Editorial

As for the past two years the last issue of the year is a thematic issue; this year it is a double issue on *Mathematical knowledge for teaching in the Nordic region*, which has been guest edited by Janne Fauskanger and Reidar Mosvold, University of Stavanger. The thematic issue consists of articles resulting from an open call. In addition, the guest editors also strongly encouraged researchers from the Nordic countries interested in MKT to submit papers, and they did! All in all, we received 15 manuscripts and are happy to present eleven of these as articles in this double issue of Nomad.

The work with the next volume's thematic issue has already started. The theme for 2015 will be *Mathematics textbook reseach* with articles produced within the *Nordic network of research on mathematics textbooks*. This is a researcher network with aim to increase the Nordic and Baltic collaboration in research on mathematics textbooks, funded by NordForsk. Researchers from Estonia, Finland, Iceland, Latvia, Norway, Sweden and international scholars from the Netherlands and Germany are members of the network which is led by professor Barbro Grevholm, University of Agder.

In this issue

In seeking to develop a practice-based theory of *mathematical knowledge for teaching* (MKT), Ball and colleagues at the University of Michigan studied the work of teaching to uncover the mathematical challenges that arise. This work led to the identification of specific tasks of teaching. The theory of MKT, defined as "the mathematical knowledge used to carry out the *work of teaching mathematics*" (Hill, Rowan & Ball, 2005, p. 373), was developed as a further refinement of Shulman's (1986) subject matter knowledge (SMK) and pedagogical content knowledge (PCK) based on analyses of these tasks of teaching mathematics. The theory of MKT is spreading around the world – and also to the Nordic countries.

Parallel to developing the theory of MKT, extensive time and money has been invested in the US in developing and validating measures of teachers' MKT. This thematic issue opens with a selection of articles focusing on issues related to these measures. The first article is written by one of the researchers from Deborah Ball's research team, Mark Hoover, and the guest editors of this thematic issue. The article has a focus on the basis for MKT and recommends increased efforts to identify

professionally defensible mathematical tasks of teaching that can serve as a common foundation for conceptualizing and measuring mathematical knowledge for teaching internationally. Although studies of adaptation of the MKT measures generally conclude that they are useable in other countries, cultural differences in teaching prompt questions about whether theories and measures of knowledge for teaching are culturally specific.

In the second article, Jóhannsdóttir and Gísladóttir report findings from a study exploring the mathematical content knowledge of the prospective teachers at the University of Iceland. The participants were measured with interviews and the MKT measures, and the findings from the study indicate that prospective teachers' MKT is procedural. The authors further conclude that the Icelandic prospective teachers have difficulty in e.g., evaluating alternative solution methods.

Whereas the second article reports from a study where MKT measures were adapted and used, Fauskanger and Mosvold dig deeper into the perils and pitfalls of using multiple-choice format in measures of teachers' MKT in the third article of the thematic issue. The article reports on a study on the connection between teachers' responses to multiple-choice MKT items – and in particular where they select the suggested solution "I'm not sure" – and their written responses to corresponding open-ended questions (long responses). The findings from analysis of teachers' responses indicate that their long responses and their multiple-choice responses do not always correspond. Some teachers who selected "I'm not sure" showed uncertainty also in their long responses, whereas other teachers revealed instrumental and even relational understanding of the content. These results are important when responses to the MKT items are to be analyzed.

Kaarstein's article also includes a critical perspective on items and measures. In her study, she focuses on how Norwegian teachers and educational researchers categorized a collection of items used in the international TEDS-M study; MKT items were included in this study. The results from Kaarstein's study imply that the item categorization depended on the item characteristics. For example, multiple-choice items were associated with mathematical content knowledge (MCK) and items asking the respondents to rewrite or reword a mathematical task were associated with mathematics pedagogical content knowledge (MPCK). Furthermore, the results indicate a common Norwegian understanding of MPCK as the teachers' and researchers' categorization largely coincided.

The group of articles following this one focuses on the work of teaching. MKT was originally developed from classroom studies and analyses of all the work teachers do in connection with teaching mathematics.

These studies were then used as a starting point for developing both the theoretical framework of MKT and the measures. Several more recent studies have returned the focus to what happens in the classroom, and the following three articles are examples of this.

The aim of Kilhamn's study is to understand the mathematical issues and demands of teaching the concept of variables, to outline a body of Specialized Content Knowledge for teaching (SCK). Kilhamn discusses in what sense x in the expression $x+2$ is a variable and what teachers need to know about variables in order to create optimal learning conditions for students. Data from two lessons in two Swedish grade 6 classrooms, with complimentary focus group interviews, were analyzed using the MKT framework. Findings suggest some aspects of SCK to be an awareness of the different roles of the algebraic letter x in the expression $x+3$, the equation $x+3=8$ and the formula $x+3=y$, an appropriate use of the terms unknown and variable, and the importance of mathematical contexts for expressions.

In the next article, Opsvik and Skorpen discuss *mathematical quality of instruction* (MQI). The MQI instrument was developed in order to measure the mathematical quality of lessons – in particular for analyzing classroom videos – and a connection has been found between MQI and MKT. Opsvik and Skorpen have translated and adapted the MQI framework for use in a Norwegian context, but they suggest a somewhat different use of the instrument in the Norwegian context. The indicators from the MQI instrument, they argue, can be relevant for observing and mentoring pre-service teachers. They suggest that the instrument is used in this way in the Norwegian context, rather than using it for evaluating the quality of individual teachers' instruction and connecting this with their MKT score.

There are several theories and frameworks that are related to MKT; one example is the *knowledge quartet* (KQ), which is a framework for mathematical knowledge in teaching. In their article, Kleve and Solem use KQ to discuss how aspects of a mathematics teacher's knowledge surfaced in a whole class discussion about decimal numbers, percentages and fractions. Their focus is the teacher's orchestration of the discussion in order to unpack the mathematical content for the students. This teacher's interactive teaching included questioning and probing students' contributions in order to make the students take part in the discussion, and these were important features of this lesson. A range of aspects of the teacher's mathematical knowledge were revealed in studying the teacher's pedagogical moves, and Kleve and Solem suggest that the interplay between the aspects of his knowledge was crucial in this lesson.

Whereas the previous three articles all related to the work of teaching mathematics, the third section of articles in the thematic issue all present suggested extensions of the MKT framework. Jakobsen, Mellone and Ribeiro focus on Norwegian prospective primary teachers' MKT when interpreting and making sense of pupils' answers in their article. They name such knowledge interpretative knowledge and consider it to be linked with certain aspects of MKT. In order to deepen these links and to access and develop such knowledge in prospective teachers, they designed a suitable set of tasks on a problem concerning fractions in order to investigate this particular kind of knowledge and clarify its features and dimensions. The results reveal the importance of developing such types of knowledge as a basis for teachers to effectively make sense of and interpret pupils' productions and to make it possible to provide effective and meaningful feedback.

Drageset in his article argues that there is a need to understand more about which types of knowledge teachers use when orchestrating mathematical discourses. He combines models for MKT with a recent framework that describes the actions that teachers typically use during classroom discourses in mathematics. By looking into what knowledge each action demands from the teacher, three areas related to mathematical knowledge for teaching are described: *doing*, *guiding* and *requesting*. "Doing" describes different ways the teachers are doing the mathematical work themselves. "Guiding" describes how the teachers help, while leaving most of the work to the students. "Requesting" describes different ways teachers asked the students to explain or contribute to the discourse. Drageset's article can then be seen as a contribution to the further development of a particular aspect of PCK: knowledge of content and teaching.

The next article represents an attempt to develop another aspect of MKT, related to the sub-category of subject matter knowledge referred to as horizon content knowledge (HCK). Smestad, Jankvist and Clark discuss how the inclusion of history of mathematics in mathematics education draws heavily on a teacher's MKT, in particular HCK, in the context of curricular changes. They discuss the role of history of mathematics in school curricula, its inclusion in textbooks, and its consequences for the mathematical knowledge needed for teaching. They address the matter from three national settings (Denmark, Norway and the United States). These settings exemplify how, in particular, teachers' HCK needs to be broader than what is necessary for only the current curriculum. Another interesting contribution in this article is the suggested distinction between a priori HCK and a posteriori HCK. The first refers to aspects of HCK that are already well developed, whereas the

second refers to a more dynamic aspect of HCK, and the authors argue that history of mathematics might contribute to this aspect of HCK in particular.

Whereas all the articles in the thematic issue so far are related to defining, describing and testing MKT, the final article goes in a different direction by discussing the implementation and teaching of MKT. In her article, van Bommel reports on a learning study conducted by a group of Swedish teacher educators. This study intended to identify critical features concerning the teaching and learning of MKT. Three seminars and 300 tests were analyzed using variation theory revealing four critical features to take into account in teaching student teachers in mathematics education: namely their need to i) formulate proper goals for a lesson, ii) outline the lesson plan in detail, iii) shift perspective from the role of being a teacher to being a mathematics teacher, and iv) understand the underlying mathematics of the lesson topic at hand. These four features are highlighted as important to the learning and teaching of MKT.

In sum, the articles in this thematic issue present different perspectives on MKT – from discussions of the foundation in analysis of tasks of teaching, via critical discussions and use of measures, into the classroom, and, finally, to different efforts to extend the MKT framework in various directions. Our hope is that the readers will find this thematic issue interesting to read, and we also hope that the different approaches used in the various articles might provide the readers with inspiration for conducting new studies of mathematical knowledge for teaching in the Nordic countries.

Workshop for doctoral students

The editors of Nomad are planning for a fourth workshop for doctoral students to be held in Gothenburg in the spring of 2015. At the moment the planning has just started. Program and other information will be published on the Nomad website as the planning proceeds.

Thanks to authors and reviewers

Finally, we wish to thank our reviewers without whom neither this thematic issue nor the two regular issues of 2014 would have been possible at all. We are sincerely grateful to all for their continued engagement. A list of all reviewers who have contributed to volume 19 of Nomad in 2014 is found below.

The Editors

References

Hill, H. C., Rowan, B. & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42 (2), 371–406.

Shulman, L. S. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher*, 15 (2), 4–14.

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