Norwegian mathematics teachers' and educational researchers' perception of MPCK items used in the TEDS-M study

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This paper presents how Norwegian teachers and educational researchers categorised a collection of items used in the international TEDS-M study. The teachers categorised the items according to their own prototype understanding of *mathematical content knowledge* (MCK) and *mathematics pedagogical content knowledge* (MPCK), while the researchers categorised them after discussing how to understand MCK and MPCK and agreeing on what categorisation criteria to use. The results show that the item categorisation depended on the item characteristics. For example, multiple-choice items were associated with MCK and items asking the respondents to rewrite or reword a mathematical task were associated with MPCK. Furthermore, the results indicate a common Norwegian understanding of MPCK as the teachers' and researchers' categorisation largely coincided.

Norway was one of 17 countries that participated in the international Teacher Education and Development Study in Mathematics (TEDS-M) (Grønmo & Onstad, 2012; Tatto et al., 2012). Despite being invited to join from the start, the Norwegian authorities hesitated to accept. As a consequence, Norway joined the survey at a relatively late stage in the process of defining the two categories of *mathematics content knowledge* (MCK) and *mathematics pedagogical content knowledge* (MPCK) (Onstad & Grønmo, 2012) and the definition of MPCK might have been less influenced by a Norwegian understanding.

The aim of this paper is to investigate how the knowledge requirements of items operationalising the MPCK category in the TEDS-M survey are perceived in Norway. To represent a Norwegian understanding, in-service mathematics teachers and mathematics education researchers were invited to categorise a collection of TEDS-M MPCK items. The teachers could be seen to represent a practical understanding of MPCK within a

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Kaarstein, H. (2014). Norwegian mathematics teachers' and educational researchers' perception of MPCK items used in the TEDS-M study. *Nordic Studies in Mathematics Education*, 19 (3-4), 57–82.

Norwegian school context and the researchers to represent a theoretical understanding. Furthermore, since the teachers were asked to categorise the items according to their own prototype understanding of MCK and MPCK and the researchers according to the norms of classical categorisation (Smith & Medin, 1981), the teachers could be said to categorise the items by a type of bottom-up approach and the researchers through a top-down approach.

Thus, the research questions of this paper are as follows:

> Given a selection of TEDS-M items, to what degree does Norwegian teachers' item categorisation coincide with Norwegian researchers' item categorisation?
>
> What emerging patterns can be found when Norwegian teachers and researchers categorise the items operationalising the MPCK category of TEDS-M?

To discuss the results of the item categorisation performed by the Norwegian teachers and the researchers, the theoretical background of this paper gives an overview of how MCK and MPCK are defined in TEDS-M before presenting a brief description of how MPCK is understood in a Norwegian context. The difference between categorising according to prototypes and according to classical theory of categorisation is briefly explained in the methodology section.

Theoretical background

MCK and MPCK in TEDS-M

The MCK framework of TEDS-M was built on the content and cognitive domains of the TIMSS 2007 and TIMSS Advanced 2008 assessment frameworks (Tatto et al., 2008; Tatto et al., 2010). The content domain consisted of topics such as *number and operations* and *algebra and functions*, and the cognitive domain categories were *knowing*, *applying* and *reasoning*.

The MPCK framework of TEDS-M was informed by the US-based framework of Ball et al. (see e.g. Ball & Bass, 2000; Ball, Thames & Phelps, 2008), the German-based framework of the COACTIV study (Kunter et al., 2013) and the framework of the TEDS-M preliminary study (Schmidt, Blömeke & Tatto, 2011).

Assuming that teaching involves situated knowledge, the conceptualisation of MPCK was based on core tasks of teaching (Döhrmann, Kaiser & Blömeke, 2012; Tatto et al., 2008). Consequently, the MPCK framework (reproduced in table 1) focused on the knowledge needed in the pre-active and interactive dimensions of teaching. MPCK thus comprised the three sub-categories *mathematical curricular knowledge*, *knowledge of planning for mathematics teaching and learning* and *enacting mathematics for teaching and learning* (Tatto et al., 2008).

Each sub-category of MPCK was again described by a number of core situations in which mathematics teachers are expected to utilise their mathematics pedagogical content knowledge. The core situations are listed under the column heading *Sample topics* in table 1, and most of the items operationalising the MPCK sub-categories, were set in teaching and learning contexts corresponding to the situations described in the sample topics (Tatto et al., 2010).

Table 1. *The sub-categories of MPCK with sample topics (Tatto et al., 2008, p. 39)*

Sub-category of MPCK	Sample Topics
Mathematical curricular knowledge (Curriculum)	Establishing appropriate learning goals Knowing different assessment formats Selecting possible pathways and seeing connections within the curriculum Identifying the key ideas in learning programs Knowledge of mathematics curriculum
Knowledge of planning for mathematics teaching and learning (Planning)	Planning or selecting appropriate activities Choosing assessment formats Predicting typical students' responses, including misconceptions Planning appropriate methods for representing mathematical ideas Linking the didactical methods and the instructional designs Identifying different approaches for solving mathematical problems Planning mathematical lessons
Enacting mathematics for teaching and learning (Enacting)	Analysing or evaluating students' mathematical solutions or arguments Analysing the content of students' questions Diagnosing typical students' responses, including misconceptions Explaining or representing mathematical concepts or procedures Generating fruitful questions Responding to unexpected mathematical issues Providing appropriate feedback

Even though the TEDS-M group aimed to achieve an understanding of MPCK that was internationally accepted, Döhrmann et al. (2012) claimed that the understanding of

> [...] teaching and learning processes [in TEDS-M] was slightly more connected to approaches predominantly taken in English-speaking countries and less to continental European traditions on subject-related reflections, called *Fachdidaktik* in German or *didactique* in French. (p. 326)

In addition to the cultural differences influencing the international definition of MPCK, there was another major issue of equal or maybe even greater importance for the TEDS-M group to review. This issue was related to the operationalisations of the MPCK categories (i.e., to item development), to the fact that MCK is considered a necessary condition for MPCK (Baumert et al., 2010). Thus, "the solution of an item in the domain MPCK generally requires MCK" (Döhrmann et al., 2012, p. 336). In core situations that the majority of the TEDS-M MPCK items were linked to, MCK and MPCK have been claimed to be inseparably linked (Döhrmann et al., 2012). Consequently, it had to be decided "from case to case whether [a TEDS-M] item set within a teaching context refer[red] to MPCK or MCK *only*" (Döhrmann et al., 2012, p. 337, emphasis added). That is, if the item context makes no further knowledge demands beyond MCK to respond correctly to the item, the context should be disregarded.

MPCK or mathematics didactics in Norway

MPCK in Norway may translate into *mathematics didactics* that resembles the German *Fachdidaktik* (Grønmo & Onstad, 2012; Sjøberg, 2001) where the understanding of mathematics and mathematics teaching is focused on the conceptual understanding of mathematical structures (Döhrmann et al., 2012; Kaiser, 2002).

Even though it is rather general, a commonly used definition of mathematics didactics in Norway is the one proposed by Gjone (1998). It stated that mathematics didactics is the theory and practice of teaching and learning mathematics. Central issues within mathematics didactics are related to questions of *what*, *why* and *how* (Gjone, 1998; Imsen, 2009).

Teacher education in Norway is required to prepare teachers for an inclusive school system (Braathe, 2010; Strømstad, 2003) in which many schools comprise both primary (1–7) and secondary (8–10) grades.

The different national Norwegian curricula have for several decades, according to Alseth, Breiteig and Brekke (2003) had a constructivist perspective on learning. And, since the 1997-curriculum teachers are obligated to teach on the basis of their students' individual background and competence (Alseth et al., 2003; Ministry of Education Research and Church Affairs, 1996).

These requirements might explain why some of the typical responses to the didactical *how* question of teaching and learning mathematics in Norway have included teaching methods such as diagnostic teaching (e.g. Brekke, 1995), teaching and learning through problem solving (e.g. Alseth, 1995) and dialogical inquiry or inquiry based teaching (e.g. Goos, 2004; Johnsen-Høines, 2009). Thus, mathematics didactics related to everyday teaching practice in Norway could be said to focus on knowledge related to individual students' conceptions and pre-conceptions and how to cognitively challenge and activate the students in order to construct mathematical knowledge.

Methodology

An online survey was used to address the Norwegian teachers' perception of the MPCK items used in the TEDS-M instruments. The online survey included a questionnaire and a test comprising a selection of TEDS-M items, mainly from the MPCK item pool.

The Norwegian researchers were invited to discuss the MPCK framework of TEDS-M before individually categorising all items included in the Norwegian booklets of the TEDS-M study.

Participants

Twenty-one in-service lower secondary mathematics teachers were sampled using a snowball or network sampling technique that entails recruiting teachers via one person in or connected to a network, or asking one teacher who asks the next one and so on (Blaikie, 2010).

All teachers were qualified to teach mathematics. The mean age of the teachers was 46 (min. 30; max. 64) and average years of teaching experience was 16.3 (min. 2; max 37).

In addition, four fellow Norwegian educational researchers from the TIMSS[1] and/or TEDS-M group were invited to join the author to discuss the TEDS-M framework and to categorise TEDS-M items. Although one researcher was working within physics education and the rest within mathematics education, all the researchers were familiar with the principles of classical categorisation. The researchers constituted a convenience or purposeful sample (Blaikie, 2010; Marshall, 1996).

Instrument

The online survey referred to in this paper was originally developed for and used in a study that aimed to investigate the relationship between teacher factors and students' achievement gain in mathematics (Kaarstein & Nortvedt, in preparation). One of the teacher factors, aside from the teachers' background, was defined as knowledge of teaching mathematics, particularly focusing on MPCK. To measure the teachers' MPCK, the online survey included a knowledge test utilising a collection of the TEDS-M MPCK items.

The test comprised 45 of the 59 MPCK items and 10 of the 150 MCK items used in the international TEDS-M survey. The 10 MCK items were included because they gave the mathematical scene or context for the MPCK items they preceded.

Even though the participating teachers in this study were teaching in lower secondary schools, the included MCK and MPCK items were selected from both the future primary and future secondary teacher item pools of TEDS-M (see table 2). Since Norwegian teachers are educated and expected to teach a wide range of grades and to teach according to their students' achievement level and competence (Ministry of Education and Research, 2003a; Ministry of Education Research and Church Affairs, 1996), including items from the future primary teacher item pool was not considered problematic.

The main reason for excluding the 14 remaining TEDS-M MPCK items was related to keeping the questionnaire and testing time within a period of two hours: Nine MPCK items designed and defined by the experts of TEDS-M to assess future primary school teachers were excluded due to similarity with included items designed to assess future secondary school teachers; Five MPCK items were excluded because the mathematical content of these items was found in the curriculum for upper secondary school in Norway.

Altogether, the 55 items in the online test encompassed 25 different contexts; hence, the items were re-labelled 1 through 25, with the addition of lower case letters (and sometimes also numbers) to identify each item within one context. The published TEDS-M items that were included in the online test are displayed in appendix A. The corresponding item IDs in TEDS-M are shown in appendix B.

Procedures

Teachers

In the online knowledge test, the participating teachers were asked to indicate whether they perceived an item or an item cluster[2] – hereafter called an *item unit* – mainly to require MCK or MPCK immediately after solving the tasks in one item unit. For example, among the items shown in appendix A, the teachers categorised items 23a and 23b as two units and items 20a–c as one unit. Radio buttons were used to prevent the teachers from selecting more than one knowledge category for an item unit.

No framework for teacher knowledge, definition of MCK and MPCK or categorisation criteria were provided, so the teachers categorised the items' knowledge requirements according to their own prototypes of the categories. To categorise according to a prototype view is to make a judgement

Table 2. *Number of TEDS-M items included in this study*

Retrieved from the item pool measuring	# of items designed to test		Total
	MCK	MPCK	
Primary school teachers	8	23	31
Secondary school teachers	2	22	24
Total	10	45	55

whether the object to be categorised is similar to the prototype or the central tendency of the category stored in the mind of the categoriser (Minda & Smith, 2011; Rosch, 1978). A prototype categorisation is thus dependent on the personal background and cultural settings within which the person or group of persons are categorising (Harnad, 2005).

Researchers

Fellow researchers were invited to join the author for a discussion of the TEDS-M framework, to reach a mutual agreement on categorisation criteria and then to categorise all items included in the Norwegian booklets of the TEDS-M study. The aim of the discussion was to come to an agreement on how the TEDS-M framework would be adjusted to match the group of Norwegian researchers' understanding of MCK and MPCK.

According to the classical theory of categorisation, a category is defined by a set of features (Smith & Medin, 1981), which means that categorising an object entails checking to determine whether the object matches the pre-set criteria. A TEDS-M item unit would hence be categorised by the researchers as MPCK if it matched the definition and categorisation criteria set for the MPCK category.

Using the adjusted framework and agreed-upon categorisation criteria, the researchers were given some items drawn from the "Item sample" appendix in the framework of TEDS-M (Tatto et al., 2008) to train for and practice item categorisation. Item categorisation disagreements in the training session were resolved before the researchers were given a couple of days to categorise individually all items used in the TEDS-M study. The researchers were not permitted to discuss their individual categorisation.

Analysis

To investigate the degree to which the researchers' classical item categorisation coincided with the teachers' prototype item categorisation, a comparison between the researchers' and the teachers' item unit categorisation was made.

Even though the teachers belong to the same national educational context, variations in item unit categorisation were expected, as they were asked to categorise the items according to a personal prototype understanding of MCK and MPCK. Hence, it was decided that if 16 (~80%) or more of the teachers categorised an item unit as, for example, MPCK, the item unit was counted as categorised as MPCK by the teachers. As for the researchers, having received categorisation training using the established categorisation criteria, agreement was understood to be all five researchers categorising the item unit as either MCK or MPCK.

To search for patterns of categorisation within the collection of TEDS-M MPCK item units, a systematic analysis of the relationships between item unit categorisation and item unit characteristics was done by applying Pearson's chi-square test for independence as described in Agresti (2007). In a Pearson's chi-square test, the objective is to test the null hypothesis that the variables included in a contingency table such as teachers' item unit categorisation by item format are independent by comparing the observed frequencies to estimated expected frequencies. Since the five researchers from the outset could be seen as a more homogenous group than the teachers, the search for patterns within the item unit categorization was done by applying the chi-square test to the teachers' item unit categorisation.

The MPCK item characteristics assigned by the TEDS-M expert group (i.e., the content domain, the curricular level, the sub-category of MPCK and the level that future teachers were expected to teach) and the author's abridged descriptions of item contexts and tasks were defined as item characteristics.

The abridged descriptions for item contexts were made for all MPCK item units embedded in a school context (i.e., including students/teachers/planning/enacting). Condensed versions of the item unit task were written for all item units. All item characteristics and attributes assigned to each MPCK item unit by the TEDS-M experts are included in appendix B. Abridged descriptions of item contexts and tasks are included in table 3.

Results and discussions

The researchers' categorisation criteria

Since the MCK framework of TEDS-M was built on the frameworks of TIMSS and all the Norwegian researchers were familiar with and accepted them, special attention was given to the MPCK framework of TEDS-M; see table 1.

During the discussion of the researchers, the sub-categories of MPCK (curriculum, planning and enacting) were recognised as reasonable and acceptable. Teachers need to utilise MPCK when preparing to teach as well as when enacting. Furthermore, planning and enacting depend on curricular knowledge, as teachers need to make plans and decisions according to a curriculum.

Accepting the TEDS-M sub-categories of MPCK, the researchers continued to discuss whether the sample topics provided for each category (i.e., the examples of core situations of teaching listed in table 1) could make up a set of categorisation criteria for the MPCK category.

Agreeing with the experts of TEDS-M that MPCK builds or adds on to MCK, the Norwegian researchers judged all but one of the MPCK sample topics to require more than MCK. The one sample topic that the researchers were not prepared to accept as a situation in which one is expected to draw on more than MCK was "analysing or evaluating students' mathematical solutions or arguments" (cf. table 1). The group of Norwegian researchers decided that analysing or evaluating a mathematical solution or argument regardless of its being a student's solution requires only mathematical knowledge. Hence, the sample topic was judged to count as a categorisation criterion for MCK. This understanding corresponds to the definition of the MCK category *specialized content knowledge* suggested by Ball et al. (2008), including mathematical knowledge unique to teaching yet not related to knowledge of students, planning, enacting, curriculum or pedagogy.

Categorising the selection of items – a Norwegian understanding?

The focus of this paper is on MPCK items, and since the teachers and the researchers categorised the seven MCK item units (labelled 3a, 4a, 6a1–4, 7a, 13a, 23a and 25a) in full accordance with the TEDS-M experts, these items are not included in the analysis.

Table 3 presents an overview of the 26 included item units defined in TEDS-M to measure MPCK. The item units are ranked according to increasing numbers of teachers categorising the item units in agreement with the TEDS-M experts (i.e., as MPCK). In addition, table 3 includes the item units' *context and/or task*, which is the author's aforementioned abridged description of the item unit context and task, and the *item format*, which is either multiple-choice (MC), complex multiple-choice (CMC) or constructed response (CR). CR items could also be referred to as open items. The number of researchers categorising the item units as MPCK is included in the rightmost column of table 3. Take, for example, the fourth row in table 3: the item unit labelled 20a–c has a complex multiple-choice format and the item context describes a situation in which *students are asked to prove a statement* and *three students' proofs are shown*. The item task then asks the respondents to *determine whether the students' proofs are valid* (see appendix A for the actual item). Item unit 20a–c was categorised as MPCK by one of the teachers and none of the researchers.[3]

Categorising according to the classical theory of categorisation, as the researchers were, is most likely the reason that they were more consistent in their categorisation than the teachers. As listed in table 3, eleven of the 26 MPCK item units were unanimously categorised by the researchers as MCK and seven units were unanimously categorised as MPCK, 18 units in all. However, allowing an item unit to be categorised "unanimously" if 16 teachers or more agreed, the teachers also categorised 18 item units unanimously: 12 item units were categorised as MCK and six units were categorised as MPCK.

Furthermore, as shown in table 3, the result of the researchers' item unit categorisation coincides with the result of the teachers' item categorisation to a relatively high degree. The teachers and the researchers agreed on categorisation for 14 of the 26 item units; five item units were categorised as MPCK and nine units were categorised as MCK. The remaining 12 item units were to various degrees perceived to require both MCK and MPCK.

The overall impression is that a relatively low number of the TEDS-M MPCK item units are perceived by the Norwegian teachers and researchers to require MPCK. In contrast, a relatively high number of item units are perceived to require MCK.

The researchers' and teachers' item unit categorisation gives an overall impression of agreement between the two groups. This agreement implies that there might be a Norwegian understanding of MPCK; it at least implies that the operationalisations of the MPCK categories are perceived to require more MCK than the TEDS-M experts intended.

Emerging patterns

To investigate the relationship between the item unit categorisation and item unit characteristics summarised in table 4 further, Pearson's chi-square tests of independence as described in the analysis section were applied, starting with the relationship between teachers' item unit categorisation and the *item context and/or task*.

However, performing a chi-square test using the 26 different item contexts and/or tasks as 26 different categories to relate to teachers' item unit categorisation was considered impractical. Instead, the author

Table 3. *Item unit format, item context and/or task and teachers' and researchers' categorisation*

Item unit	Item format	Item context and/or task	#T	#R
10a,b	CMC	Indicate type of growth for sequence	1	0
11a–d	CMC	Indicate whether sufficient justification for equivalence of statement is given	1	0
14	MC	Teacher provides students with problems; find mathematical idea highlighted in the collection of problems	1	3
20a–c*	CMC	Students asked to prove a statement, 3 students' proofs shown; determine whether students' proofs are valid	1	0
9	MC	A teacher is preparing to teach; find the equation or inequality not representable by a scale pan	2	0
15	MC	Teacher asks 1 student to describe her method; find true statement about student's method	2	1
19	MC	Examining students, 1 student's solution shown; find most appropriate statement about student's solution	2	0
2*	MC	1 student's statement shown; represent student's statement using algebraic notation	3	0
22	MC	Students given a problem, 2 students' solutions shown; indicate the most appropriate of the two students' solutions	3	0
8a–d	CMC	4 students' solutions shown; decide whether the indicated student strategies are correct	4	0
16a–c	CMC	Given a specific change in curriculum, decide whether topics still could be taught to lower secondary students	4	1
21a–d*	CMC	Teacher wanting to prove a formula for a group of students; decide whether stated knowledge is needed to understand the proof	4	0
18a–d	CMC	Teachers wanting to tie meaningful story-problems to fraction division; decide whether 4 different story-problems adequately represent the fraction division displayed	6	2
24	MC	Teacher wanting to show geometric property for a group of students; decide most helpful knowledge to understand teacher's demonstration	6	3
1a	CR	1 student's solution shown; describe what you think the student might have been thinking	7	5
5	MC	3 students' solutions are shown; decide whether students' methods could be used in any similar problems (i.e., check generalisability of method)	8	0
12	CR	Teacher presenting problem to students, 1 student's solution shown; describe student's misconception	8	5
6b	CR	List 2 key mathematical skills students need to have to construct a diagram	13	3
17*	CR	Give a reason that might account for the students' experienced difference in difficulty level of the two mathematical problems shown	13	4
25b1–3	CMC	3 students' solutions to problem solved in item 25a; evaluate and assign points to each student's answer	13	0
4b*	CR	Suggest easier problem involving same processes/operations as in item 4a	18	5
13b	CR	Suggest a question to help student introduced in the item text prior to question in 13a to improve her statement	18	5
23b*	CR	1 student's solution shown; describe what you think the student might have been thinking	18	4
3b	CR	1 student having difficulty with notation in item 3a; reword question to assess same understanding but more accessible for student	19	5
7b*	CR	Problem posed to students; rewrite question of item 7a without changing the skills required to solve the problem	19	5
1b	CR	Give examples of teaching practice to reduce the misconception of student in item 1a	20	5

Notes. * Published items; see appendix A. #T = number of teachers and #R = number of researchers categorising the item unit as MPCK.

categorised the 26 item units on the basis of the researchers' discussion and the description of mathematics didactics in Norway.

Defining reasonable item context categories to analyse is, according to Döhrmann et al. (2012), considered futile since it must be decided from case to case whether an item set in a school context is to be categorised as MCK or MPCK. The categorisation depends on whether the "correct solution of the item merely requires mathematical knowledge" (Ibid., p. 337). Therefore, it was decided to define categories on the basis of the item units' tasks.

Five categories were defined. The first category (I) relates to the sample topic that the Norwegian researchers judged to be a categorisation criterion for the MCK category rather than MPCK. Item units included in this category have tasks instructing the respondents to *justify*, *validate or check for correctness/appropriateness* of students' solutions or methods. Category II reflects important aspects of diagnostic teaching that are included in the sample topics for the sub-categories of planning and enacting in the TEDS-M MPCK framework: item unit tasks in category II require the respondents to have *knowledge about student thinking and misconceptions*. The third item task category included item units where the respondents were asked to *reword/rewrite* mathematical problems for students or *suggest questions to guide student thinking*. Category III tasks hence require knowing how to differentiate according to students' individual achievement level or competence. Category IV tasks require knowledge about different *mathematical representations*, *mathematical ideas and skills* needed to solve or understand a problem, knowledge needed for instance to initiate and guide discussions in the classroom as is the purpose of inquiry based teaching (Goos, 2004). The last category (V) included only one task (item unit 10a,b), which did not fit into any of the previous categories.

Table 4 shows an overview of the item task categories I–V. The table further shows which item units the author included in the different categories, and finally, it shows the observed and expected distribution of teachers' item unit categorisation crossed with item unit task categories.

According to the chi-square statistics shown in table 4 ($\chi^2 = 147.0$, df = 4, $p < .0001$), the variables are dependent. Comparing the observed frequencies included in columns three and four in table 4 to the expected frequencies included in the parenthesis in the same columns, it can be stated that categories II and III are associated with MPCK and that categories I, IV and V are associated with MCK.

It should be noted that even though table 4 shows an association between, for example, category I and MCK, it can only be inferred that category I item units were categorised as MCK more often than expected and as MPCK less than expected.

Table 4. *Teachers' item unit categorisation by item task categories*

Item unit task include instructions to	Item units included in category**	Frequency of teachers' item unit categorisation		Total
		MCK	MPCK	
Category I: justify, validate proofs/solutions, decide correctness/appropriateness of method/strategy	**5, 8a–d, 11a–d, 15, 19, 20a–c, 22, 25b1–3**	134 (101.8)*	34 (66.2)	168
Category II: describe/know student thinking, misconceptions; translate students' solutions	1a, 1b, **2**, 12, 17, 23b	56 (76.4)	70 (49.6)	126
Category III: reword or rewrite a question; write a question to guide student to improve	3b, 4b, 7b, 13b	10 (50.9)	74 (33.1)	84
Category IV: find different representations; list prior knowledge and skills needed, mathematical ideas	6b, **9, 14, 16a–c, 18a–d, 21a–d, 24**	111 (89.1)	36 (57.9)	147
Category V: indicate type of growth for sequence	**10a,b**	20 (12,7)	1 (8.3)	21
Total		331	215	546

Notes. *Estimated expected frequencies for hypothesis of independence. **Item units having a multiple-choice format are highlighted; non-highlighted item units have a constructed response format.

Highlighting the item units with a multiple-choice format, table 4 also shows that all but one item unit included in the MPCK associated categories II and III have a constructed response format and that all but one item unit included in the MCK associated categories I, IV and V have multiple-choice format. Thus, activities such as validating mathematical proofs, indicating the correctness of solutions, checking whether methods are generalisable and judging the appropriateness of mathematical solutions were more often perceived to require MCK rather than MPCK.

The constructed response item units 1b, 3b, 4b, 7b, 13b and 23b (in categories II and III) were all preceded by item units categorised unanimously by the TEDS-M experts, the Norwegian teachers and researchers as MCK. These units add to the mathematical context of the preceding item units; thus, the strong connection between MCK and MPCK in the school context might be disconnected. Consequently, mathematical problem solving activities are seemingly less prominent in part b of these contexts. The focus has shifted from MCK to knowledge of students' previous knowledge and misconceptions, what students find easy or hard and how to guide students. All of these activities are closely linked to planning and teaching mathematics. Moreover, some activities also resemble elements described and included in diagnostic teaching and are thus understood as a part of mathematics didactics in Norway.

Categorisation, of course, depends on a complex mixture of all item characteristics and the (educational cultural) background of the categorisers, and considering all combinations of teachers' item unit categorisation and item characteristics would make the paper too long. However, before moving on to the concluding remarks, table 5 is included to show the results of all the chi-square tests performed.

Table 5. *Association between teachers' item unit categorisation and item unit characteristics, including chi-square statistics*

Item characteristic	Association		Chi-square statistics		
			χ^2	df	*p*
Item unit format	Multiple-choice* Constructed response	MCK MPCK	158.2	1	< .0001
Content domain	Algebra Number Geometry Data	MCK MCK MPCK MPCK	46.8	3	< .0001
Item unit task	Category I Category II Category III Category IV Category V	MCK MPCK MPCK MCK MCK	147.0	4	<.0001
Curricular level	Novice Intermediate Advanced	MPCK MPCK MCK	33.2	2	< .0001
Sub-category of MPCK	Curriculum Planning Enacting	MCK MPCK MCK	89.8	2	< .0001
Level expected to teach	Primary Secondary	MPCK MCK	4.9	1	= .0266

Note. *The multiple-choice variable includes both complex multiple-choice and multiple-choice format.

Concluding remarks

The analysis performed in this paper indicates that the 21 participating Norwegian teachers' prototypical categorisation of the 26 selected TEDS-M MPCK item units is related to item unit characteristics such as item format and item task. The overall impression is that many items were not understood to require MPCK as defined by the TEDS-M group and that the Norwegian teachers' perception of knowledge requirements was by and large supported by the five educational researchers' classical categorisation, indicating a common Norwegian understanding of MCK and MPCK.

However, the combination of chosen sampling techniques and the low number of participants in this study make generalisation problematic. Thus, recruiting more participants, including primary and upper secondary school teachers and mathematics teacher educators, would have contributed to a more accurate picture of how the MPCK item units are perceived in Norway.

The emerging patterns of the teachers' and researchers' categorisation indicate that item units with a constructed response (open) format – including questions requiring knowledge about student thinking, misconceptions, how to improve or vary teaching, or how to guide students – were perceived as MPCK. On the other hand, when the item units with a multiple-choice format required knowledge related to different mathematical representations, knowing what kind of mathematical knowledge and skills students need to understand mathematical concepts or procedures or evaluating students' mathematical solutions (i.e., checking correctness or validating statements or proofs), the majority of the teachers and researchers categorised them as MCK rather than MPCK.

Fourteen of the 26 included item units in this paper were operationalisations of the TEDS-M MPCK sub-category of enacting, and according to Döhrmann et al. (2012), TEDS-M items measuring enacting were predominantly operationalisations of the sample topic "analysing and evaluating students' mathematical solutions or arguments", which is the very sample topic that Norwegian researchers judged to be a categorisation criterion for MCK, arguing that this is pure mathematical knowledge needed only when teaching mathematics.

The item task categories I and II (see table 4) included 13 of the 14 enacting item units. Category I included item tasks related to the evaluation of students' mathematical solutions, whilst category II included item tasks related to the analysis of students' solutions or arguments, associated with MCK and MPCK, respectively. One might ask if the reason for this emerging pattern could be related to the Norwegian teachers' and researchers' understanding of the *evaluating* and *analysing* concepts.

Had the Norwegian researchers accepted *all* MPCK sample topics suggested by TEDS-M as categorisation criteria (see table 1), the resemblance between the categorisation of the Norwegian teachers and researchers most likely would not have been as high as reported here. On the other hand, had the TEDS-M experts agreed with the Norwegian researchers' decision to move the sample topic about analysing and evaluating students' mathematical solutions or arguments from the MPCK framework to MCK, the overall agreement between all groups might have been very high.

Hence, one suggestion of this paper is that more attention to the placement of the sample topic "analysing and evaluating students' mathematical solutions or arguments" is needed. Does this particular sample topic show a typical teaching situation requiring MCK, MPCK or both? Would it help if this one sample topic were to be divided into two new topic samples in which *analysing* is the focus of one and *evaluating* is the focus of the other?

Furthermore, regarding the pattern indicating that multiple-choice format items require MCK while constructed response format items require MPCK, other questions may be raised. For instance, might the multiple-choice format constrain the way tasks or problems are posed? Is it possible that adequate multiple-choice item distractors are more difficult to design for MPCK items?

Additionally, when having an MCK item unit set the mathematical scene for the following MPCK item unit included in the same item context (e.g., item 23 in appendix A), the teachers categorised to a much higher degree in agreement with the TEDS-M experts. This might suggest that MPCK items should follow an MCK item to loosen the tight linkage between MCK and MPCK.

Even though commendable efforts have been made by all the contributors to and experts involved in the TEDS-M study to define and measure future teachers' mathematical content knowledge and mathematics pedagogical content knowledge, the operationalisation of the MPCK category in particular is in need of further research.

A final remark about possible implications of the results reported here could be directed towards mathematics teacher education in Norway. The present teacher education curricula state that future teachers should know and draw on national and *international* research related to the teaching profession (Ministry of Education and Research, 2003b, 2010). Thus, possible cultural differences related to MPCK should be made explicit for the teacher students.

References

Agresti, A. (2007). *An introduction to categorical data analysis*. Hoboken: Wiley-Interscience.

Alseth, B. (1995). Undervisning i problemløsningsstrategier [Teaching problem solving strategies]. *Nordic Studies in Mathematics Education*, 3 (3), 7–26.

Alseth, B., Breiteig, T. & Brekke, G. (2003). *Evaluering av Reform 97. Endringer og utvikling ved R97 som bakgrunn for videre planlegging og justering – matematikkfaget som kasus* [Evaluating Reform 97. Changes and developments in R97 as the basis for further adjustments – the case of school mathematics]. Notodden: Telemark Research Institute.
Ball, D. L. & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83–104). Westport: Ablex.
Ball, D. L., Thames, M. H. & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59 (5), 389–407.
Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., et al. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, 47 (1), 133–180.
Blaikie, N. (2010). *Designing social research. The logic of anticipation* (2nd ed.). Cambridge: Polity Press.
Braathe, H. J. (2010, March). *Dilemmas of streaming in the new curricula in Norway*. Paper presented at the Sixth International Mathematics Education and Society Conference, Berlin, Germany.
Brekke, G. (1995). *Introduksjon til diagnostisk undervisning i matematikk* [Introduction to diagnostic teaching in mathematics]. Oslo: Centre for teacher education and school service, University of Oslo and Telemark Research Institute, Notodden.
Döhrmann, M., Kaiser, G. & Blömeke, S. (2012). The conceptualisation of mathematics competencies in the international teacher education study TEDS-M. *ZDM*, 44 (3), 325–340. doi: 10.1007/s11858-012-0432-z
Gjone, G. (1998). Innføring i matematikkdidaktikk [Introduction to mathematics didactics]. In G. Tufteland (Ed.), *Matematikk I. For allmennlærerutdanningen* [Mathematics I. For the generalist teacher education] (pp. 82–102). Oslo: Universitetsforlaget.
Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education*, 35 (4), 258–291. doi: 10.2307/30034810
Grønmo, L. S. & Onstad, T. (2012a). Hva er TEDS-M 2008? [What is TEDS-M 2008?]. In L. S. Grønmo & T. Onstad (Eds.), *Mange og store utfordringer. Et nasjonalt perspektiv på utdanning av lærere i matematikk basert på data fra TEDS-M 2008* [Many major challenges. A national and international perspective on mathematics teacher education based on the data from TEDS-M 2008] (pp. 9–16). Oslo: Unipub.

Grønmo, L. S. & Onstad, T. (Eds.). (2012b). *Mange og store utfordringer. Et nasjonalt og internasjonalt perspektiv på utdanning av lærere i matematikk basert på data fra TEDS-M 2008* [Many major challenges. A national and international perspective on mathematics teacher education based on the data from TEDS-M 2008]. Oslo: Unipub.

Harnad, S. (2005). To cognize it to categorize: congnition is categorization. In H. Cohen & C. Lefebvre (Eds.), *Handbook of categorization in cognitive science* (pp. 19–44). Amsterdam: Elsevier Ltd.

Imsen, G. (2009). *Lærerens verden: innføring i generell didaktikk* [The teacher's world: introduction to didactics] (4 ed.). Oslo: Universitetsforlaget.

Johnsen-Høines, M. (2009). Dialogical inquiry in practice teaching. *Nordic Studies in Mathematics Education*, 14 (1), 39–59.

Kaarstein, H. & Nortvedt, G. (in preparation). Experiences from developing a tool to investigate relationships between teacher factors and student achievement gain in mathematics.

Kaiser, G. (2002). Educational philosophies and their influence on mathematics education – An ethnographic study in English and German mathematics classrooms. *Zentralblatt für Didaktik der Mathematik*, 34 (6), 241–257. doi: 10.1007/BF02655723

Kunter, M., Baumert, J., Blum, W., Klusmann, U., Krauss, S. & Neubrand, M. (Eds.). (2013). *Cognitive activation in the mathematics classroom and professional competence of teachers. Results from the COACTIV Project*. Dordrecht: Springer.

Marshall, M. N. (1996). Sampling for qualitative research. Family Practice, 13(6), 522–526. doi: 10.1093/fampra/13.6.522

Minda, J. P. & Smith, J. D. (2011). Prototype models of categorization: basic formulation, predictions, and limitations. In E. M. Pothos & A. J. Wills (Eds.), *Formal approaches in categorization*. Cambridge University Press.

Ministry of Education and Research (2003a). *Rammeplan for allmennlærerutdanningen* [National curriculum regulagtions for teacher education program for grades 1–10]. Retrievedd from http://www.regjeringen.no/upload/KD/Vedlegg/UH/Rammeplaner/Lærer/Rammeplan_2003_allmennlaererutd.pdf

Ministry of Education and Research (2003b). *Rammeplan for praktisk-pedagogisk utdanning* [National curriculum regulations for practical-pedagogical education]. Retrieved from http://www.regjeringen.no/upload/kilde/kd/pla/2006/0002/ddd/pdfv/175796-2rammeplan_2003_ppu.pdf

Ministry of Education and Research (2010). *National curriculum regulations for differentiated primary and lower secondary teacher education programmes for years 1–7 and years 5–10*. Retrieved from http://www.regjeringen.no/upload/KD/Vedlegg/UH/forskrifter/National_Curriculum_Differentiated_Teacher_Education.pdf

Ministry of Education Research and Church Affairs. (1996). *Læreplanverket for den 10-årige grunnskolen* [Curriculum for the 10-year compulsory school]. Oslo: Nasjonalt læremiddelsenter.

Mullis, I. V. S. & Martin, M. O. (Eds.). (2013). *TIMSS 2015 Assessment frameworks*. Chestnut Hill: TIMSS & PIRLS International Study Center.

Onstad, T. & Grønmo, L. S. (2012). Rammeverk og metoder [Framework and methods]. In L. S. Grønmo & T. Onstad (Eds.), *Mange og store utfordringer. Et nasjonalt og internasjonalt perspektiv på utdanning av lærere i matematikk basert på data fra TEDS-M 2008* [Many major challenges. A national and international perspective on mathematics teacher education based on the data from TEDS-M 2008] (pp. 197–225). Oslo: Unipub.

Rosch, E. (1978). Principles of categorization. In E. Rosch & B. B. Lloyd (Eds.), *Cognition and categorization* (pp. 27–48). Hillsdale: Lawrence Erlbaum.

Schmidt, W. H., Blömeke, S. & Tatto, M. T. (2011). *Teacher education matters. A study of middle school mathematics teacher preparation in six countries*. New York: Teachers College Press.

Sjøberg, S. (2001). Innledning: skole, kunnskap og fag [Introduction: school, knowledge and subject]. In S. Sjøberg (Ed.), *Fagdebatikk* [Debating subject didactics] (pp. 11–48). Oslo: Gyldendal.

Smith, E. E. & Medin, D. L. (1981). *Categories and concepts*. Cambridge: Harvard University Press.

Strømstad, M. (2003). 'They believe that they participate ... but': democracy and inclusion in Norwegian schools. In J. Allan (Ed.), *Inclusion, participation and democracy: What is the purpose?* Dordrecht: Kluwer Academic Publishers.

Tatto, M. T., Schwille, J., Senk, S., Ingvarson, L., Peck, R. & Rowley, G. (2008). *Teacher education and development study in mathematics (TEDS-M): Policy, practice, and readiness to teach primary and secondary mathematics. Conceptual framework*. East Lansing: Michigan State University.

Tatto, M. T., Schwille, J., Senk, S., Ingvarson, L., Rowley, G. et al. (2012). *Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: findings from The IEA teacher education and development study in mathematics (TEDS-M)*. Amsterdam: International Association for the Evaluation of Educational Achievement (IEA).

Tatto, M. T., Senk, S., Bankov, K., Rodriguez, M. & Peck, R. (2010). *TEDS-M 2008 assessment frameworks: measuring future primary and secondary teachers mathematics and mathematics pedagogy knowledge*. Retrieved from http://teds.educ.msu.edu/reports/

Notes

1 Trends in International Mathematics and Science Study (see e.g. http://timssandpirls.bc.edu or Mullis & Martin, 2013).

2 Item clusters are found in all the MPCK items with a complex multiple-choice (CMC) format. Item 20 has a CMC format, which means that the item has one stem followed by a list of similar tasks (here labelled a through c; see appendix A). As all tasks in the CMC items of TEDS-M were scored separately yet categorised by the TEDS-M experts as operationalisations of the same category, it was decided to have the teachers categorise all CMC items in the reported study as separate units.

3 The teachers' and the researchers' item unit by item unit categorisation can be seen in appendix C.

Hege Kaarstein

Hege Kaarstein was a member of the Norwegian TEDS-M group and she is currently working as a researcher for the Norwegian TIMSS 2015 group at the Department of Teacher Education and School Research, University of Oslo. The focus of her research is measurement of mathematical knowledge for teaching.

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Appendix A. Published TEDS-M items

[Amy] is building a sequence of geometric figures with toothpicks by following the pattern shown below. Each new figure has one extra triangle.

Variable t denotes the position of a figure in the sequence.

In finding a mathematical description of the pattern, [Amy] explains her thinking by saying:

I use three sticks for each triangle.

Then I see that I am counting one stick twice for each triangle, except the last one, so I have to remove those.

Variable n represents the total number of toothpicks used in a figure.
Which of the equations below best represent [Amy's] statement in algebraic notation?

Check <u>one</u> box.

A. $n = 2t + 1$ □
B. $n = 2(t + 1) - 1$ □
C. $n = 3t - (t - 1)$ □
D. $n = 3t + 1 - t$ □

Item 2
(MFC 108 in Brese & Tatto, 2012, p. 7). Categorized as MPCK by TEDS-M.

(a) A machine uses 2.4 litres of fuel for every 30 hours of operation.
How many litres of fuel will the machine use in 100 hours if it continues to use fuel at the same rate

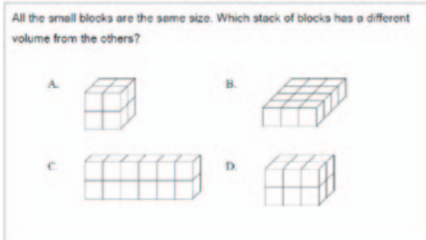
Check <u>one</u> box.

A. 7.2 □
B. 8.0 □
C. 8.4 □
D. 9.6 □

(b) Create a different problem of the same type as the problem in (a) (same processes/ operations) that is EASIER for <primary> children to solve.

Item 4a and 4b
(MFC 206 a,b in Brese & Tatto, 2012, p. 11). 4a was categorized MCK, 4b MPCK.

The following problem was given to <primary school> children.



(a) What is the correct answer to this question?

Check one box.

A. Stack A □
B. Stack B □
C. Stack C □
D. Stack D □

(b) How could the question above be rewritten so that it assesses the same skills but **WITHOUT** using the word **VOLUME**?

Item 7a and 7b

(MFC 307 a,b in Brese & Tatto, 2012, p. 19). 7a was categorized MCK, 7b MPCK.

If B represents the weight (in grams) of each box, □, pictured below, and ⏢ represents a one-gram weight, the equation $3B + 4 = 10$ can be pictured by the pan balance shown below.

An inequality such as $3B + 4 < 10$ or $3B + 4 > 10$ would show one side of the pan balance lower than the other.

Ms [Clarke] is preparing to teach a unit on solving linear equations and inequalities.

If X represents the weight of a given box, which of the following sentences can **NOT BE REPRESENTED** by a pan balance?

Check one box.

A. $13 = 4X + 5$ □
B. $3X + 10 = 4$ □
C. $3X + 3 = 2X + 15$ □
D. $9 + 6X < 21$ □

Item 9

(MFC 312 in Brese & Tatto, 2012, p. 24). Categorized as MPCK.

The following problems appear in a mathematics textbook for <lower secondary school>.

1. [Peter], [David], and [James] play a game with marbles. They have 198 marbles altogether. [Peter] has 6 times as many marbles as [David], and [James] has 2 times as many marbles as [David]. How many marbles does each boy have?

2. Three children [Wendy], [Joyce] and [Gabriela] have 198 zeds altogether. [Wendy] has 6 times as much money as [Joyce], and 3 times as much as [Gabriela]. How many zeds does each child have?

(a) Solve each problem.

(b) Typically Problem 2 is more difficult than Problem 1 for <lower secondary> students. Give one reason that might account for the difference in difficulty level.

Item 17
(MFC 604 a,b in Brese & Tatto, 2012, p. 64 and 67). 17a was included in the online knowledge test without asking the teachers to solve or categorise the problems. In TEDS-M, 17a was categorized as MCK and 17b as MPCK.

Some <lower secondary school> students were asked to prove the following statement:

When you multiply 3 consecutive natural numbers, the product is a multiple of 6.

Below are three responses.

[Kate's] answer

A multiple of 6 must have factors of 3 and 2.
If you have three consecutive numbers, one will be a multiple of 3.

Also, at least one number will be even and all even numbers are multiples of 2.

If you multiply the three consecutive numbers together the answer must have at least one factor of 3 and one factor of 2.

[Leon's] answer

$1 \times 2 \times 3 = 6$

$2 \times 3 \times 4 = 24 = 6 \times 4$

$4 \times 5 \times 6 = 120 = 6 \times 20$

$6 \times 7 \times 8 = 336 = 6 \times 56$

[Maria's] answer

n is any whole number

$n \times (n+1) \times (n+2) = (n^2 + n) \times (n+2)$

$= n^3 + n^2 + 2n^2 + 2n$

Canceling the n's gives $1 + 1 + 2 + 2 = 6$

Determine whether each proof is valid.

Check one box in each row.

		Valid	Not valid
A.	[Kate's] proof	☐	☐
B.	[Leon's] proof	☐	☐
C.	[Maria's] proof	☐	☐

Item 20 a-c
(MFC709 a-c in Brese & Tatto, 2012, p. 75). Categorized as MPCK.

A mathematics teacher wants to show some <lower secondary school> students how to prove the quadratic formula.

Determine whether each of the following types of knowledge is needed in order to understand a proof of this result.

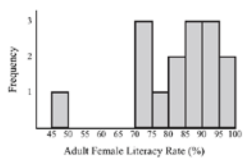
Check one box in each row.

	Needed	**Not needed**
A. How to solve linear equations.	□	□
B. How to solve equations of the form $x^2 = k$, where $k > 0$.	□	□
C. How to complete the square of a trinomial.	□	□
D. How to add and subtract complex numbers.	□	□

Item 21 a-d

(MFC 712 a-d in Brese & Tatto, 2012, p. 79). Categorized as MPCK.

The following graph gives information about the adult female literacy rates in Central and South American countries.



Suppose you ask your students to tell you how many countries are represented in the graph. One student says, "There are 7 countries represented."

Check one box.

	Right	**Wrong**
(a) Is the student right or wrong?	o	o

(b) In your opinion, what was the student thinking in order to arrive at that conclusion?

Item 23a and 23b

(MFC 806 a,b in Brese & Tatto, 2012, p. 82). 23a was categorized as MCK, 23b MPCK.

Reference

Brese, F. & Tatto, M. T. (2012). *TEDS-M 2008. User guide for the international database. Supplement 4*. Amsterdam: IEA Secretariat.

Appendix B. TEDS-M item ID and attributes

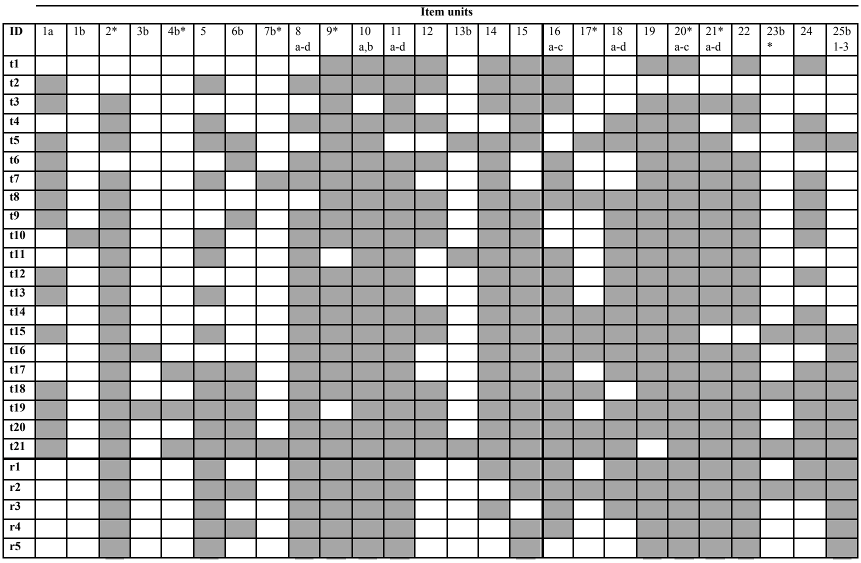
Item unit label	Item ID in TEDS-M	Item attributes assigned by TEDS-M experts				
		Content domain	IF**	SC**	CL**	L**
1a	MFC 105a	Data	CR	E	I	p
1b	MFC 105b	Geometry	CR	P	I	p
2*	MFC 108	Algebra	MC	E	I	p
3b	MFC 201b	Data	CR	P	I	p
4b*	MFC 206b	Number	CR	P	I	p
5	MFC 210	Number	MC	E	I	p
6b	MFC 302b	Data	CR	C	I	p
7b*	MFC 307b	Geometry	CR	P	N	p
8a-d	MFC 311a-d	Geometry	CMC	E	I	p
9*	MFC 312	Algebra	MC	C	A	p
10a, b	MFC 405a, b	Algebra	CMC	C	A	p
11a-d	MFC 406a-d	Algebra	CMC	E	A	p
12	MFC 409	Data	CR	E	I	p
13b	MFC 413b	Geometry	CR	E	I	p
14	MFC 506	Number	MC	C	I	p
15	MFC 512	Number	MC	E	I	p
16a-c	MFC 603a-c	Algebra	CMC	C	I	s
17*	MFC 604b	Algebra	CR	E	N	s
18a-d	MFC 611a-d	Number	CMC	P	N	s
19	MFC 702	Geometry	MC	E	N	s
20a-c*	MFC 709a-c	Number	CMC	E	N	s
21a-d*	MFC 712a-d	Algebra	CMC	C	I	s
22	MFC 715	Data	MC	E	I	s
23b*	MFC 806b	Data	CR	E	N	s
24	MFC 810	Geometry	MC	C	N	s
25b, 1-3	MFC 812b 1, 3, 4	Algebra	CMC	E	N	s

Notes. * Published items (see appendix A).

**

IF=Item format	SC= Sub-category of MPCK	CL= Curricular level	L=Level future teacher was expected to teach
MC= multiple-choice	C=curriculum	N=novice	p=primary school
CMC= complex MC	P=planning	I=intermediate	s=secondary school
CR=constructed response	E=enacting	A=advanced	

Appendix C: Teachers' and researchers' categorisation of the MPCK item units



Notes. *Published item units, see appendix A.
Teachers' ID are labelled t1-t21 and the researchers' ID are labelled r1-r5. Item units 1-15 were designed to measure future primary school teachers whereas item units 16-25 were designed to measure future secondary school teachers. One cell in the table represents one person's categorization of the item unit indicated in the column heading above. A white cell in the table represents an item unit categorized as MPCK. A grey cell represents an item unit categorized as MCK.