An analysis of mathematical modelling in Swedish textbooks in upper secondary school

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A new national curriculum has recently been implemented in the Swedish upper secondary school where one of the goals to be taught is modelling ability. This paper presents a content analysis of 14 "new" mathematical textbooks with the aim to investigate how the notion of mathematical modelling is presented. An analytic scheme is developed to identify mathematical modelling in the textbooks and to analyse modelling tasks and instructions. Results of the analysis show that there exist a variety of both explicit and implicit descriptions, which imply for teachers to be attentive to complement the textbooks with other material.

Mathematical modelling is described as an essential part of mathematics education in many countries around the world, but the use of modelling activities in day-to-day teaching in mathematics classrooms is generally still limited (Niss, Blum & Galbraith, 2007). The present situation in Sweden is no exception in this respect, where the role of mathematical modelling is emphasised in the national Swedish upper secondary curriculum, though teachers seem not to give priority to integrating mathematical modelling into their everyday mathematics teaching (Frejd, 2011). There are many *epistemological* and *systemic* reasons for this. *Epistemological* reasons relate to teachers' conceptions of mathematics and of modelling, the relation between learning mathematics and developing modelling competency, etc., while systemic reasons concern the curriculum, the national tests, the textbooks, etc.

In a study exploring some epistemological reasons related to teachers' conceptions about the role of mathematical modelling in mathematics education, 18 upper secondary mathematics teachers from different parts of Sweden were interviewed (Frejd, 2011). Half of these teachers considered some of the modelling items discussed during the interviews

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Frejd, P. (2013). An analysis of mathematical modelling in Swedish textbooks in upper secondary school. *Nordic Studies in Mathematics Education*, 18 (3), 59–95.

not to be about mathematics and should not be part of mathematics education. Also, almost all teachers said that mathematical modelling is more used in physics or in chemistry education than in mathematics education (ibid.).

A *systemic* reason according to Ärlebäck (2009) is that the notion of mathematical modelling is described in vague terms in the Swedish curriculum, which makes it difficult for teachers to interpret what it is that they shall teach. Another systemic reason that may influence teaching about modelling is how modelling is assessed in high stakes assessment (Niss, 1993), such as the wide scale national course tests used in Sweden. Assuming that the Swedish national course tests inform teachers what to teach, it appears that the tests present a rather narrow view of mathematical modelling. Frejd (2011) concluded, based on an analysis of test items from a sample of the national course tests, that these tests assess modelling only fragmentally.

The way the notion of mathematical modelling is presented in mathematical textbooks is another systemic reason. From a historical perspective only some isolated tasks with a modelling character were found in textbooks from the late 1990s (Jakobsson-Åhl, 2008). According to Ikeda (2007), the lack of adequate mathematical textbooks is a common obstacle to teach modelling in lower secondary school, and Cabassut and Wagner (2011) found that modelling was described only implicitly in tasks in primary textbooks in France and Germany. However, there seem to be very few research studies that systematically have analysed how textbooks treat mathematical modelling in upper secondary school. The aim of this paper is to investigate how Swedish mathematical textbooks for upper secondary school interpret and explain the notion of mathematical modelling.

Mathematical modelling and textbooks

This section includes a description of *the Swedish curriculum* in relation to mathematical modelling, a clarification of *the notion of mathematical modelling* based on research literature in mathematics education and a *brief literature overview* of research focusing on analysing mathematical textbooks connected to mathematical modelling.

The Swedish curriculum

The Swedish government has introduced and implemented a new curriculum for upper secondary school valid from 2011, which emphasizes mathematical modelling as one of seven teaching goals: to develop students' ability to "interpret a realistic situation and design a mathematical model, as well as use and assess a model's properties and limitations" (Skolverket, 2012, p. 2). This formulation indicates an explicit call for using realistic modelling activities in the mathematics classroom. When teaching of mathematics in Sweden to a large extent depends on textbooks, both as a guide for teachers on what to teach and for students to work individually with exercises (Jablonka & Johansson, 2010), it is highly relevant how the mathematical topics are described, presented and interpreted by the textbooks authors, in order to support students to develop the abilities described in the seven goals. The mathematics curriculum in the Swedish upper secondary school is organised by five sequential courses (Mathematics 1 to 5). The courses come in up to three different versions depending on type of study programme: Mathematics 1a (the vocational educational programs); Mathematics 1b (e.g. the arts program and the social science program; and Mathematics 1c (the technology program and the natural science program). The seven general teaching goals, including modelling, are the same for all the different courses.

The notion of mathematical modelling

There are many definitions/ descriptions of the notion of mathematical modelling used in mathematics education research (e.g. Blum, Galbraith, Henn & Niss, 2007; Frejd, 2011; Garcia, Gascón, Higueras & Bosch, 2006; Jablonka & Gellert, 2007; Kaiser & Sriraman, 2006). For this paper, the description of mathematical modelling by Blomhøj and Højgaard Jensen (2003) is used. Their description consists of six sub-processes depicted in (a) to (f) in figure 1.

The modelling process in figure 1 is described as "an ideal modelling process focusing primarily on the structural aspects of the process" (Blomhøj & Højgaard Jensen, 2003, p. 123) and may be used as a tool for analysing mathematical modelling (ibid.). The double-headed arrows and the vertical line emphasise that it is neither a linear process nor a cyclic process, since the sub-processes may be repeated and performed in any order. In the *perceived reality* a situation or a phenomenon is generally described and to clarify and specify the object of investigation a *formulation of a task* in the *domain of inquiry* is needed, where the domain of inquiry with its characteristics and conditions frame the task. In order to set up a mathematical model that corresponds to the task in the domain of inquiry "the modeller" (a person who is trying to solve a modelling problem) is required to select the relevant objects, relations and idealisations, called *systematization*, which creates a *system*. The system is

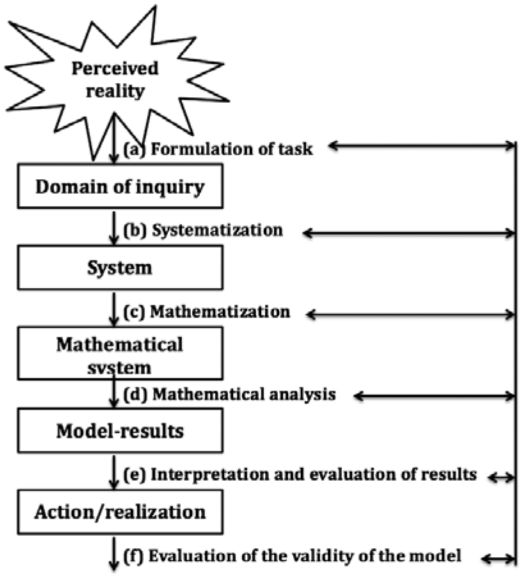


Figure 1. *The modelling process by Blomhøj and Højgaard Jensen (2003, p. 125)*

translated into a mathematical representation, the *mathematization process*, into a *mathematical system*. The mathematical system is analysed, the *mathematical analysis*, or in other terms using mathematics to solve the corresponding mathematical problem to produce mathematical results and conclusions, called *model results* in figure 1. The "modeller" is then supposed to make an interpretation of the results in the initial domain of inquiry, which means that *the results are interpreted and evaluated* in relation to the formulated task in the domain of inquiry and in relation to the empirical data (if they exist). This sub-process is used to establish suggestions for or against some *actions* due to the problem, or to support or not whether new insight gained from the investigated phenomena is valid. The *evaluating the validity of the model* is a second validation, which requires new data and refers to the questioning of the entire modelling process.

In addition, Blomhøj and Højgaard Jensen (2003) refer to figure 1 and write that "[b]y mathematical modelling competence we mean being able to autonomously and insightfully carry through all aspects of a mathematical modelling process in a certain context" (p. 126); a definition I have adopted for this paper. The main reasons for using this framework are: (1) Blomhøj and Højgaard Jensen (2003) focus on structural aspects of the modelling process which can be used as a tool to compare and contrast to other descriptions of modelling; (2) The modelling process as described by Blomhøj and Højgaard Jensen as a non linear and non cyclic process is empirically observed (Ärlebäck, 2009; Borromeo Ferri, 2006; Oke & Bajpai, 1986), but not always depicted in other modelling schemes (see Perrenet & Zwaneveld, 2012 for examples of different schemes); (3) Blomhøj and Højgaard Jensen's framework is a similar view of modelling that forms the basis of the construction of the guiding questions used in Frejd (2011) to examine national course test items; and (4) It is an elaborated research perspective, which does not contradict the description of modelling in the Swedish curriculum (Skolverket, 2012). Instead it may be seen as an extension of the Swedish teaching goal, the modelling ability, with more clear and defined details, which leaves less space for interpretations. In the following I will refer to the framework by Blomhøj and Højgaard Jensen (2003) simply as the framework.

Research on mathematical modelling in textbooks

There are recent empirical investigations presenting evidence that the mathematical textbooks play an important role in the mathematics classroom. For instance, the TIMSS 2007 indicates that a primary source for teaching and learning mathematics is the textbook (see http://timss.bc.edu/timss2007). This may affect how teaching and learning take place in the classroom. Johansson (2005, p. 48), based on a literature review, presents five consequences of using textbooks as a primary source (i.e. arguments why it is important to analyse textbooks). These five consequences are: a) The teachers most likely present the mathematical topics in the textbooks; b) The teachers most likely do not present mathematical topics that are not included in textbooks; c) The textbooks' instructional approach influence the teachers' teaching strategies; d) The sequence of the instructions given by the teachers often have similarities to the textbook; and e) The textbooks are the main source in planning the presentation of the mathematical content according to the teachers.

A search in the database ERIC (Educational Resources Information Center, EBSCO) using the terms "mathematical modelling" together with "textbooks" gave six references, and the same search in MathEduc gave 17 references, but none of the references focused on textbook analysis with an explicit aim to analyse how mathematical modelling is interpreted and explained in different textbooks. Nevertheless, there are other research studies with related aims. Gatabi, Stacey, and Gooya (2012) analysed and compared textbook problems (grade nine) used in two textbooks in Australia and one in Iran related to mathematical literacy. Their analysis is based on the PISA concept of mathematical literacy, which is related to a modelling cycle. They found that only few textbook problems did require the students to interpret and check their answers and they conclude

> [f]inally, even though mathematical modelling is at the heart of the mathematical literacy [...], we must acknowledge that neither in this study nor in our other experiences with school textbooks in Australia or Iran, have we seen many examples of problems that really meet the criteria for a genuine modelling problem. (ibid., p. 418)

Rowlands (2003) analysed how A-level textbooks used in mechanics treated modelling. He found that the descriptions of modelling in the mechanics textbooks was not specific to mechanics, but seemed to stem from a more general approach of mathematical modelling found in mathematics (i.e. a cyclic process with aspects like Blomhøj and Højgaard Jensen's (2003) description). This is, according to Rowlands (2003), inappropriate in the beginning of A-level since the students need to grasp that ”the modelling process is structured according to the scientific model – to which students have to be induced” (p. 103) and thus ”[t]he general mathematics modelling procedure is only appropriate after the student has learnt mechanics and is familiar with the translation component” (p. 103–104). Other research studies that are analysing textbooks and with some connections to modelling relate to mathematical applications (Lu & Bao, 2012) as well as word problems and problem solving (see for instance Fan & Zhu, 2007; Jakobsson-Åhl, 2008; Kongelf, 2011; Mayer, Sims & Tajika, 1995).

As discussed in the ”modelling section”, the differences and similarities between problem solving, applications and mathematical modelling are not clear. In addition, the notion of problem solving has different meanings in research literature in mathematics education (Lesh & Zawojewski, 2007; Lester, 1994; Stanic & Kilpatric, 1989). Already Schoenfeld (1992) stated that ” 'problems' and 'problem solving' have had multiple and often contradictory meanings through the years – a fact that makes interpretation of the literature difficult” (p. 337). However, from the theoretical framework adopted for this study and other similar ”modelling cycles” with a focus on the entire process (Blum & Niss, 1991; Niss et al., 2007), the distinction between modelling, application and problem solving refers to the problem situation and how the problem is described. According to Niss et al. (2007), a modelling problem is an open ended real life problem, where the real life setting is the starting point for the modeller to find the mathematical system based on the perceived reality, direction clockwise in figure 1. An application problem is described, in contrast to a modelling problem, as a problem dressed up with everyday language with some connection to reality, where the mathematical model is implicitly defined. In other words, the focus of an application problem is the invented perceived reality based on the mathematical system, direction counter clockwise in figure 1. Problem solving includes solving problems related to non real life situations, which is a clear distinction to mathematical modelling. However, there also exists a combination of problem solving and applications, which is the notion of applied problem solving sometimes used in the same manner as modelling, and sometimes used to solve any extra-mathematical problem (Niss et al., 2007). Niss et al. (2007) categorise common types of problems as: *word problems*, *standard applications* and *modelling problems*. Word problems are characterized as intra mathematical problems dressed up with a given context.

> Olle, Pelle and Lasse are together 55 years of age. Lasse is half as old as Olle and Pelle is five years older than Olle. How old are the boys?
>
> (Johansson & Olsson, 2011, p. 79; my translation)

The stated word problem above is nothing more than an ordinary commonly used ”problem solving” task that is easily solved by an equation. Standard applications are characterized by having a given implicit model.

> A quadratic room has the area of 25 m^2. What size does the room have?
>
> (Szabo et al., 2011, p. 78, in 1c; my translation)

The implicit model given in the example above is the second degree equation describing a quadratic area. Modelling problems are characterised by involving in principle the entire modelling process.

> How long time does it take to walk up the stairs to the top floor [in a 54 stories building]? (Alfredsson et al., 2011, 1c, p. 142; my translation)

The problem above found in Alfredsson et al. (2011) is similar to a Fermi problem used by Ärlebäck (2009) and includes the entire modelling process. A Fermi problem is a open problem with real-world connections that can be understood and solved at different levels of complexity (for more information see Ärlebäck, 2009). However, as Ärlebäck (2009) states, ”the notions modelling, application and problem solving are connected with each other and overlapping in many ways” (p. 181; my translation).

The complexity of defining problem solving and application problems makes it difficult for researchers to differentiate between the types of problems used in mathematical textbooks. Kongelf (2011) is referring to eight textbook studies using the following definition: ”a problem is [defined] as a situation that requires a decision and/or an answer, no matter if the approach of solution is readily available or not to the problem solver" (p. 11). The consequence for the researcher using such a broad definition as above is to examine "all" textbook exercises, and this can be seen in Fan and Zhu (2007) who analyse "all kinds of problems including the so called 'routine' problems and 'non routine' problems, 'conventional' problems (e.g. project tasks, open-ended problems), among others" (p. 64). Similar reasoning is done by Lu and Bao (2012), but from an application perspective where they analyse application problems when mathematics is applied both in the extra-mathematical world and the intra-mathematical world. This description seems to entail a broad analysis for the researcher as well. However, they do not state that the analysis included "all" textbook problems and neither do they discuss what types of textbook exercises are possible to exclude from the stated description.

To summarise the results from some textbook studies (Fan & Zhu, 2007; Kongelf, 2011; Lu & Bao, 2012; Mayer et al. 1995) there seem to be differences between how mathematics textbooks present problem solving and applications in different countries and in particular how explicitly general problem solving strategies and heuristic approaches are treated. However, it is not always easy to compare different textbook studies, since there is a "lack of common and explicit criteria for textbook comparisons" (Charalambous, Delaney, Hsu & Mesa, 2010, p. 120). In the next section, an approach for analysing mathematical modelling in mathematical textbooks is described, explained, and justified.

Methodology

There are plenty alternatives for how to analyse textbooks. Jakobsson-Åhl (2008) used a historical epistemological perspective to explore variations in Swedish textbook descriptions of algebraic content during the second half of the twentieth century, by using a phenomenographical approach with no pre-defined categories. Dowling (1996) investigated sociological features in textbooks and used a sociological approach with concepts and expressions from researchers like Bernstein and Foucault. Haggarty and Pepin (2002) compared textbooks from three different countries, using a broad analytic scheme with pre-defined questions listed in the five categories "authority of the text, the authors' view of mathematics, the interconnectedness of content knowledge, the analysis of content knowledge and pedagogical intentions" (pp. 131–133). A common approach suggested by Johansson (2005) and Robson (2002) is the method of content analysis to examine school textbooks, frequently used for analysing textbooks (Fan & Zhu, 2007; Kongelf, 2011; Gatabi et al, 2012; Lu and Bao, 2012) focusing on structural organisation of content (counting frequency of words, expressions, strategies, etc.) and it is a transparent method (Robson, 2002).

In order to search for characteristic features of how modelling is interpreted and explained in Swedish textbooks in relation to the stated framework with a focus on structural aspects of the modelling process, I have chosen to follow Robson's (2002) guidelines for a content analysis:

> start with a research question, decide on a sampling strategy, define the recording unit, construct categories for analysis, test the coding on samples of text and assess reliability, and carry out the analysis.

Initial phases of the content analysis

The research question posed to address the aim of this paper is:

- How is the notion of mathematical modelling presented and treated in textbooks in the first mathematical course in upper secondary school in Sweden?

The reasons for posing this particular research question are that it is operational to analyse what is *presented* in text, pictures, problems etc. to seek answers to how modelling is *treated*. A descriptive approach together with the framework may give information about how textbooks present and treat the notion of mathematical modelling, without interfering with the authors' intentions (the authors may interpret mathematical modelling in one way and describe it in another way even if this was not the intension). Second, the first mathematical course is mandatory for all students and it may be their first experience with mathematical modelling, since the curriculum for compulsory school does not include mathematical modelling as an ability (Skolverket, 2011). The mathematical content in the first course also includes topics like statistics, functions, etc. (Skolverket, 2012), which gives the textbook authors a basis for developing modelling problems.

The *sampling strategy* focused on identifying mathematical textbooks used in the first course at the upper secondary school. The sample of textbooks analysed in this paper was decided based on a search from a webpage, made by the Swedish National centre for mathematics education (see www.ncm.gu.se), listing Swedish publishers selling mathematical textbooks. In Sweden there is no authority that examines the quality of mathematical textbooks and textbooks for Swedish upper secondary school are all commercially produced. It is the school principals' responsibility that students get access to "textbooks and teaching aids of good quality" (Skolverket, 2010, p. 15; my translation), which means that the teachers at the schools in collaboration with the principals that decide on what books to use.

To *define the recording unit* is according to Robson (2002) to explain what parts of the content are to be analysed and recorded, like words, themes, paragraphs etc. This paper will use several recording units; first words (examining all text in any form in the book for words like mathematical model, modelling, modelling ability and real world problems / real world situations); second paragraphs and worked examples (explicit explanations or illustrations of the modelling process and solved examples in relation to mathematical models and modelling); and finally tasks (textbook tasks and exercises related to the first and second recording unit).

Constructing categories for the analysis is the process of organizing the content and making the analysis of the content transparent, so that it is possible to replicate. However, Robson (2002) does not give any general descriptions on how to construct categories due to the vast amount of possibilities in which this can be done. In this paper the construction of categories, explained in the next section, are based on the research question and are depicted in an analytic scheme (see figure 2).

A description of the analytic scheme

The *first* part is to identify texts that treat modelling in the textbooks and how explicitly the textbooks use the words modelling and models. The statement by Niss et al. (2007, pp. 6–7), "if we want students to develop applications and modelling competency as one outcome of their mathematical education, applications and modelling have to be explicitly put on the agenda of the teaching and learning of mathematics", indicates that it is important that modelling is described explicitly in textbooks, and the argument by Chávez (2003) that the number of pages of a particular mathematical topic has an impact of the time the teacher teaches this topic, provide reasons for counting pages, counting how frequently the notions models and modelling are being used and identifying where in the text the words may be found.

The *second* part of the analytic scheme examines how textbook instructions (instructional texts and worked examples) describe different sub-processes of modelling related to the framework, as well as examines the aim of the instruction. The words models and modelling may be presented more or less clearly in textbooks, here characterised as *explicit* and *implicit* descriptions. Explicit descriptions are when the textbook authors have used the words models, modelling, etc and these are explicitly elaborated and/or explained. Implicit descriptions are when

Analytic scheme

Identification of modelling

– Number of words
– Place in text

– Model, modelling, real world problem
– Where in the text the words are placed (e.g. intro, tasks, solved examples, other readings)

Ananlysis of modelling instructions

– Explict/ Implict descriptions of words
– What is the aim with the text or the worked problems?
– Formulating a task in the domain of inquiry
– Selecting the relevant objects, relations and idealising
– Translation into a mathematical representation
– Using mathematics to solve the corresponding mathematical problem
– Making an interpretation of the results in the initial domain of inquiry
– Evaluating the validity of the model
– Identify if the words modelling etc are explicitly or implicitly elaborated and/or explained?
– Is the main aim of the text on lerning a topic in mathematics or mathematical modelling?

– Does the task needs to be reformulated in order to get a more accurate question?
– Is it unclear from the outset what kind of facts are most relevant to formulate a mathematical model or is it stated explicit or implicit in the task text?
– Is it expected to assign variables, parameters, or constants to solve the problem in order to set up a mathematically formulated statement (a mathematical model) describing the problem addressed?
– Do the students need to use the model to calculate a mathematical result?
– Are the results interpreted and evaluated in relation to the formulated task in domain of inquiry and in relation to the empirical data?
– Is more data given in the text so that is possible to evaluate the mathematical model ?

Ananlysis of modelling item

– What is the aims with the item
– What type of models are supposed to be used?

– Is the main aim of the item on lerning a topic in mathematics or mathematical modelling?
– Linear functions, exponential functions, statistical models, etc.

Figure 2. *The analytic scheme used in the content analysis*

the words are used but not given any clear meaning. The aim of the instruction in the textbooks is a corner stone of what is transmitted to students and teachers about modelling. In mathematics education modelling is an activity that serves two separate aims, either described as an aim in itself (to develop students' modelling competencies) or as an aim to develop a broader mathematical knowledge (used as a didactical tool to learn mathematics) (see e.g. Blum & Niss, 1991). In relation to the aim of the instruction in the textbooks, it is also accurate to examine how the structure of the modelling process is described in the textbooks, which may influence teachers' and students' conceptions of modelling. By using the six sub-processes in the framework six analytic questions were developed with the purpose to analyse what sub-process are presented in the textbooks and what is left out (see figure 2).

The *third* part concerns how the modelling *items* are presented. The items and exercises in the textbooks are only text statements, which means that there exist no solved solutions with descriptions of sub-processes used. This is an exploratory study focusing on how modelling is presented, and without having students' solutions an analysis of items in terms of sub-processes does not seem appropriate, since it would require methods that also include analyses of expected solutions. However, the types of models the students are supposed to use may be found in the textbook and may give information about what type of mathematics is in the foreground. The aim of the items will indicate if primary focus is on modelling ability or on mathematics. For transparency the method of analysis is illustrated in the next section.

Two examples analysed with the analytic scheme

To *test the coding on samples of texts* is included in the guidelines for doing a content analysis (Robson, 2002) and this section illustrates the analysis of a sample text. The sample is chosen to present the analysis of one explicit and one implicit description. In figure 3 the first example is given, an *explicit* description.

> Activity
>
> How many and for how long?
>
> To "model" in mathematics means to make a mathematical model, which is a simplification of a real situation. You start by making some assumptions and estimations and then use approximations in the calculations.
>
> Work in groups
> – Write down the estimations and computations you make in your group.
> – Discuss if the answer is reasonable.
> – Finally, compare your answer and your solution with other groups.
>
> 1. How many steps do you take when you walk to school?

Figure 3. *An example of an explicit instruction from Alfredsson et al. (2011, p.224 in 1b, my translation)*

In the first part of the analysis, the instruction text as illustrated in figure 3 is identified by the words "mathematical model" (counted once), "to model" (is interpreted as modelling and counted once), and "real world situation" (interpreted to be related to a real world problem and counted once). The words are placed in the introduction text under the label "Activity" (a description to clarify the meaning of modelling and instructions to students/ teachers who perform the activity) and this activity is described in one page.

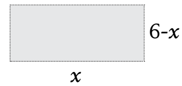
In the second part, both the notions model and modelling are analysed as *explicit*, because it is written that modelling means ”to make a mathematical model” and a mathematical model is ”a simplification of reality” (see figure 3). The aim of the activity is ”to model” (do modelling) and there are some written recommendations on how to work. Comparing the text with the framework using the questions in the analytic scheme, it is concluded that the text does not include statements that the question needs to be reformulated (*formulating a task in the domain of inquiry*). However, it is written in the text that assumptions and estimations are needed, which is analysed as it is unclear from the outset what kind of facts are most relevant to formulate a model (*selecting the relevant objects, relations and idealisations*). In addition, the text describes that students are supposed to ”model” and that includes ”to make a mathematical model” (assign variables/parameter and or constants to formulate a mathematical model addressing the problem, *translation into a mathematical representation*), ”use computations” (the students are supposed to use the model to calculate a result, *using mathematics to solve the corresponding mathematical problem*), ”discuss if the answers are reasonable” within the working group (the results are interpreted and evaluated in relation to the problem, *making an interpretation of the results in the initial domain of inquiry*) and ”finally, compare your answer and your solution with other groups” (more data is suggested to evaluate the mathematical model, *evaluating the validity of the model*). To summarise, the notions are described *explicitly*, the aim of the text is focusing on modelling and all aspects from the framework seem visible except for *formulating a task in the domain of inquiry*.

In the third part, the item 1 ”How many steps do you take when you walk to school?” (see figure 3) is used in the section modelling activity and it is not clear neither implicitly nor explicitly what type of mathematical model students are supposed to use (some type of estimation model), so the aim is focusing on modelling.

The second example, an *implicit* description, is presented in figure 4. In the first part of the analysis, the words ”mathematical model” are found in the beginning of the worked example. It is stated that ”she makes the following mathematical model” (see figure 4) and mathematical model is counted (once).

In the second part, the notion mathematical model is analysed as *implicit*. The term is not defined or given an explicit meaning and is not repeated in the text to clarify the meaning. It is not clear whether the mathematical model refers to the rectangular figure, the area, the function or something else. It is up to the reader to interpret ”mathematical model” by the content and the figures in the text.

Moa has 12 meters of fence and wants to enclose a rectangular area as large as possible. She makes the following mathematical model.



Assume that one side is x meters.

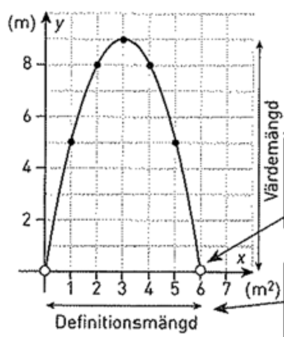
The other side is then (6-x) meters, because the two sides should be 6 meters together.

The area she calls y and gets $y(x) = x \cdot (6\text{-}x)$. One may say that the area y is a function of the side x. Moa computes the area for different values of x:

x (m)	**0**	1	2	3	4	5	**6**
y (m^2)	**0**	5	8	9	8	5	**0**

Moa realises that the side x needs to be larger than zero but less than 6. She writes $0<x<6$. This is called the domain of the function.

The graph below describes the area.



The area, y, is larger than zero and at most 9 m^2. That is the range of the function: $0<x\leq9$

The open circles mean that the point is not included in the domain

The side of the rectangle, x, is larger than zero, but less than 6, which is called the domain: $0<x<6$

From the graph it is possible to observe the maximum area = $9m^2$. That area is found for x=3, which means a square with the side 3 meters. We also observe that the area varies between 0 and 9. This is written as $0<x\leq 9$ and is called the range of the function.

! DEFINITION: Domain and range

If y is a function of x, the domain is all permissible x-values.

The range is the permissible values for y.

Figure 4. *An example of an implicit instruction from Sjunnesson et al. (2011, p. 201 in 1c, my translation).*

The title of the page is *Domain and range* (not possible to see in figure 4) and the page ends with definitions of domain and range, which therefore seem to be the objective with the worked example. Therefore, the aim is categorised as purely mathematical. The worked example does not include statements that the question needs to be reformulated in the domain of inquiry and it seems to be quite clear for "Moa" that she needs to analyse the sides of the rectangle to find the function for the area (it is clear what facts are most important). However, it is written in the worked example that Moa makes "an assumption" (i.e. assumption here means assign the variable x to one side), which is analysed as it is required to assign a variable to formulate a model. In addition, the worked example describes that Moa "makes the following mathematical model" (formulate a mathematical model addressing the problem). As seen in figure 4, Moa uses the function (a mathematical model) to calculate several results presented in a table and she interprets the result and evaluates the domain of the side. Notable is that the text then leaves Moa and her table unchecked and instead the authors of the worked example present a graph of the area function and write that the maximum value for the area can be found through observations of the range. Even though, there are some confusing descriptions between Moa's evaluation and some more general instructions to the reader about evaluation, there are descriptions about evaluation in relation to the formulated task in the domain of inquiry and in relation to the empirical data. The mathematical model (which may be assumed to be the area function) is not evaluated since it is not discussed at all. To summarise, the notion of mathematical model is described *implicitly*, the aim of the worked example is focusing on pure mathematics and the aspects from the framework visible in the worked example are: *translation into a mathematical representation*, *using mathematics to solve the corresponding mathematical problem*, and *making an interpretation of the results in the initial domain of inquiry*.

This example of an implicit description of mathematical model /modelling does not include a third part, because it does not include unsolved items.

Final phases of the content analysis

The *reliability of the coding* was assessed by letting an independent researcher do the same analysis of the second and the third part of the analytic scheme. The first part of the analytic scheme (counting the words and identify where in the text they are placed) was controlled by myself by doing the analysis a second time at a later occasion. The independent researcher was given the analytic scheme and the two analysed examples as an instruction for how to proceed. All the worked examples, model items and modelling items were scanned and mailed to the independent researcher for the analysis. The researcher was asked to check half of the worked examples and a fourth of the items. The selection of the sample was established with the use of software from internet that generated random numbers. Overall the result from the analytic scheme can be described as consistent between coders.

The only differences in coding in the second part were in one worked example about scaling and in one code in one example about the decrease of value of a car. The independent researcher argued for that the scaling problem (i.e. calculate the length of a car model in the scale 1:18 if the real car is 4.30 m, which was solved by 430/18 cm, see Alfredsson et al., 2011, 1b, p. 225) is a problem about proportional reasoning and includes *translation into a mathematical representation* ($1/18 = x/4.30$), *using mathematics to solve the corresponding mathematical problem* ($4.30/18 \approx 0.239$), *making an interpretation of the results in the initial domain of inquiry* (23.9 cm). This is one way to discuss about the problem situation, while another way, used in this paper, is to argue that the mathematical model discussed in the problem is actually a representation of a real object, a mathematical model of a physical object, which is a different meaning of a mathematical model and thus does not refer to mathematical modelling in relation to the framework. The second discrepancy between coding is in the sub question about the decrease in the car value;

> What question may be answered by the inequality $98000 \cdot 0.70^x < 20000$
> Solve the inequality graphically and answer the question.
> (Alfredsson et al., 2011, 1c, p. 304; my translation)

The independent researcher had followed the instructions and concluded that the task needed to be reformulated in order to get a more accurate question. However, the instruction about reformulating the problem refers to the framework's first sub process where the realistic problem situation is described in a general manner and needs to be more specified. The situation above is not describing a realistic situation, instead it is to find a situation based on a model (an inequality), it is an example of application according to Niss et al. (2007).

The discrepancies in coding in the third part about the aim of the items related to a minor misunderstanding about coding parts of the process as modelling, which were discussed and agree upon. Also some items that include modelling characteristics, but include guiding questions to be followed, were coded by the independent coder as mathematical modelling. However, following the framework, where modelling competency includes all aspects of the process, it is important that the students themselves find these questions. The coding of types of mathematical models were coherent in terms of exponential, linear and power functions, but not as coherent when it came to another level of models such as economic models, geometric models, statistic models, etc. This was not surprising, since the economic, geometric and statistic models are not as well defined as different types of functions.

The reliability test is concluded as being successful in terms of coding in relation to the framework, with almost identical coding, and in relation to the aims of the modelling activities, were the issue of a holistic modelling approach was brought up to discussion and agreed upon. However, the coding of the mathematical models was difficult since the developed framework included different levels of mathematical models (like linear functions and economic models).

At the time of this study, there were four identified publishers having published new textbook series in line with the new mathematics courses. The published series are *Exponent 1b* and *1c* (Gennow et al., 2011), *Exponent 1a* (Johansson & Olsson, 2011), *Matematik 1a* (Viklund et al., 2011), *Matematik 5000* (Alfredsson et al., 2011), *Matematik M* (Sjunnesson et al., 2011), *Matematik* Origo (Szabo et al., 2011). This textbook study include 14 textbooks, including all textbooks from the different series, except one textbook from Viklund et al. (2011), to which I had no access.

Results and analysis

The structure of the results in this section will be presented in three parts related to the Analytic scheme. The first part is connected to identification of modelling, the second part refers to the analysis of modelling instructions, and the third part is relating to the analysis of modelling items.

Results from the first part of the analytic scheme

This section presents table 1, which displays the frequency of the *words model, modelling, real world problems* (in Swedish modell, modellering and verkliga problem/situationer) as well as the number of pages where these words are found, and table 2 illustrating where these words are placed in the analysed textbooks.

The word *model* (model and mathematical model), as displayed in table 1, is frequently appearing in Matematik 5000, in Matematik Origo and in Matematik 1a but is less frequent or absent in Exponent and Matematik M. The word *modelling* (modelling and modelling ability) is only stated in the introduction in the Exponent series and it occurs a few times in the Matematik 5000 series. The words *real world problems* (real world problems and real situations) occur less than ten times per textbook in

Table 1. *The frequency of the words model, modelling and real world problems*

Textbooks	The freq. of words (in numb. of pages)		
	Model	Modelling	Real world problem
Mat 1a Ora	34 (6)		
Mat 1a Gre	34 (6)		
M1 a	2 (1)		
M1 b	2 (1)		
M1 c	14 (5)		8 (8)
Exp 1a		1 (1)	
Exp 1b	2 (2)	1 (1)	
Exp 1c	2 (2)	1 (1)	
5000 1a red	78 (22)	5 (4)	1 (1)
5000 1a yel	65 (21)	4 (3)	1 (1)
5000 1b	57 (18)	5 (4)	2 (2)
5000 1c	45 (21)	8 (6)	3 (3)
Origo 1b	49 (15)		9 (7)
Origo 1c	49 (15)		9 (7)

Matematik 5000, Matematik M and in Matematik Origo. Notable is that the word *model* in Matematik 1a is counted 34 times on 6 different pages, which is due to the fact that one worked example included the word model 13 times and in the consecutive page there were items to be solved that included the word model 8 times.

Table 2 shows that the words *mathematical model* are most frequently used in items for students to solve in the analysed textbooks, except for Origo where the words most frequently are placed in introductions to chapters or activities. No worked examples including *mathematical model* are found in M1a, M1b and the Exponent series. The 5000 series and the Origo series place *mathematical model* a few times in other texts, which refers to table of contents, index, summary, diagnose testing and in a mind map. The word *modelling* is only used in the introduction to the textbooks in the Exponent series (i.e. modelling ability) and in the introduction to an activity in the 5000 series (see the analysed explicit example in the method section). The frequency of the word modelling in items in table 2 is marked with brackets, because they do not include the word modelling. However, these items are explicitly described as a modelling activity or as a modelling ability. The word modelling is also placed in other texts in the 5000 series, which refers to the index and the table of

Table 2. *The frequency of where in the text the words model, modelling and real world problems are placed.*

Textbooks	The frequency of place in text											
	Model				Modelling				Real world problem			
	A[1]	B	C	D	A	B	C	D	A	B	C	D
Mat 1a Ora	2	1	5									
Mat 1a Gre	2	1	5									
M1 a			1									
M1 b			1									
M1 c		1	4						5		3	
Exp 1a					1		(20)[2]					
Exp 1b	1		1		1		(17)					
Exp 1c	1		1		1		(21)					
5000 1a red	7	2	14	5	1		(4)	2				
5000 1a yel	7	2	11	5	1		(4)	2	1			
5000 1b	2	3	15	5	1		(4)	3	1			1
5000 1c	5	3	11	5	2		(8)	2	2			1
Origo 1b	7	2	5	2					2	2	2	
Origo 1c	7	2	5	2					2	2	2	

Note. 1. A = Chapter introduction; B = Worked examples; C = Items; D = Other.
2. Number in brackets show items explicitly described as a modelling activity without use of the word *modelling*.

contents. The words *real world problems* or *real world situations* are not occurring very frequently. The word is found five times in introductions to the five chapters in M1c as a goal to be learned. It is the same text for all chapters, which is "Here you may learn [...] interpret a realistic situation and chose proper solutions to solve mathematical problems" (Sjunnesson et al., 2011, M1c, p. 6; 60; 110; 172; 242). The 5000 series includes *real situation* in the introduction to the modeling activity, which is also found in two items in Origo and in three items in M1c. Origo uses *real word problems* in the introduction to a section called "Equation as a mathematical model" (Origo 1b, p. 96; my translation) and in two worked examples, i.e. "we translate a real world problem to a mathematical model" (Origo 1b, p. 104; my translation).

The results from the second part of the analytic scheme

This section gives details how the textbooks treat modelling in terms of explicit and implicit descriptions, the aim of the instructional texts and worked examples, and the analysis of sub-process. First, examples will be presented to illustrate how a particular book series has treated the notions, and finally a summary will be given in table 3.

Exponent

No explicit description of meanings of *modelling* or *models* is found in the Exponent series. The word *model* is only found in one item and in the introduction to probability. One may read that ”[p]robability models are used among other things in biology for population growth and the spreading of disease, in physics to describe radioactive decay and in finance for insurance premiums and fund investments” (1b, p. 241).

The word *modelling* (i.e. modelling ability) is only used in the textbooks’ aim for students to practice the seven teaching goals listed in the curriculum. All items (often whole sets of items) are marked with one or more of the seven teaching goals in a ”ladder” (see figure 5, where number 4 to the right refers to the modelling ability). About 20 places in respective textbook have been ”marked” as focusing on modelling by the authors.

ÖVNINGAR

2029 Lös ekvationerna
a) $5x = 45$
b) $6x = 20{,}4$
c) $3{,}7x = 18{,}5$
d) $13{,}2x = 125{,}4$
e) $12 = 8x$
f) $14{,}7 = 21x$

2030 Lös ekvationen
$x + 2x + 3x + \ldots + 100x = 15150$

2031 Vilka av ekvationerna har lösningen $x = 3{,}5$?
a) $7x = 24{,}5$
b) $8x = 28$
c) $9x = 40{,}5$
d) $12x = 42$

1 2 3 4

2032 Lös ekvationerna
a) $x + 19 = 40$
b) $x - 9 = 14$
c) $12 = x + 9$
d) $52 = 77 - x$
e) $21{,}7 + x = 19{,}2$
f) $x - 44 = 11{,}5$

Figure 5. *Exercises marked with modelling ability (Johansson & Olsson, 2011, p. 93)*

It is not evident what meaning the textbook authors give to modelling ability by looking at these tasks in figure 5. One interpretation is that the item 2030, which is dealing with an arithmetic series, is their suggestion of a modelling item. A further discussion of the marked examples is found in the third part of the result.

Matematik 1a

No explicit descriptions of the notions mathematical *models* and *modelling* are found in Matematik 1a (Viklund et al., 2011). However, the authors use and describe the word model with a ”non-mathematical meaning” and refer to how the organization of the content is described (all these meanings are omitted in table 1). The use of the word *models* (related to mathematics) is found in the introduction texts to problem solving strategies and functions. The authors write that a good help to solve problems is "to use laboratory equipment or models" (Viklund et al., 2011, p. 25, green; my translation) and later they write that "THEN YOU HAVE TO KNOW [...] to use linear and exponential models to describe a course of events and understand these models in programme-specific subjects and in everyday life" (Viklund et al., 2011, p. 208, green; my translation). It is up to the reader to interpret the meaning of models in the first introduction to problem solving strategies and in the second it implicitly refers to linear and exponential functions.

There is one worked example that includes the word model, which is presented in both textbooks and is about a company that is going to use control technologies for reducing emissions of zinc. Two models are given in the text (model 1: $U = 0{,}1 \cdot 0{,}9^x$ and model 2: $U = 0{,}1 - 0{,}005^x$) for the reduction of emission of zinc (U in mg/m^3) after some time (x years). The questions raised are (a) to describe both models with words, draw the graphs to the given models with a graphical calculator and (b) read with help of the calculator how large the emission of zinc is after 5 years with model 1 and (c) when the emission is similar independent of two models. The solutions to (a) and (b) indicate that the aim of the worked example is focusing on mathematical aspects (describing meanings of a given exponential functions and linear function). Another aim seems to focus on how to use a graphical calculator (i.e. what buttons to push). The only sub-processes visible from the framework is *using mathematics to solve the corresponding mathematical problem* and *interpretation of the results in the initial domain of inquiry*, since the students are supposed to use the model to calculate a result with the use of a graphical calculator and interpret the answer in the calculator window.

Matematik 5000

The Matematik 5000 series is the only series in this sample having explicit descriptions of both the words *model* and *modelling* (to model), which were discussed in the explicit example related to figure 3.

The word *model* is also found in introductions to chapters, as headings like linear and exponential models (only in the 1a red and yellow) and probability models. In addition, some worked examples include the word model. For instance, one worked example is about the decreasing value of a computer that initially costed 6000 SEK; either the value decreases with 1200 SEK every year or with 25 % of its value every year. The solution is displaying a table, a graph and a formula. In relation to the linear model it is described that "after 5 years the value is 0 [and] [t]he formula is only valid for 5 years" (1a red, p. 270; my translation). The aim of this worked example is towards mathematics with focus on different representations. The models are implicitly predefined since the sections before are dealing with linear and exponential functions. The sub-process visible are *translation into a mathematical representation* (different representations), and *making an interpretation of the results in the initial domain of inquiry* (the validity of the linear model). However, the sub-process *using mathematics to solve the corresponding mathematical problem* is not found since no particular value from the mathematical model is asked for.

Matematik M

In the Matematik M series (Sjunnesson et al., 2011) the *descriptions of modelling* and *models* are implicit. The word *model* is not frequently used. In textbooks 1a and 1b the word appears twice in a section about scale, and in textbook 1c the word appears 14 times, most frequently in items. The only worked example (the implicit example in the analysis section) is relating to functions and has a focus on mathematics.

Matematik Origo

The meaning of the notions of *models* and *modelling* are explicitly described in the introduction to the chapter about algebra and equations (identical descriptions in both textbooks). A mathematical model is described to be useful when solving problems from other school subjects and everyday problems and is defined as a "formula or an equation describing reality in a simplified way" (p. 74 in 1c; my translation). However, as in the other analysed textbooks the word *model* is mainly found in the section dealing with linear and exponential functions. The notion of *modelling* is described without the word modelling as "a method for problem solving with equations as a mathematical model, which includes three steps" (p. 75 in 1c; my translation) illustrated by a circle with one-way arrows (see figure 6).

The first phase as seen in figure 6, *translation*, is to assign a variable to the wanted number and then translate the real world problem into an equation. The second phase, *work within the model*, is to solve the equation and the third phase, *interpreting*, is to interpret the mathematical solution and formulate an answer. If the answer does not answer the initial question then you are supposed to go back to phase one. *Translation*, *work within the model* and *interpreting* are also parts the framework. However, the sub-process *formulating a task in the domain of inquiry; selecting the relevant objects, relations and idealisations* seem to be lacking. In addition, *evaluating the validity of the model* is not based on new data. Instead it is

A method for problem solving with equation as mathematical model may include three steps.

1. Translation
Here you assign a variable to the wanted number, for example x. Then you describe the real problem by translating it to an equation.
2. Work within the model
Here you solve the equation in an ordinary way and you can ignore the original problem. This step is easier to do from a mathematical point of view.
3. Interpretation
Here you interpret the solution of the equation and formulate an answer to the original problem. It is important to evaluate whether the mathematical solutions can answer the original question.

If the mathematical solutions do not answer the original question, you have to go back to step 1 and see if it is possible to improve the model.
To solve problems with mathematical models can therefore be compared to working in a circle.

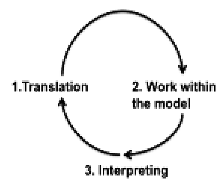


Figure 6. *Modelling in Szabo et al. (2011, p. 75 in 1c, my translation)*

described in figure 6 that ”[i]f the mathematical solutions do not answer the original question, you have to go back to step 1 and see if it is possible to improve the model”. However, phase one is to assign a variable for a wanted number and translate the original problem to an equation, and it is not clear to the reader where or how it is possible to improve the model in this phase, which makes the phase between *interpreting* and *translating* a bit odd.

To describe the three phases a worked example of a real world problem is given (a standard application according Niss et al., 2007). ”A quadratic room has the area of 25 m^2. What size does the room have?” (p. 78 in 1c; my translation). Phase one is to assign the variable x to the side and set up the equation $x^2 = 25$, phase two to solve the equation, and phase three is to interpret that only the positive root is adequate. The phases seem to correspond to the sub-processes: *translation into a mathematical representation*, *using mathematics to solve the corresponding mathematical problem*, and *making an interpretation of the results in the initial domain of inquiry*. However, even if the analysed worked example is related to the instructional text describing a problem solving strategy (modelling), the aim of the worked example is a standard application focusing on mathematics.

The second worked example is to compare two options for bike rentals. In the solution the text book authors state that ”[w]e translate a real problem to a mathematical model” (1b, p. 104; my translation). However, this time the mathematical model is not an equation but an inequality. The analysed focus on the ”rental” example is on mathematics, since the previous worked example on the same page is illustrating how to solve inequalities like ”$2 + x < 3x - 4$” (1b, p. 104) and the ”rental” example seems to be a standard application.

Notable is that the heading ”mathematical models” (1b, p. 119), found in a mind map of the chapter ”Algebra and equations” and has a list of two points (mathematical problems and a method for problem solving), is later found as a problem solving activity related to functions. The introduction text to that activity declares that ”[w]ith help of mathematics is it possible to make *models* that describe real situations in a simplified way. These are very useful, but they almost always give a limited view of reality” (1b, p. 195; my translation). Here a critical stance towards mathematical models is indicated. However, the authors do not discuss what *models* are, more than giving the example of equations and listing a few problems related to linear and exponential functions.

Summary of the results from the second part of the analytic scheme

Table 3 displays whether models and modelling are described implicitly (Im), explicitly (Ex) or is absent (no mark) in the analysed textbooks. In addition, the table shows the aim and the frequency of the analysed instructional texts and worked examples that include models and modelling. For example, Ma2 means that the aim of the text is focusing on mathematics and is found two times. The six columns to the right in the table refer to the frequency of identified sub-process from the framework.

From table 3 one can observe that three of the textbook series (Mat, M1 and Exp) only use the term model as implicit descriptions in instructional texts or worked example, which means that it is up to the students to interpret the notion. The other two textbook series (5000 and Origo) give explicit descriptions of models as ”formula or an equation describing reality in a simplified way” (Origo, p. 74 in 1c; my translation) or as ”a simplification of reality ”(5000, p. 224 in 1b; my translation). The notion of modelling is not discussed at all in two of the textbooks (Mat and M1). In the Exponent series modelling is described implicitly in some items as an ability and in the Origo series the word modelling is not used (implicit description), but the notion of modelling is described explicitly

Table 3. *Summary of findings from of the second part of the content analysis*

Textbooks	Model	Modelling	Aim of text	Ma/ Mo	Formulate	Select	Translate	Using math	Interpret	Evaluate
Mat 1a Ora	Im		Ma1					1	1	
Mat 1a Gre	Im		Ma1					1	1	
M1 a	Im									
M1 b	Im									
M1 c	Im		Ma1				1	1	1	
Exp 1a		Im								
Exp 1b	Im	Im								
Exp 1c	Im	Im								
5000 1a red	Ex	Ex	Ma2	Mo1		1	2	1	2	1
5000 1a yel	Ex	Ex	Ma2	Mo1		1	2	1	2	1
5000 1b	Ex	Ex	Ma3	Mo1		1	2	3	4	2
5000 1c	Ex	Ex	Ma3	Mo2		2	3	4	5	3
Origo 1b	Ex	Ex/Im	Ma3				3	3	3	
Origo 1c	Ex	Ex/Im	Ma3				3	3	3	

as a problem solving activity with three phases. The 5000 series describes modeling explicitly as a group activity to develop mathematical models, including making assumptions, estimations, calculations and evaluation.

Only the 5000 series has a particular instructional text (there is one activity in 5000 1a and 1b, and two activities in 1c) with an aim to teach modelling. All other instructional texts or worked example seem to focus on other topics in mathematics. As seen in the table 3 the frequency of the analysed texts is at most five (5000, 1c).

Relating the instructional texts or worked example to the framework, it is found that focus is *towards translation into a mathematical representation, using mathematics to solve the corresponding mathematical problem* and *making an interpretation of the results in the initial domain of inquiry*. Only the 5000 series is considering *selecting the relevant objects, relations and idealisations* and *evaluating the validity of the model*. No series is discussing *formulating a task in the domain of inquiry*.

The results from the third part of the analytic scheme

This section presents the results concerning the aims with the analysed items and types of mathematical models the students are supposed to use to solve the items. Similar to the previous section, examples will first be presented to illustrate what types of items a particular series has used. The result is then summarised in table 4.

Exponent

The word *model* is found in one item in textbooks 1b and 1c. The section including this item is called ”Different ways to describe functions” (p. 205, 1b; my translation). The item is labeled as a ”Challenging problem” and is marked with the *modelling ability*. The heading of the item is postage charges and the students are given a table with two columns with maximum weight and price. The question is how to draw a graph relating to the table. Three questions are specified as; ”What does it mean if you draw a graph as a continuous line?; What does it mean if you draw a graph as ”stairs” (horizontal lines); What model shows the connection realistically?” (p. 209, 1b; my translation). The aim of the item is analysed as focused on *mathematics*, since the students are supposed to interpret a predefined *graphical mathematical model of continuous and stepwise functions* in a section about different ways of describing functions. The other items (about 20) in the textbooks marked with modelling (i.e. modelling ability) are labelled as exercises, challenging problems and group activities.

In figure 5 one example of a set of exercises is marked with modeling ability and these exercises and all other exercises can be characterized as focusing on mathematics and most of them are closely related to the chapters about linear and exponential functions.

Group activities marked as modelling is exemplified with ”critically examine a given diagram”, ”buying a car” and ”arranging a concert”, and they often include guiding questions to be followed (see figure 7).

Buying a car

Task

To buy a car and calculate the monthly cost. Expect service, operation and maintenance for five years. The car runs about 15000 km per year and it must be fully paid after 5 years.

Procedure

Select a car priced 100000-200000 SEK
Select a creditor (bank, car salesman or other)
Find out the current interest (home assignment)
Include down payment (at least 20%)
Consider tax deduction (30%)
Include the service costs, car care, taxes, insurance and tyres
Expected operating costs (petrol, oil, etc.)
Make a table of the costs

Report

Report in a table (e.g. Excel)
Describe and show the calculations
Report the monthly cost
Explain what mathematics you have used

Figure 7. *An example of group activity (p. 145, in 1a, my translation)*

The guidelines described in figure 7, e.g. select a car priced 100000–200000 SEK, do not seem to be a part of the used framework. The students themselves are supposed to ask these questions and there is no instruction/guideline in relation to the evolution of a solution (or the mathematical model). A consequence of these guidelines to be followed is that this activity is not considered as modelling, because the model is already implicitly given since the wanted variables are found in the guidelines. All the group activities in the exponent series seem to aim towards mathematics except one item. This group activity is called ”Billion” (1a, p. 18) and students are supposed to develop ten questions in relation to a billion. The text gives some example, where one example of question is characterized as a Fermi problem (i.e. ”how long time does it take to count to one billion” (1a, p. 18)).

Matematik 1a

There are two types of items, *tasks* and *missions*. The tasks are ”regular” textbook items categorised in three levels of difficulties and the missions

> are more extended tasks that you can choose to do [...] [t]here is an advantage to solve the missions in groups and gives you a possibility to develop different mathematical abilities, such as solving mathematical problems, both theoretically and practically, and communicating mathematics. (orange, p. 3; my translation)

Most of the counted numbers of the word model (33 out of 34 times) are found in the section about functions and almost all of these are described in one solved example, one mission and four tasks (31 out of 33 times). Most of the tasks analysed refer to linear functions and exponential functions.

The missions including the word model, placed in the section about exponential and linear functions, are called ”student accommodations at after-school recreation centre is needed” (green, p. 233) and ”hotel rooms are needed” (orange, p. 233). The two missions are actually the same mission, because after the first line in the missions the texts are almost identical. There are two employees that independently of each other develop a linear model ($N = 150 + 20t$) and an exponential model (increase of 9 % every year) to estimate the need of ”rooms” or ”accommodations” in the next five years.

Matematik 5000

The word models are found in tasks and in an investigation task. All these tasks and the investigation task are about linear and exponential functions (one task is about power functions) focusing on mathematics. The items include aspects of comparing and using predefined models and to set up linear and exponential models based on text problems.

Examples of modelling problems are: how many persons can fit on a football field and how long time does it take to walk the stairs to the top floor in a 54 floor high building? These tasks may be characterized as Fermi problems, and the last task above is similar to an example used by Ärlebäck (2009) in a research study to introduce modelling.

Matematik M

In textbooks 1a and 1b there is only one item about scaling that includes the word model. However, textbook 1c includes more items (four) where the word model is a part of the text. These items are categorized in the book as tasks, test tasks and activity.

The two tasks and the test task in the textbook are about using predefined power functions and exponential functions in some context. The activity is a practical task, found at the end of the section exponential functions, where the students are going to observe how the bounce height of a bouncing ball decreases with the number of bounces. The students are supposed to find a function describing the situation. However, the activity is structured with guidelines and quite similar to the group activity in the Exponent series. For example the first guideline is ”[i]nvestigate the height (h) of the bounces as a function of x for at least six bounces and fill in the table” (1c, p. 222; my translation).

Matematik Origo

The five items related to the word model are found in tasks, chapter test, problems and investigations, and ¤-tasks.

The one task in the section ”percentage changes” is about the change in value of a painting, where the painting either increases in value and then decreases (model 1) or the value is decreasing and then increasing (model 2). The students are to find out if there is a difference between these percentage models. Two items are described in a ”chapter test” related to functions where one of the test items is for the student to ”[d]escribe the difference between a linear model and an exponential model” (1b, p. 200; my translation). The item called ”problems and investigations” is a set of five implicit given exponential and linear functions for students to set up and compare (for example ”1. The world’s population is now about 6 billions and is growing by 1.5 % per year. 2. A new born baby weighs about 4000 g and increases its weight by about 400 g per week” (1b, p. 195; my translation). The ¤-task is a theme task where it is possible for students to develop abilities and knowledge required for the higher grades. The theme in the ¤-task, where the world model is found, is about Stockholm marathon and the students are supposed to draw a linear function based on a table with distance and time and then answer some questions like ”why doesn't the line go through the origin [...] give an equation for the straight-line as $y = kx + m$ [...]” (1b, p. 193; my translation).

All the five items summarized above do not seem to focus on modelling in terms of the framework, since the models are given implicitly to the students in relation to the specific chapters (percentage or functions).

Summary of the results from the third part of the analytic scheme

Table 4 (below) shows the *aim* of the analysed items in terms of mathematics (Ma) or modelling (Mo). The textbook series with an aim at modelling are the 5000 series and Exponent 1a, with items described as Fermi problems. These Fermi problems are categorised as estimation models and they are found in the column O*ther* in table 4. Three of the textbook series (Mat, M1 and Origo) have quite few items dealing with models and these mathematical models are mainly exponential and/or linear functions (an item may include both an exponential and a linear function). The 5000 series also mainly use exponential and linear functions as models. The Exponent series on the other hand uses a variety of mathematical models, were the most frequent category is Other as seen in table 4. For example geometric sums, probability models, geometric models, binary models and rational functions are used.

Table 4. *Summary of findings from of the third part of the content analysis*

Textbooks	Aims	Exp	Linear	Power	Scale	Percentage	Statistics	Economics	Other
Mat 1a Ora	Ma	5	3						
Mat 1a Gre	Ma	5	3						
M1 a	Ma				1				
M1 b	Ma				1				
M1 c	Ma	3	1	1					
Exp 1a	Ma/Mo		1		2		2	4	11
Exp 1b	Ma	4	4	1		1	1	2	5
Exp 1c	Ma	6	3	1		3	1	1	9
5000 1a red	Ma/Mo	9	8		2				4
5000 1a yel	Ma/Mo	8	6		1				4
5000 1b	Ma/Mo	6	8	1	2				4
5000 1c	Ma/Mo	5	8	1					8
Origo 1b	Ma	1	3	1		1			
Origo 1c	Ma	1	3	1		1			

Discussion

There is a variety of descriptions between the analysed textbooks both in terms of how frequently the words models, modelling etc have been used and how the notions have been presented and treated.

Regarding frequency, the word *model* is at most found 78 times on 22 pages in textbook 5000 1a red, but is not found at all in Exponent 1a. The descriptions of the meanings of the word *model* also differ between the different textbook series. Three of the textbook series (Mat, M1 and Exp) describe model with implicit descriptions in instructional texts or worked examples. The other two textbook series (5000 and Origo) give explicit descriptions of models as "formula or an equation describing reality in a simplified way" (Origo, p. 74 in 1c; my translation) or as "a simplification of reality" (5000, p. 224 in 1b; my translation). However, since the descriptions of the word model are somewhat vague it may open up for different interpretations by different readers. How the textbooks use the notion may also be a bit confusing for the students. For example the 5000 series asks students to find a mathematical model or formula (Ma 5000 1b, p. 339) without discussing similarities and differences between mathematical models and formulas as well as uses everyday meanings of models, such as T-shirt models (size or brand) (Ma 5000 1b, p. 31). Origo describes models as formulas or an equation, but gives examples of models with inequalities. The finding that most of the model tasks refer to linear and exponential functions is not surprising. Linear and exponential functions are functions emphasised in the curriculum (Skolverket, 2011) as a part of the first mathematics course. Nevertheless, the textbooks do not use this as an argument to give a more clearly elaborated presentation of mathematical models described as functions.

The word *modelling* (i.e. modelling and modelling ability) is only found in the introduction in the Exponent series and a few times in the Matematik 5000 series. How the notion of modelling is described ranges from more explicit descriptions such as a cyclic problem solving method (Szabo et al., 2011) and an activity to solve Fermi problems (Alfredsson et al., 2011) to more implicit descriptions in form of tasks for the students to solve (Gennow et al., 2011) as well as tasks that include the word model without further explanations (Viklund et al., 2011; Sjunnesson et al., 2011). The use of Fermi problems as a way to introduce modelling as found in Alfredsson et al., (2011) "may provide a good and potentially fruitful opportunity to introduce mathematical modelling at upper secondary school level if this activity is followed up appropriately" (Ärlebäck, 2009, pp. 44–45). However, beside the Fermi problems, the items related to models and modelling that were analysed seem to aim towards mathematics. Also the analysis of the instructional texts or worked examples indicates a focus on mathematics (i.e. *translation into a mathematical representation*, *using mathematics to solve the corresponding mathematical problem* and *making an interpretation of the results in the initial domain of inquiry*). Only the 5000 series considers other aspects of modelling such as *selecting the relevant objects, relations and idealisations* and *evaluating the validity of the model*. No series is discussing formulating a task in the domain of inquiry. A holistic approach towards modelling includes all aspects of modelling, which seems not to be a part of the investigated textbook series and is quite similar to the result in Frejd's (2011) investigation of modelling items in national course tests. The test items mainly focused on the intra-mathematical aspects of modelling, such as using already existing models to calculate results or assigning variables to formulate mathematical statements to calculate results.

The use of models not connected to linear and exponential functions have potential for further development by textbook authors. The statistics section (which is a part of the first courses) seems to be promising, since it is stated in the curriculum to "[e]xamine how statistic methods and results are used in society and in science" (Skolverket, 2012, p. 9). Many of the decisions made in society are based on statistic and economic models, but it is not always the best model chosen in practice, since there are more stakeholders with their own purposes that come into play (i.e. politicians, companies, negotiators, etc) (Jablonka, 2010). Textbook authors may construct modelling items where students are to gather their own statistical data to develop mathematical models that may be used for decision making. An environmental example could be for students to develop a mathematical model to decide if the school would benefit from building solar panels to gain cheap electricity. Items like that may develop critical discussions in the classroom about how mathematical models are used in the society and what aspects are important to include in the model. Such critical discussions may contribute to develop critical citizens in a democracy (Skovsmose, 2006).

There may be many reasons why the descriptions of modelling found in the textbooks varied. Some suggestions are that the modelling ability in the curriculum is not interpreted as a holistic ability by the textbook authors, the authors emphasise other aspects of mathematics as more important, or the authors did not have enough time to reflect about the new curriculum and relied on older textbooks. The textbooks are commercially produced, which may be another reason, where the most important drive might be economic interest to gain a large proportion of the market share instead of pedagogical influences (Cháves, 2003). According to Frejd (2011), teachers do not emphasise modelling in their everyday teaching but rather see modelling as a part of physics, and thus it may not be economically proficient to include it in the textbooks. The modelling activities as highlighted as Fermi problems in the 5000 series are ”extra activities” and one may wonder what happens to these extra modelling activities, when the Swedish mathematical textbooks often include too many pages for a class to finish during classroom time (Johansson, 2005). It is up to the teacher to decide what types of group activities, challenging problems etc. he/she would give priority to in the classroom, and if the teacher in the Swedish upper secondary school believes that modelling is a part of physics there is a risk that these modelling activities might be excluded.

The results about the variety of descriptions also indicate that all students do not get the same possibilities to develop the modelling ability as stated in the curriculum if the books are being followed. This implies that the teachers need to complement their teaching of modelling with other teaching material in order to meet up the curriculum goals, in particular if one follows the principle in Blomhøj and Kjeldsen (2006) that ”the pedagogical idea behind identifying mathematical modelling competency as a specific competency is exactly to highlight the holistic aspect of modelling. Modelling competency is developed through the practice of modelling.” (p. 166). It can be argued that qualitative teaching may compensate for inadequacy of mathematical textbooks but according to Schoenfeld (1998) there is no evidence that it does. This brings forward the question if any textbook series more adequately than another is presenting models and modelling. If the students are left to themselves to work in the textbooks, as some evidence indicates as the current situation in Sweden (Jablonka & Johansson, 2010), the content of modelling should be presented explicitly (Niss et al., 2007). Only two of the textbook series present explicit descriptions of modelling either as a cyclic problem solving method (Szabo et al., 2011) or as an activity to solve Fermi problems (Alfredsson et al., 2011). However, modelling is still only presented in a marginalized way, which means that the textbooks only do not enough support students to develop a holistic modeling ability. In order for a modelling ability to be developed, modelling needs to be explicitly taught with support from a teacher (Niss et al., 2007).

Conlusion

The most prevalent teaching method in the Swedish upper secondary school is individual work with mathematical textbooks, complemented by short teacher introductions (Jablonka & Johansson, 2010). According to Reys, Reys and Chávez (2004) the textbook is the main source for how and what the teachers will teach and in the end what the students will learn. Therefore, it is of relevance how and what is described in mathematical textbooks about different mathematical notions.

In this study it is concluded that mathematical modelling, which is one of the seven main abilities to be taught in the national curriculum (Skolverket, 2012), is not treated as a central notion in the analysed textbooks. None of the textbooks analysed does really support the fulfilment of the Swedish curriculum concerning mathematical models and modelling. The descriptions of both mathematical models and modelling vary between the analysed textbooks both in terms of how frequently the words models and modelling have been used and how the notions have been described. Except for a few Fermi problems, the aim of the analysed models and modelling items is towards intra-mathematical aspects of modelling. That result seem to mirror the results in Frejd (2011) where national course test items and secondary teachers' conceptions of mathematics also seem to focus mainly on intra-mathematical aspects, even though Frejd's (2011) results were established before the new curriculum with the modelling ability was implemented. There seems to be a large potential for Swedish textbook authors to further develop the presentation of the notion of mathematical modelling and mathematical models. The five textbook series, assumed to be representative of the textbooks used in Swedish mathematics education at upper secondary level, do not strongly, and to very different extent, support the current curriculum in terms of modelling ability and the consequences will be that students who want to meet the standards in the curriculum to develop a holistic modelling ability need to get support from a teacher with complementing material.

This study may also contribute to the ongoing research, from a methodological point of view, with an emerging framework for analysing textbooks with focus on modelling. It provided information about similarities and differences of how models and modelling are described in different textbooks, but at the same time it also revealed that in terms of analysing items for students to solve, the method needs to be further developed.

Acknowledgments

I would like to thank my supervisor Christer Bergsten and the reviewers for valuable comments for the final version.

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