Exploring the mathematical knowledge of prospective elementary teachers in Iceland using the MKT measures

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This article reports findings from a study carried out with prospective teachers at the University of Iceland. The study explores the mathematical content knowledge of participants, with a special focus on the understanding of numbers, operations, patterns, functions, and algebra. The mathematical knowledge is measured with interviews and a survey, translated and adapted from the MKT measures designed by Ball and the research team at the University of Michigan. The findings indicate that prospective teachers' knowledge is procedural and related to the "standard algorithms" they learned and used in elementary school. Findings also indicate that prospective teachers have difficulty evaluating alternative solution methods, and working with and understanding fractions.

Elementary school teachers play an important role in mathematics education where they provide the foundation of computation and mathematical reasoning. Teachers' understanding of mathematics must be solid if mathematics is to be meaningful to students and for the teaching to be effective. Effective teaching requires an understanding of the underlying meaning of concepts and procedures, as well as justifications for the ideas and procedures presented and the ability to make connections between topics (Ball et al., 2005; Oakes & Lipton, 2002). Teachers' understanding of mathematics therefore plays an important role in the real mathematical thinking that occurs in the classroom and it has been argued that mathematical content knowledge is related to the mathematical quality of teachers' instructions as well as their teaching style (Baumert et al., 2010; Charalambous, 2010; Hill & Ball, 2009; Hill et al., 2008; Shulman, 1987; Ma, 1999).

For at least two centuries, teachers' knowledge has been measured and assessed in different ways. The main purpose of these assessments has

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been the certification process, to evaluate individual teacher's knowledge or performance (Hill, Sleep, Lewis & Ball, 2007). However, the notion that teachers needed some special kind of mathematical knowledge has been on the rise for the past 25 years (Hill et al., 2007).

In 1986, Shulman introduced the term "pedagogical content knowledge", as the special kind of knowledge teachers needed. He described it as:

> A second kind of content knowledge is pedagogical knowledge, which goes beyond knowledge of the subject matter per se to the dimension of subject matter for teaching [...]. Within the category of pedagogical content knowledge I include for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstration – in a word, the ways of representing and formulation the subject that make it comprehensible to others. (Shulman, 1986, p. 9)

Since Shulman, educational research has often distinguished between three types of teachers' knowledge: content knowledge, pedagogical content knowledge and basic pedagogical knowledge. Both types of content knowledge are thought to attribute to the quality of teaching, and affect students' learning and motivational development. Studies have indicated that content knowledge alone is not sufficient to guarantee quality teaching, but they also have implied that teachers lacking in mathematics content knowledge are less equipped to explain and represent topics in such ways that makes sense to the students. This lack of conceptual understanding cannot be compensated with general pedagogical skills. Pedagogical content knowledge cannot exist without content knowledge, but pedagogical content knowledge is needed to facilitate learning (Baumert et al., 2010).

Mathematical knowledge for teaching (MKT)

Deborah Ball and her colleagues at the University of Michigan have been working on a practice-based theory of the mathematical knowledge needed for teaching based on Shulman's concept of pedagogical content knowledge (Delaney et al., 2008; Hill, Schilling & Ball, 2004; Ball, Thames & Phelps, 2008). They seek to understand the mathematical knowledge teachers' use in the classroom, in order to map and measure it (Hill, Ball & Schilling, 2008; Hill et al., 2004). Their research led to the development of the *Mathematical knowledge for teaching* (MKT) measures. The purpose of the measures is to research the nature and role of mathematical knowledge for teaching, which is defined as: "the mathematical knowledge needed to carry out the *work of teaching mathematics*" (Hill, Rowan & Ball, 2005, p. 373). Teaching is described as "everything that teachers must do to support the learning of their students" (Ball et al., 2008, p. 395).

> Examples of this "work of teaching" include explaining terms and concepts to students, interpreting students' statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and providing students with examples of mathematical concepts, algorithms, or proofs. (Hill et al., 2005, p. 373).

Since elementary teachers teach elementary mathematics, the developers of the MKT measures considered material used in K–12 classrooms when creating items. The items aim to reach mathematical knowledge beyond simple calculations. They also aim to assess teachers' performance in explaining mathematical concepts, interpreting students responses, assessing students' understanding and their difficulties, and identifying common students errors, and their developmental sequences (Hill et al., 2004; Hill et al., 2007, Hill et al., 2008).

As a result of their work, the research team at the University of Michigan has identified four measurable domains of teachers' mathematical knowledge. Within subject matter knowledge there are: common content knowledge (CCK) and specialized content knowledge (SCK). Within pedagogical knowledge there are: knowledge of content and students (KCS), and knowledge of content and teaching (KCT). The research team has also suggested two other knowledge aspects; knowledge at the mathematical horizon, within subject matter knowledge and knowledge of the curriculum within the pedagogical content knowledge. These two aspects have not yet been included in the measures.

CCK is the mathematical knowledge and skills used in settings other than teaching (e.g. correctly solving mathematics problems, using terms and notations). SCK is the mathematical knowledge and skills unique to teaching (e.g. looking for patterns in errors, assessing generalizability of nonstandard solution methods). KCS is the awareness of the interaction between the students and the content (e.g. what topics or concepts confuse, interest or motivate students, what they find hard/easy), while KCT is the merger of mathematics and teaching (e.g. in what order to teach content, how to represent it, what examples to use) (Ball et al., 2008).

The MKT measures were developed to measure and research teachers' MKT in the United States. However, the measures have been used for prospective teachers as well. Johnson (2011) used the MKT measures when she investigated the development of MKT among senior elementary education majors. McCoy (2011) used the measures in her study of the relationship between mathematics teacher efficacy and the growth in specialized mathematical knowledge among prospective elementary teachers. Consideration is needed when the MKT measures are used with prospective teachers, especially in terms of interpreting results regarding SCK. It is reasonable to assume that SCK grows throughout the teaching career, making it plausible for SCK to be less among prospective teachers compared to practicing ones.

Since the MKT measures relate to the task of teaching instead of teaching practice, they are considered more universal and suitable for translation (Ng, Mosvold & Fauskanger, 2012). It is important to compare the mathematics curriculum of the target country to the NCTM standards (NCTM, 1991, 2000), where teaching practices and mathematical content to be taught in the United States are listed. Although there might be differences in teaching culture between countries, it is reasonable to assume that the mathematical knowledge for teaching overlaps. The question "Why can't you divide by 0?", is a content knowledge question based on the definition of division. Questions like it seem to be universal, and teachers from all over should know its answer (Delaney et al., 2008). In spite of best efforts in translation and adaptation it is always plausible that in some aspects of the measures, cultural bias can never fully be eliminated (Mosvold & Fauskanger, 2013).

The MKT measures have been used to measure teachers and prospective teachers mathematical knowledge for teaching outside the U. S. They have been adapted and translated for use in Ghana, Indonesia, Ireland, Korea, Norway, and Iceland (Hambleton, 2012; Jóhannsdóttir, 2013). The MKT measures were also used, among other measures, in the development of items used in a large study, examining preparation of elementary- and middle-school teachers in mathematics in 17 nations in 2008. Results from that study showed a significant difference in achievement between the participating countries in terms of mathematical knowledge of prospective teachers (Blömeke & Kaiser, 2014).

The current study

The main purpose of this study was to determine the level of mathematical content knowledge among prospective elementary teachers in Iceland guided by the following research question:

– What is the level of mathematical content knowledge among prospective Icelandic elementary teachers?

- More precisely, (a) what is the level of their common content knowledge? and (b) What is the level of their specialized content knowledge?

The study investigated the mathematical knowledge by using items from the MKT measures (Jóhannsdóttir, 2013). The items were selected and adapted to be in line with the Icelandic school community and the Icelandic national curriculum for mathematics. The items in the study were supposed to reflect both whether prospective teachers could answer basic mathematics problems and how they solved special mathematical tasks that could arise in teaching.

Methods, background and sample

Icelandic teacher education

New laws regarding teacher education in Iceland were passed July 1st 2008 and came into effect on July 1st 2011. The new laws lengthened the certification process of teachers from three years to five years. After 2011, prospective teachers in elementary and lower secondary education in Iceland need to complete five years of preparation at a teachers college where three-years is theoretical and professional undergraduate program (180 ECTS credits) and two- years is a graduate program (120 ECTS credit) that accumulates in a Master of Education degree.

The University of Iceland, School of education has the largest teacher education program in Iceland and is responsible for the education of the majority of prospective teachers. In the undergraduate program, prospective teachers can choose between three lines: (1) General teacher education, meant for those who intend to teach grades 1–10 and minor in two subjects, (2) Subject teacher education, for those who wish to major in one subject and get certified to teach both primary schools and secondary schools and (3) Early childhood education, meant for those who intend to teach grades 1–4.

All three lines share mandatory core courses that add up to 100 ECTS credits, none of which are mathematics content courses. Within the general teacher education line, prospective teachers can choose to minor in mathematics for 40 ECTS credits and within the Subject teacher education line, students can major in mathematics for 80 ECTS credits. The graduate programs are 120 ECTS units, where prospective teachers can choose mathematics education for 40 ECTS units (University of Iceland, 2012).

Sample

The participants in this study were 38 prospective teachers in a five-year teacher education program in the School of education at the University of Iceland. At the time of the study the participants accounted for 44 % of the undergraduate students in elementary education, and 38 % of masters students (Gudmundsdottir, 2012). Fifteen of the participants in this study were in their second or third year of their undergraduate studies in elementary education, and 23 of them in their first year of the masters program. Nineteen participants had early childhood education as their major, eight had mathematics, and eleven had other subjects as major. Over 89 % of the participants in this study were female (Jóhannsdóttir, 2013). The sample was a convenience sample. All undergraduate students present in the core course *Teaching mathematics to young students* on the day of the study were offered to take part and all 23 of them accepted. Master students present at the School of education during the time of the study were invited to take part and 15 accepted. Every participant in the study was asked if s/he was willing to be interviewed; 10 out of the 38 participants agreed. Two of the interviewees were male and eight were female. Nine were in their first year of graduate studies and one was in his/hers last year of undergraduate studies (Jóhannsdóttir, 2013). More detailed demographics are listed in table 1.

Quantitative measures

The focus of this study was teachers' knowledge of the mathematical topics, numbers and operations (NOP), and patterns, functions and algebra (PFA). The quantitative data from this study was collected with a survey consisting of translated and adapted items from the MKT measures. The MKT items are multiple-choice, and each item has a reported difficulty and discriminating level. Each MKT item either stands alone, stem item, or has other problems attached to it, leaves. The number of items on an MKT assessment should be at least 15–20, since longer tests are more reliable. For items to discriminate among participants their slope should be above 0.5 and item difficulty should be well targeted (Learning Mathematics for Teaching, 2011a). Following these guidelines items were translated and adapted using a framework developed by Delaney et al. (2008). Emphasis was placed on choosing items that reflected the knowledge specified in the Icelandic national curriculum in mathematics (Jóhannsdóttir, 2013).

The final set of items selected for the Icelandic survey contained 24 items (51 counting the leaves), 12 from each topic: NOP and PFA. Most of the items dealt with mathematical topics usually covered in grades 1

Table 1. Participants

	n	%
Gender		
Females	34	89.5
Males	4	10.5
Age		
20–29	21	55.2
30–39	12	31.6
40+	5	13.2
Level of studies		
B.Ed	15	39.5
M.Ed	23	60.5
Major in school of education		
Early childhood education	19	50.0
Subject teaching (Mathematics)	8	21.1
General education/Subject teaching (Other than mathematics)	11	28.9
Mathematics courses in high school/college		
2–3	10	27.8
4–6	13	36.1
7+	13	36.1

through 5. Topics covered included: subtraction, division, fractions (multiplying, dividing, simplifying), alternative algorithms, positive and negative numbers, perimeter, area, patterns, writing equations, and functions (Jóhannsdóttir, 2013).

Extensive research has been conducted to investigate whether the MKT items reliably measure teachers' mathematical knowledge for teaching (Hill et al., 2007). The reliabilities reported for the original scales used in this study were within acceptable limits (Hill et al., 2004). Since the MKT items used in the study were translated and adapted, their reported reliability was compromised. To ensure the reliability of the survey used in this study, alpha was calculated for its different parts. Knowledge wise, the items fell into two categories, CCK and SCK. Topic wise the items also fell into two groups, NOP and PFA. Table 2 shows the calculated alpha for each of the topics, as well as for each of the knowledge domains, CCK and SCK. The values of alpha for these groups of items were well within the range of acceptable values (Tavakol & Dennick, 2011).

Table 2. *Calculated Alpha for scaled measures*

	Number of items, including the leaves	Reliability (Cronbach's alpha)
Number and operations	21	0.79
Patterns, functions and algebra	30	0.86
Common content knowledge	25	0.82
Specialized content knowledge	26	0.85
All items	51	0.90

Qualitative measures

Once participants had completed the survey, they were invited to take part in an individual interview. Of the 38 survey participants, 10 agreed to be interviewed. The interviews were semi-structured; audio recorded and lasted from 30 to 50 minutes. The interviews were designed by the researcher to give a deeper insight into participants thinking process while solving mathematical problems. During the interviews participants were asked to solve four problems: subtraction with regrouping, double-digit multiplication, division without remainder, and division of fractions. Once the participants had solved each problem, they were asked about their use and choice of methods and words, and also if they could think of other solution methods and come up with an appropriate story in context with the problems (Jóhannsdóttir, 2013).

Data

The data from the surveys and transcripts from the interviews were analyzed in order to answer the research question of this study. Item response theory (IRT) was used to find each participant's trait level and each item's difficulty. A participants' trait level is their level on the psychological trait being assessed by the test items. Participants' trait level is one of the factors affecting how they answer a particular item. Another factor influencing participants' probability of answering a certain way is item difficulty. Item difficulty is calculated based on trait level (Furr & Bacharach, 2008). When the items difficulty matches the participant's trait level, the participant has a 50 % chance of answering the item correctly (Allen & Yen, 1979). Trait level and difficulty scores are standardized with mean 0 and standard deviation of 1. Test items vary in their ability to differentiate between persons with different trait levels. The discrimination value implies the connection between the item and the trait being measured by the test (Furr & Bacharach, 2008).

Using IRT when dealing with such a small sample is rare, but not unheard of when using the simplest models (Reeve & Fayers, 2005). As previously stated, the MKT items come with calculated difficulty; based on the teachers that pilot tested them. In order to compare other groups to the original test group, similar attempts to calculate item difficulty need to be made. Even though comparison is not the focus of this paper, such comparison was made in a study it was based on. Findings from studies utilizing the MKT measures have to be reported using standardized scores, so using IRT to do so for this study is convenient. Because of the small sample size, the IRT results should be interpreted with caution, but they do give an idea about the ranking of the prospective teachers with regards to each other and to the items in the study.

Analysis

To evaluate elementary mathematical CCK among prospective Icelandic teachers the responses from the survey were examined. The items in this category dealt with the following topics: fractions, division, percentages, perimeter, area, patterns (numerical & non-numerical), formulas, functions, expressions, system of equations, rules in mathematics, mathematical facts, and mathematical concepts. Descriptive statistics and information from the IRT analysis were used to study the CCK of the prospective teachers (Jóhannsdóttir, 2013).

Data from the interviews was used to further shed a light on prospective teachers' level of CCK; in particular information regarding the proper use of mathematical language and notations (Jóhannsdóttir, 2013).

In order to evaluate the prospective teachers' level of SCK, responses from the survey were graded with a special focus on items regarding SCK. Items within this knowledge domain included the following topics: rules in mathematics, alternative solution methods, mathematical explanations, the making of story problems, use of visual aids and models, and mathematical definitions. Item difficulty was examined to identify topics relatively easy and difficult for participants. Results from the interview were compared to results from the survey to provide a better understanding of the prospective teachers' SCK.

Results

The survey

Results from the survey are discussed in terms of standardized scores, as outlined in the Terms of use for the MKT instrument (Learning

Mathematics for Teaching, 2011b). The results from the survey reflected a great variety in the achievement of the prospective teachers participating in this study. The trait levels were approximately normally distributed, with the range of 3.68. There was not a statistically significant difference between scores from the two scales, NOP and PFA.

Common content knowledge

Twenty-five of the items on the survey were meant to measure CCK. The average item difficulty calculated for the items was -0.19 ($SD = 0.95$). There was a great variance in participants' trait levels on the common content knowledge items (ranging from -1.66 to 4.00, $M = 0.23$, $SD = 1.08$). One participant's trait level of 4 skewed the distribution, but without it the distribution was approximately symmetrical.

Results from the survey indicated that the topics described in table 3 were relatively difficult for participants, with item difficulty ranging from 0.43 to 2.14. The survey's results implied that the topics described in table 4 were fairly easy for participants, item difficulty ranging from -2.46 to -0.43.

Table 3. *Difficult common content knowledge topics*

Topic	Item difficulty
Identifying surjective function	2.14
Properties of multiplication	1.30
Properties of positive and negative numbers	0.65
Multiplying fractions	0.54
Algebra problem, needing a system of equations to solve	0.43

Table 4. *Easy common content knowledge topics*

Topic	Item difficulty
Formula for perimeter	-2.46
Visual representation of a percentage of an area	-2.14
Non-numerical patterns (forms)	-1.32
Properties of subtraction	-1.17
Bijective functions	-0.44
Number patterns	-0.43

Interviews

Participants were asked to solve four problems in the interviews. Table 5 shows the problems as well as interviewees' performance in solving them.

Most of the interviewees could solve the mathematical problems posed, with the exception of division of fractions. Six of the interviewees solved that problem correctly. One said: ”Are you kidding me!” when showed the problem and said s/he honestly did not have a clue about how to solve it. The remaining three interviewees tried to solve the problem without success. Two of them converted the fractions to decimals while doing so, and two found a common denominator for the fractions during the solution process.

Table 5. *Problems posed in interview*

Problem	Right	Wrong	Tried but didn't finish	Didn't try
74 – 26	9	1		
79 x 48	8	2		
1035 ÷ 5	9	1		
$2\frac{1}{4} \div \frac{1}{2}$	6	2	1	1

Specialized content knowledge

There were 26 SCK items in the survey. Their average item difficulty was calculated 0.211 (*SD* = 0.84). The distribution of participants' trait levels in the SCK part of the survey was approximately symmetric. Participants' trait levels ranged from -2.48 to 2.04, or a difference of 4.52 and the mean was -0.21 (*SD* = 1.04).

Results from the survey indicated that the following topics, listed in table 6, were rather difficult for participants. The topics described in table 7 seemed to be fairly easy for participants, their item difficulty ranging from -1.10 to -0.43. A common denominator for difficult items within each knowledge domain, CCK and SCK, was fractions.

The interviews

The participants' skill to make word problems to go with the problems they solved and their explanations of mathematical concepts and operations were used to examine interviewees' specialized mathematical knowledge.

Table 6. *Difficult specialized content knowledge topics*

Topic	Item difficulty
Alternative method to divide fractions*	1.89
Explanation for equivalent fractions	1.67
Division rules	1.67
Visual model for multiplication	1.17
Alternative subtraction method	1.03

* 5 participants skipped this item. They were counted as wrong.

Table 7. *Easy specialized content knowledge topics*

Topic	Item difficulty
Evaluating different expressions for area	-1.10
Decomposing numbers	-0.90
Describing a situation with an equation*	-0.90
Finding a story to fit a model of a whole number divided by a proper fraction	-0.60
Evaluating partial division method	-0.43

* 8 participants chose the ”I don’t know” answer possibility for this item.

Subtraction, 74 – 26

Half of the interviewees said that 6 could not be subtracted from 4. Six of them used ”standard algorithm with borrowing” to solve the problem 74 – 26. Three of the interviewees that initially used an alternative method to solve the subtraction problem, said they would use the ”standard algorithm” when teaching others.

Multiplication, 79 x 48

Eight of the participants began using ”standard algorithm” to solve the problem. Seven of them gave examples of another possible solution method, but two of them ran into trouble while trying to apply that method. One interviewee was able to connect the problem 79 x 48 to binomial multiplication, (80 – 1)(50 – 2).

Three interviewees had difficulty finding a story problem to go with the multiplication problem. Two of them found a story in the end, but one could not. Apart from one area model story, all of the first stories produced by each interviewee represented repeated addition model. One interviewee came up with a multiplication story regarding area when first asked to find a story problem, but when prompted, five other interviewees came up with an area story.

Division, 1035/5

Everyone could come up with a story/stories for the division problem. All of the interviewees made up stories about equal sharing and seven of the stories had to do with dividing 1035 kronas equally between five people. One interviewee came up with a division problem representing repeated subtraction model. When asked about geometry/area problem in connection with the division problem, three interviewees made up story problems where the area was known and it was supposed to be divided in to five parts. The remaining six interviewees could not think of a story problem.

All of the interviewees mentioned the ”standard algorithm” using the division bracket as a way of solving the problem. Half of them mentioned more than one way of solving the problem and usually mentioned partial division as an alternative to the ”standard algorithm”. Two of the interviewees used division algorithm to explain why they began working from left when solving the division problem, using the division bracket. The other interviewees could not explain the reasoning behind the algorithm, or tried to explain it without referring to mathematics.

One of the interviewees could explain why dividing by zero was undefined and used the definition of division to do so. Two of the interviewees gave mathematically wrong explanations. One said that when dividing by zero the answer would be zero, and another explained that when dividing by zero nothing happened to the other number. One of the interviewees gave no explanation and the remaining five explained that dividing by zero was not possible, by some version of ”you cannot split between no one”.

Division of fractions, $2\frac{1}{4} \div \frac{1}{2}$

Six of the interviewees successfully solved the division of fractions problem. Two of them could explain both how and why they solved the problem the way they did, while the remaining eight referred to the way they had originally learned it as a reason for their way of solution. Three of the interviewees came up with a proper story for the division of fractions problem. One came up with a story fitting $2\frac{1}{4} \div 2$. The rest of the interviewees did not try to find a story problem.

Discussion

The question posed in the study addressed the level of elementary mathematical knowledge of prospective teachers, both regarding CCK and SCK. It came as no surprise that prospective teachers majoring in mathematics scored significantly higher on the survey than did prospective teachers majoring in other subjects. This statistical difference was detected both for the CCK part and the SCK part of the survey.

Results both from the survey and the interviews indicated that the prospective teachers' knowledge was procedural and related to the "standard algorithms" they had learned in elementary school. This was particularly evident during the interviews. The prospective teachers were observed while solving problems, where they were instructed to "think aloud" during the solution process. Most of them could successfully solve the problems, but could not explain the reason why they chose a particular method or why the method worked. The most common answer for why a certain way was chosen to solve a problem or a certain step was taken in the solution process, was "Because that is the way I learned to do it as a kid."

Three problems stood out as very difficult in the SCK part of the survey. Two of them dealt with alternative solutions methods in subtraction and division of fractions, and the third with an explanation for a division rule. In order to have a 50 % chance of solving these problems correctly, each participant's trait level had to be between one and two standard deviations above the average. The high item difficulty calculated for these items could not be explained by participants' unfamiliarity with the representation of the problems. This difficulty with alternative solution methods was also present in the interviews. When interviewees were asked to think of another method of solving a problem, most of them were unable to do so.

The difficulty the prospective teachers had with fractions was also in line with results from the interviews. Four of the interviewees could not solve the division of fractions problem, and were confused about the use of common denominator, language (numerator, denominator) and the relationship between fractions and decimals. The results indicated that these prospective teachers were confusing division by fractions with whole-number division and did not have a deep understanding of what a fraction was (Cramer & Whitney, 2010).

Results from the interviews indicated that the relationship between multiplication and division was unclear for the prospective teachers. Six of them came up with a story problem regarding area for multiplication, where two sides of a rectangle were known and the area needed to be found. Only one interviewee was able to come up with a story problem for area and division, where the area and one side of a rectangle were known and the other side needed to be found. The other interviewees were not able to make the connection between the division problem and the previous multiplication area problem. These results support the theory of prospective teachers' knowledge being procedural rather than conceptual.

The findings from this study regarding prospective teachers' level of mathematical content knowledge are in line with prior research that has indicated that people can perform mathematical calculations (procedural knowledge) without the understanding of concepts and the underlying principle (Ball, 1990; Tirosh & Graeber, 1989, 1990).

Conclusions

The MKT measures used in this study were designed to measure the mathematical knowledge of teachers and prospective teachers in the United States. Even though measures were taken to ensure the validity of the translation and adaptation of the items used in this study, there may still be areas where cultural differences could have skewed results. For example the Icelandic prospective teachers did considerably worse in identifying surjective function than their peers in the United States (Jóhannsdóttir, 2013). This could stem from cultural differences in mathematics education, as surjective functions are not given much room in the Icelandic curriculum in mathematics.

It has to be kept in mind that the participants in this study were prospective teachers so the results from the study cannot be generalized to practicing teachers. The sample in this study was a small convenience sample, and participation was voluntary. Therefore the sample in this study could represent prospective teachers more confident in their mathematical content knowledge, than those who chose not to take part in the study. For further studies it is recommended to strive for a more representative sample of prospective teachers.

The findings from this study indicate that the level of elementary mathematical knowledge among prospective teachers in Iceland is based on recollection and reproduction of basic skills and concepts. For example, when asked why putting a zero was necessary before multiplying with 4 in the problem (79 x 48) an interviewee said: "Because I was told that each time you go down one line you are suppose to add one zero, if you go down two lines you put two zeros." This explanation was in line with many of the explanations given by the prospective teachers during the interviews, indicating that during their own mathematics education, emphasis was on procedures rather than conceptual understanding.

The findings also showed that Icelandic prospective teachers had considerable difficulty with fractions, which is in line with research done in the United States (Ball, 1990). While operations with fractions are not a big part of the elementary school curriculum, one of the things worth pondering over is whether teachers should or should not be able to answer a student's questions not covered by the curriculum. Mathematics educators seem to agree on that "teachers must know in detail and from a more advanced perspective the mathematical content they are responsible for teaching [...] both prior to and beyond the level they are assigned to teach" (National Mathematics Advisory Panel, 2008).

Examining prospective elementary teachers' mathematical knowledge provides some information regarding their mathematics education (Simon, 1993). Prospective teachers developed much of their mathematical understanding, procedures and approach to mathematics, through their mathematics education prior to entering any teacher education program. It is important to understand that teacher preparation programs are critical; not only for future teachers, but also for the children they will be teaching. If prospective teachers enter the teaching profession with an inadequate mathematics background it stands to reason that they will produce students that are similarly weak. Some of those students then go on to become future teachers and the cycle continues. The Icelandic curriculum stresses the importance of multiple solution methods, students' reasoning, and the connection between mathematics and students' daily lives. It also stresses the importance of understanding and skills going hand in hand in mathematics education. It is clear that many Icelandic prospective teachers participating in this study did not have a sufficient understanding of concepts or the underlying principles for teaching mathematics. This gives rise to the question of whether or not the courses a prospective teacher takes, and the experiences they have while in their preparation programs is sufficient to teach mathematics to students, even at the lowest level.

Like in many studies exploring prospective teachers' mathematical knowledge in a teacher education program, this study measured their cumulative knowledge, not only the mathematical knowledge acquired in the program. That being said, the teacher education program is responsible for prospective teachers' mathematical knowledge when they enter the teaching profession and therefore need to provide sufficient mathematical content such that prospective teachers are ready and well prepared to teach according to the National Curriculum and Standards.

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