (1 % of the tasks), but not as a model to support the understanding of the continuous character of decimal numbers. In NA the number line is only used to visualize operations (see figure 11), to identify what decimal numbers are located in a particular position, and to visualize that multiplying with a number smaller than 1 gives a smaller result (see figure 12). In the teacher guide, NA also offers examples of how to use the number line during instruction.

Use of the place value chart

As earlier shown in figure 7, the contemporary textbook WiN offers a place value chart to relate the place value in monetary decimal numbers and measurement decimal number numbers to the place value in bare decimal numbers. WiN also offers the place value chart to compare decimal numbers with a different number of digits behind the decimal point, but this only is done with measurement decimal numbers, not

1. *Say out loud:* 40,004 87,95 326,08 3,507 0,75
4000,4 30,007 32,608 3507 0,009

2. *Write down the numbers of exercise 1 in the place value chart. Do not forget the decimal comma.*

D	H	T	E	t	h	d
		4	0,	0	0	4

Figure 13. *Use of place value chart in NA (1969), grade 5, student book B*

with bare decimal numbers. Another way in which a place value chart is used in WiN, is to separate the digits of bare decimal numbers that are given in words.

NA also offers a place value chart, which is mostly used to separate the digits of bare decimal numbers (see figure 13). The oldest textbook series, FA, offers a place value chart only once. The set of tasks that includes this is similar to that of NA as shown in figure 13, with the difference that the FA tasks deal with place values from millions to ten thousandths.

Use of different calculation methods

The contemporary textbook WiN offers directions in the teacher guide on how to give room to students to use their own ways of solving tasks with decimal numbers. The teacher guide also provides examples of possible calculation methods that students may use. However, WiN does not pay attention to different calculation methods with decimal numbers in the student books with the exception of two tasks in which students have to choose whether or not to use a calculator (see table 3).

The older textbook NA does offer different calculation methods with decimals in the student books. This involves 11 sets of tasks with in total 96 tasks (7 % of all calculation tasks) in which students are offered examples of different ways of calculating with decimal numbers, followed by tasks where they are invited to choose the easiest way (see figure 14).

1. *Is this allowed?*
15 : 2,5 = 30 : 5 = .. of 3 × (5 : 2,5) = 3 × 2 = ..
of (10 : 2,5) + (5 : 2,5) = .. + ..
21 : 3,5 = 42 : .. = ..

2. *Calculate in your head, and use the easiest way.*
35 : 2,5 = 31,5 : 3,5 = 45 : 2,5 = 38,5 : 3,5 =
20 : 2,5 = 28 : 3,5 = 32,5 : 2,5 = 56 : 3,5 =

Figure 14. *Use of different calculation methods in NA (1969), grade 6, student book B*

In addition, NA offers in 67 tasks (2% of all calculation tasks) a choice between mental calculation and written calculation (see table 3).

In the oldest textbook FA, both the student books and the teacher guide pay attention to different calculation methods. In the student books, in 19 tasks (1% of the calculation tasks) students are explicitly asked to solve the task in different ways. In the teacher guide, possible different ways of calculating tasks are given as well. FA also offers in 21 tasks (1% of the calculation tasks) a choice between mental calculation and written calculation (see table 3).

Use of own productions

All three textbook series provide tasks in which students are asked to come up with own productions. In all cases this applies to decimal numbers in contexts, and with only one exception, these own productions are related to monetary decimal numbers. Examples from the two older textbooks are: "Come up with five subtraction tasks. The answer is always f 32.76" (NA, 1969) and "Come up with a problem that has a gain of f 4.75 as an answer" (FA, 1958). An example from the contemporary textbook WiN is a set of tasks where several products are on display in a shop window with their prices and the students have to decide what products they will buy for a certain amount of money. In this way they themselves can choose which decimal numbers they use to calculate with. The set of own production tasks that is not related to monetary decimal numbers is shown in figure 15. Here, the students may locate measurement decimal number of their own choice, which gives them a lot of freedom in making the tasks more or less difficult.

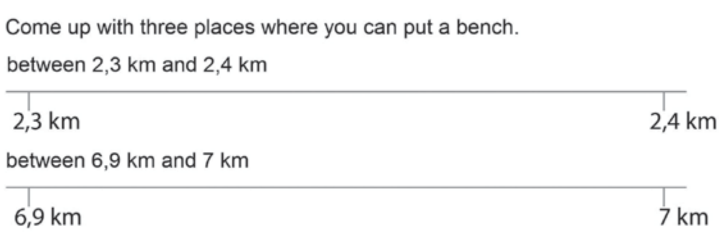


Figure 15. *Use of own productions in WiN (2009), grade 5, student book B.*

Conclusions

Regarding our first research question, as could be expected, most of the RME characteristics included in our analysis framework were found to be present in the researched contemporary RME-oriented textbook WiN. The results concerning our second research question are more noteworthy: onsets of several RME characteristics were already found in the two older textbook series that date from before the RME reform started. Table 4 summarizes our findings.

Regarding the content perspective our analysis revealed that in the contemporary textbook WiN, a substantial part of all tasks with decimal numbers concerns measurement decimal numbers. This type of decimal numbers is offered to support students' understanding of place value, comparing and ordering decimal numbers, rounding off decimal numbers, relating decimal numbers to fractions, continuing a counting sequence of decimal numbers, and understanding the continuous character of decimal numbers. The two older textbooks, FA and NA, also offer tasks with measurement decimal numbers, but the proportion of these tasks is very low. In FA, the oldest textbook, measurement decimal numbers are only meant for understanding place value, whereas in NA more aspects of understanding are addressed.

With respect to the perspective of the performance expectations, our textbook analysis revealed that in the contemporary textbook WiN a substantial proportion of decimal numbers tasks is about either mental calculation or estimation. The attention to mental calculation was also found to be present in both older textbooks, FA and NA, whereas substantial attention for estimation was only found in NA. Yet the didactical use of estimation to support the correct placement of the decimal point in precise calculations, is present in all three textbook series.

Our textbook analysis gave mixed results for the perspective of learning facilitators. The RME characteristic of using contexts as a source for learning is present in the contemporary textbook WiN, but absent in the two older textbooks FA and NA. The number line as a model is used in WiN and in NA, but not in the oldest textbook FA. In the contemporary textbook WiN the number line is applied for multiple purposes: locating and determining decimal numbers, visualizing operations and the continuous character of decimal numbers, and for relating decimal numbers to fractions. The place value chart is utilized in all three textbook series, although in the oldest textbook FA only once. The way the place value chart is used is the same in all three textbook series, namely for writing down decimal numbers and determining the place value of the digits. All three textbook series use different calculation methods as a learning facilitator. In the contemporary textbook WiN, directions are given for this only in the teacher guide, whereas in the older textbook NA different calculation methods are included in the student books. Remarkably, in the oldest textbook, FA, both the student books and the

Table 4. *RME Characteristics in WiN and Onsets of RME Characteristics in FA and NA*

	FA (1958)	NA (1969)	WiN (2009)
Content			
Tasks with measurement decimal numbers	Proportion of all tasks with decimal numbers		
	2%	4%	23%
	Used for supporting understanding of		
	Place value	Place value Comparing and ordering decimal numbers Rounding off decimal numbers Relating decimal numbers to fractions	Place value Comparing and ordering Rounding off Relating decimal numbers to fractions Continuing a counting sequence of decimal numbers Continuous character of decimal numbers
Performance expectations			
Tasks on mental calculation	Proportion of all calculation tasks with decimal numbers		
	24%	13%	24%
Tasks on estimation	Proportion of all calculation tasks with decimal numbers		
	10%	17%	28%
Learning facilitators			
Use of contexts	For application	For application	As a source for learning For application
Use of number line	No	Yes, to Fill in decimal numbers Visualize operations with decimal numbers	Yes, to Locate and determine decimal numbers Visualize operations with decimal numbers (only once in a student book) Visualize the continuous character of decimal numbers Relate decimal numbers to fractions
Use of place value chart	Only once	Yes	Yes
Use of different calculation methods	Yes, present in Student books Teacher guides	Yes, present in Student books	Yes, present in Teacher guides
Use of own productions	Yes, for tasks with Monetary decimal numbers	Yes, for tasks with Monetary decimal numbers	Yes, for tasks with Monetary decimal numbers Measurement decimal numbers (only once)

teacher guide support the use of different calculation methods. The use of own productions is addressed in all three textbook series.

By including only one contemporary textbook and two old ones in our textbook analysis, the results of our study of course do not give a complete picture of past and current approaches to teaching decimal numbers in Dutch primary school textbooks. Nevertheless, we feel our study contributes to a deeper knowledge of the RME characteristics included in the currently most used textbook WiN. Our study reveals what makes WiN belong to the RME approach and what RME characteristics are not so prominent in WiN. Different from what we expected, our study brought into view that students in WiN are hardly invited to choose by themselves what type of calculation to use to solve a problem with decimal numbers. Also, we did not anticipate that the use of different calculation methods with decimal numbers with the purpose of finding relations between these methods, is not supported in WiN, whereas this is done in FA and NA, the two older textbooks involved in our analysis.

The fact that the use of different calculation methods was found in FA and NA, also indicates another finding of our study, namely that particular RME characteristics were already present in the two textbook series dating from the time before RME came into being. This finding suggests that the RME reform was not a complete break with the past and that the roots of RME go back farther than its start in the late 1960s. In this way, our historical excursion has opened our eyes (again) to using old textbook series as a source to revisit and improve our contemporary textbooks series.

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Textbook series

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Notes

1 The following data about the market share of the two types of textbooks are based on: Bokhove, Van der Schoot & Eggen, 1996; De Jong, 1986; Janssen, Van der Schoot, Hemker & Verhelst, 1999; Janssen, Van der Schoot & Hemker, 2005; Van den Heuvel-Panhuizen, 2010.

2 Teacher guides are books especially for the teacher that contain the same tasks as the student books and give in addition all kinds of didactical information, such as goals and directions and suggestions for instruction.

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