# Making sense of a "misleading" graph

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Given the importance of a critical-analytical disposition in the case of graphical artefacts, this paper explores *graphicacy* based on students' answers to an item from PISA survey test. Primarily, results from the written test were analyzed using PISA's double-digit rubrics or coding. In evaluating these categories, it is observed that just a small percentage of students are able to produce answers that reflect a critical-analytical approach with respect to the use of statistical/mathematical operators and forms of expressions. Secondly, video observation shows that students tend to employ what is perceived as an "identification approach" while discussing the task. Whereas elements of mathematical and statistical ideas can be identified in the students' discussion, these are not explicitly stated and are largely submerged in everyday concerns and forms of expression.

The importance of using *graphical representations* (henceforth referred to as *graphical artefacts*) in enhancing teaching of mathematics is well recognized (e.g. Arcavi, 2003; Bloch, 2003). Whereas in mathematics, graphical artefacts per se may not be sufficient for expressing mathematical concept(s) and therefore students may need to be conversant with the manifestation of these concepts in other semiotic systems or settings (see for example Bloch, 2003). Graphical artefacts are central to statistical literacy. In the present study statistical literacy is perceived as the ability to critically read, interpret and communicate data effectively with the help of statistical tools and forms of expression (cf. Gal, 2002; Watson & Kelly, 2008). In statistics, graphical artefacts are vital components of the process that defines statistical enquiry i.e. the initial question, determining a sample, data collection and data analysis (cf. Garfield & Gal, 1999; Watson, 2006). Thus, it is expected that instructional designs involving graphical artefacts ought to pay considerable attention to the factors and

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Olande, O. (2013). Making sense of a "misleading" graph. *Nordic Studies in Mathematics Education*, 18 (1), 5–30.

processes generating, as well as surrounding the production and interpretation of graphical displays. In this way it might be possible to meet the anomaly that was observed by Gal (1998), that the mathematics and statistics education community has overemphasized the development of skills that enable students to generate and act on data at the expense of skills about evaluating and communicating the meaning and implication of data. In the context of graphical artefacts, the generation, evaluation and communication are perceived as embedded in the construct *graphicacy.*

## *Graphicacy*

The construct of graphicacy describes the ability to understand and present (communicate) relationships that cannot be successfully communicated by words or mathematical notation alone (Balchin & Coleman, 1965). In the construct graphicacy, Balchin and Coleman aimed at including "graphing" as one of the underpinnings of sound education alongside the ability to *read*, *write* and do *arithmetic*. Thus their effort would be perceived as an attempt at expanding the "literacy" sphere to include graphing practice. However, the demands of contemporary society seem to be shifting away from a broad based view of the construct of literacy to placing emphasis on what can be referred to as the essential domain of specific literacy e.g. mathematics literacy, reading literacy, statistical literacy and scientific literacy (cf. OECD, 2006). However, given the multi-disciplinary nature of graphing, it might be challenging to confine it to a particular subject domain. Thus, while appreciating that in the subject domain of statistics, graphicacy might be a subset of statistical literacy. In the present study the construct graphicacy is retained based on the assumption that in a school context, students might not grasp the graphical artefacts from a strictly subject specific exposition

Another integral aspect of graphicacy is data integrity; the ethical-communicative considerations demanded of good graphing practice. Bertin (1967/1983) laid down what has been considered to be a comprehensive theory of graphing for both analytical and presentational purposes (see Wainer's foreword in Bertin, 1967/1983). In this theory, Bertin expounds on and illustrates the need to produce clear, simple and easy to understand diagrams and graphs. However, it seems that the theme of data integrity has been emphasised at the expense of other important aspects of graphicacy. According to Tufte (2001), much of the 20$^{th}$ century thinking about statistical graphs has been preoccupied by how some charts might fool naive viewers while other important issues, such as use of graphics for serious data analysis, are largely ignored (a case of a typical misleading diagram is reproduced in figure 1).

Thus, graphicacy is in the present study perceived as including the ability to interpret and produce graphical artefacts. Central to this activity is the need to uphold data integrity with respect to the production and communication of data as well as critical-analytical disposition. Thus, a number of factors need to be taken into account when investigating graphical comprehension, or graphicacy.

## Graph comprehension

Studies exploring students' graphic comprehension seem to arrive at various levels of graphicacy (e.g. Bertin, 1967/1983; Curcio, 1987; Gal, 1998, see also Friel, Curcio & Bright, 2001). Bertin (1967/1983) identified three levels of questioning that have impact on the level of reading or interaction with graphical artefacts: the first kind of question emanates from the graphical system, using it as a reservoir from which one extracts a piece of information. The second level of questioning typifies a form of internal information processing involving reducing the length of the components in order to discover groups of elements or categories of data, while the third level of questioning tends to reduce the information in the graphical system to a single, ordered relationship among components. These questions lead to three levels of graphic reading viz., *elementary*, *intermediate* and *overall* levels of reading respectively. Some correspondence is made between Bertin's levels of interaction with graphical artefacts and Curcio's (1987) levels of graphic comprehension. Curcio's levels are referred to as *reading the data*, *reading between the data* and *reading beyond the data* (see also Friel et al., 2001). These levels of graphic comprehension can generally be characterized as follows. Level one is associated with activities arising from say, solving a task that requires the identification of the month which receives the highest precipitation from a graph showing annual rainfall pattern for a given city. Level two includes such questions that require moving beyond isolated elements of the graph, for example answering the question: how was the rainfall pattern during the months of April through August. The third level is perceived as assuming a summative, inferential level and can be characterized by questions such as, what is your general impression of the town's annual rainfall pattern? Based on the data, what are the prospects of it raining in the town in October? The characterization of the three levels of graphic comprehension provided here is general since a closer look at the different versions of the "three level" graphical comprehensions, shows internal differences (see Friel et al., 2001).

The common denominator for the three levels of interaction with the graphical artefacts outlined above is that they arise from the kind of question posed. There are some questions which are perceived to attract surface interaction and some that attract various levels of deep engagement with the data. Gal (1998) explicitly identifies two questions that can be posed with regard to the interpretation of graphs and tables: *literal reading questions* and *opinion questions*. Literal reading questions are related to activities where students point to specific features or points on the graph or compare such points. Opinion questions on the other hand call for opinions where the focus is on the quality, reasonableness and relevance of evidence used to further a thought sequence. In putting forward the two types of questions Gal, in effect condenses the three levels of questioning mentioned earlier (see e.g. Bertin, 1967/1983; Curcio, 1987) to two (cf. Friel et al., 2001). Similar two-way approaches of characterizing interaction with graphical artefacts are also reported by Postigo and Pozo (2004) who posit that research on the learning of graphs makes a distinction between local and global interpretation of graphs (e.g. Ben-Zvi & Arcavi, 2001). In this distinction, global interpretation of a graph is perceived as being more difficult than local interpretation since it involves an abstraction process which a majority of the students have not achieved. Thus, an assumption might be made that literal questions probably lead to local interpretation of graphs.

Given the role that the type of questioning plays in interaction with graphical artefacts, it is the case that the types of questions that students meet in educational situations might have an impact on their general disposition with regards to interaction with graphical artefacts. Roth, Pozzer-Ardenghi and Han (2005), postulate that students generally adapt to certain ways of doing things by developing structured dispositions. While these structured dispositions have a function to instil in students a subject specific discourse, they might in some cases be uncritically employed which might be thus interpreted as indoctrination. Thus, a student who is used to certain ways of solving a particular type of problem will probably find it easier in the first instance to apply the same methods when confronted with a similar problem.

### *Critical analytical approach*

In the present study, a two-way classification of interaction with graphical artefacts is adopted. The approach draws inspiration from the recognition that a graphical artefact is part of a sign system where aspects of familiarity with context, content and necessary tools of interaction are seen as necessary for the sense making process (cf. Diezmann 2009; Monteiro & Ainley, 2007; Roth, 2009). Thus, whereas the type of questioning has effect on the quality or level of interaction with the graphical artefact, other aspects, such as tool use, also play an integral role in this interaction and subsequent sense making process. In this approach the first category, referred to as identification approach (to task solving) is characterized by an interaction activity where the visual-familiarity dimensions attached to the graphical artefact play a fundamental role in the sense making process. This refers to an interaction activity that is in the first instance guided by visual perceptual elements and where the use of familiar or elementary operators (e.g. addition, subtraction) and forms of expression are employed. The second category, the critical-analytical approach, is characterized by the presence of a deep engagement with the graphical artefact going beyond mere visual perception. In this category, one is critical and reflective with respect to the content and context and there is active selection and use of domain specific operators (e.g. proportion, fraction from mathematics and variability, sample space from statistics) and this might be challenging in normal circumstances (cf. Adjiage & Pluvinage, 2007; Jones, 2005; Pantziara & Philippou, 2011). Here one is critical in terms of questioning the production and communication of the graphical artefact, as well as analytical in terms of selection and application of appropriate subject specific tools and forms of expression. Monteiro and Ainley (2004) introduced the construct critical sense which they defined as the ability to look behind the data and deeply analyse information rather than simply accepting the initial impression given by the graph. In the context of the present study a critical-analytical approach is perceived as encompassing a reflective inquisitive disposition as well as the ability to make the appropriate choice of mathematical or statistical operators to solve problems at hand, as well as the use of appropriate language to express the results and processes involved. Thus, a critical-analytical approach is perceived as being in harmony with the notion of *mathematical literacy* conceived of as a way of thinking, reasoning and working ”mathematically”; allowing students effective engagement in everyday situations as active empowered and participatory citizens (Walls, 2009).

As a way of developing interpretive skills and statistical literacy, students need to pose critical and reflective questions such as those touching on the reliability of the measurements used, representativeness of the sample and the sensibility of the conclusions in light of the data collected and sample selection (Garfield & Gal, 1999). Consequently, instructional designs which promote a critical and reflexive approach to graphicacy fulfils one of the general aims of mathematical and statistical literacy as mentioned earlier, namely to equip students with such skills that are necessary for them to become critical-analytical citizens (cf. De Lange, 2006; Gal, 2002; OECD, 2003). Thus, one of the aims of institutionalized teaching and learning ought to be to equip students with structured dispositions that are critical-analytical (cf. Roth, Pozzer-Ardenghi & Han 2005). In fostering statistical literacy, the use of material from the everyday world e.g. media graphs and situations that students can easily identify with, has been suggested (cf. Gal, 2003; Rumsey, 2002; Watson, 1997). Material resources from the real world are not only often realistic and meaningful, but also have an advantage as a source for nurturing critical-analytical dispositions. This is in view of the fact that in the real world, graphical artefacts and statistical information may be presented in distorted ways to serve egocentric interests. Also, depending on context, the graphical displays might have emotional overtones or connotations. Thus a critical-analytical approach to interacting with graphical artefacts may help go beyond factors that may otherwise obscure an objective analysis of data involved. Some researchers (see e.g. Monteiro & Ainley, 2007; Roth et al., 2005) suggest that graphs can be interpreted differently depending on the context and that sometimes the values attached to the content and context of the graph in an out of school contexts may cloud the application of school based conventional practice.

### *A misleading graph*

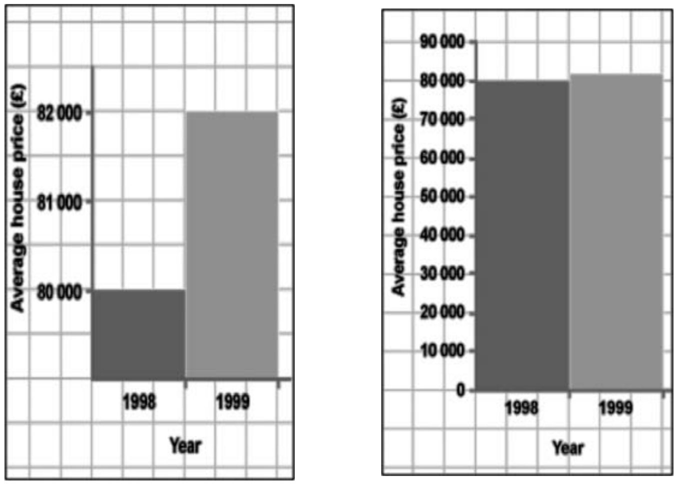
Regarding the use of material resources from the real world, the use of data from mass media has been suggested as a good source (e.g. Watson, 2006). Thus, it is not uncommon in school mathematics contexts to find illustrations exemplifying media coverage. A typical example (see figure 1) was retrieved from a mathematics resource website: GCSE *bitesize*, a BBC resource for students and teachers at the upper level of the compulsory school (see BBC, 2010). The approach used by the BBC to illustrate and correct misleading statistics is not unique for this website; a similar approach is not uncommon in some mathematics textbooks used in Sweden.

The information attached to the item was that statistics, though invaluable for providing evidence to support findings and making decisions, can be misleading and may be used in just such a way.

Figure 1a illustrates a classical case of misleading graphs, while figure 1b shows how this can be ”corrected”. The question attached to figure 1a is:

> What is wrong with this bar chart? How should the information be represented?

In connection with figure 1a, two things stand out. Based on the visual appearance (initial impression), figure 1a can be interpreted as twisted and thus misleading. However, the graph can also be perceived as an




a) Graph showing a "typically misleading graph"

b) Perceived "correct" display of the data in a)

Figure 1. *Depiction of a misleading graph and how this is rectified*

attempt to highlight or magnify a particular aspect of the prices which is not captured in figure 1b. In this case an acceptable correction would probably have included a broken line in the $y$-axis and then the discussion would be about the kind of deductions one can derive from the graph. Based on the appearance and construction of the graph (the discrepancy in the vertical scale), figure 1a is normally explained as intended to give the impression that values have increased by multiples of the smaller bar, e.g. in figure 1a, that house prices have tripled in one year.

As much as it is important for students to be conversant with the conventions of graph construction, an approach that focuses mainly on this aspect of graphicacy may deprive students of the discussion that can promote a critical-analytical disposition. The approach illustrated above may be perceived as belonging to a superficial reading where the focus is on the elements of the graphic material and identification of their different aspects (cf. Postigo & Pozo, 2004). This is expected since the question attached to the graphic is of the type *literal reading question*. It provides hints on where corrections need to be made and it does not suggest the need for a judgement or opinion (cf. Gal, 1998). It is by discussing possible underlying factors that may have generated the graphical display that the task is lifted from the visual perceptive dimension i.e. focusing mainly on the initial impression to the application of critical-analytical approach. In the present study a graphical item used in the OECD PISA survey 2003 is used to explore and illustrate students' graphicacy.

## Aims of the study

In exploring aspects of students' graphicacy, the present study applies the PISA survey item Robberies (PISA code M179Q01, see figure 2). This task calls for a reaction to information attached to a graphical artefact appearing in the newspaper indicating the occurrences of robberies. The focus of the study is to explore the approach and depth of analysis to reach a solution that the Swedish students employed as they solved the question posed, that is, the evaluation of the critical-analytical aspects of the students' responses. Here, the ability to select appropriate and optimal operators and being able to communicate solutions effectively was perceived as indicating elements of critical-analytical approach. In this way it is expected to be able to identify students' graphicacy when confronted with graphical artefacts in an educational setting.

The item was of the type constructed open ended. It had the characteristic of opinion question items as defined by Gal (1998) and it is therefore different from the BBC question in figure 1. As mentioned earlier, opinion questions elicit responses that provide more information giving insight on the solution approach. For this item the OECD average success rate (*p*-value) from PISA 2003 survey was just 30 %. It is worth mentioning that PISA tasks are based on a number of components such as the contexts or domains in which problems are located and the competencies needed to engage in the tasks (OECD, 2003). Thus, there are tasks based on public, educational, private and scientific situations, etc. each of which might involve different mathematical ideas such as change and relationship, uncertainty, space and shape among others. Given that this item constitutes the personal domain and was considered as requiring basic statistical literacy specific ideas associated with uncertainty, it was pertinent to further explore the nature of the students' responses. To achieve this, the following questions guided the investigation:

- What is the nature of the students' graphicacy, as manifested in their written and verbal responses to the given task?
- What forms of expression do students use when faced with a statistical/mathematical task containing graphical artefacts?

By posing these questions it is intended not only to explore the levels of graphical comprehension but also the possible factors that may influence the quality of students' solutions to typical items containing graphical artefacts.

## Method

The study is reported in two sections based on the two study approaches used to access and collect data. The first section of the study utilizes the results from the PISA 2003 survey double-digit rubrics. Double-digit rubrics allow for different credit awards: full or partial credits depending on the operational procedures used by the student(s). Operational procedure means not only providing descriptive solutions but also the use of mathematical language and symbols (cf. NCTM, 2000). In the double-digit rubric, the first digit shows the extent to which the response is correct while the second digit indicates the content of the response (cf. OECD, 2003). This digit has also been referred to as the ”didactical digit” (cf. Allerup, Lindenskov & Weng, 2006). A significant feature of the double-digit rubric is that the codes are prepared from field trials such that the codes can be perceived as providing categories of the different approaches to solving the task used by the student(s). A detailed description of the double or two-digit rubrics is provided by Lie, Taylor and Harmon (1996; see also Dossy, Jones & Martin, 2002). Since these scores are retrieved from the OECD PISA database the secondary analysis done might also be considered as background information.

The second section includes an observational study (video), conducted in the autumn term 2009. The students involved were students in the first year of upper secondary school (year 11). In the present study video observations from two study programmes are presented: *Child care recreational programme* (vocational study programme preparing students for a career at nursery schools, kindergarten and at after school recreational centres) and the *Natural science programme* (theoretically oriented study programme in mathematics, physics, chemistry, biology etc). Conducting the video observation study was perceived as offering a collaborative environment with a resulting possibility of encouraging peer scaffolding. The decision to include only students from these two programmes in the present study out of video observations involving 17 students from a total of five programmes, was based on aspects of phenomenography. According to the phenomenographic perspective on research, it is of interest to identify qualitatively different ways of experiencing, understanding and conceptualizing a phenomenon. These ways of experiencing are then summarized into categories expressing the different ways of ”interacting” with the phenomenon. Thus in interviews or observational situations it is possible to attain a ”saturation point” such that adding more material, subjects or interlocutors would probably not add more information to the results or observation at hand (cf. Limberg, 2008; Marton, 1981; Person, 2006).

There is some level of correspondence between the double-digit rubrics and the categories created from a phenomenographic analysis; namely that both coding by means of double-digit rubrics and phenomenography seek to capture the different ways students interact with a given mathematical task (cf. Lie et al., 1996).

### *Sample selection*

One of the salient requirements for students participating in the PISA survey is that they should be about 15 years old (OECD, 2005). At this age it is expected that the students are approaching the end of their compulsory schooling and thus it is of interest to assess how prepared they are to face societal challenges (cf. OECD, 2003). For Sweden, a representative sample of 4624 students were examined for the PISA 2003 survey, of these about 1400 students had the possibility to answer the item of interest for the present study. For a comprehensive account of the selection procedures and administration of the PISA test please refer to *PISA 2003 Technical report* (OECD, 2005).

In Sweden by the age of 15, most students are in their final year of compulsory secondary school. Since the observation study was conducted at the beginning of the academic year, it was not feasible or effective to select students in their final year. From the context of the school syllabus, we perceived a risk that these students might not previously have encountered the concepts or topical strands in mathematics that were needed to solve the study task. Therefore, for the observation study, the selected students were freshmen at the upper secondary school level and as such there was no doubt that they would have attained the basic level of requirement for secondary school in the Swedish compulsory school system.

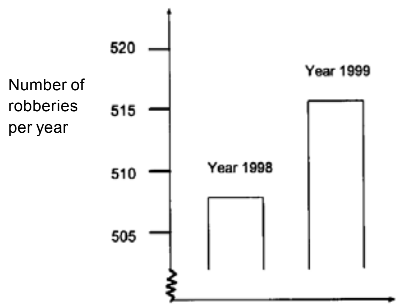
According to the Swedish curriculum documents (cf. Skolverket, 2009) regarding graphicacy, students are expected by the end of compulsory school, to be able to interpret, compile, analyze and evaluate data in tables and diagrams. This basic requirement applies to all students. This is significant since the students participating in the video observations were selected from different study programmes at upper secondary school. No strict selection criterion was employed, thus the sample selection for the video observation can be seen as a convenience sampling.

### *Item M179Q01-Robberies*

This item is one of the PISA released items and thus could be freely used in the study.

Question 1: ROBBERIES M179Q01- 01 02 03 04 11 12 21 22 23 99

A TV reporter showed this graph and said:
”The graph shows that there is a huge increase in the number of robberies from 1998 to 1999”




Do you consider the reporter’s statement to be a reasonable interpretation of the graph?
Give an explanation to support your answer.

Figure 2. *Robberies, the selected item from the PISA survey test*

This is a constructed open ended item which means that the students had the opportunity to offer supporting arguments for their solutions instead of simply identifying values or structural elements in the graphical artefact. A closer scrutiny of the students’ responses to this item was done to identify the prevalent mathematical operator and forms of expression. This was achieved by using the double-digit rubrics scores for the item.

For the observation study, elements of PISA’s double-digit rubrics were used with a focus on the use of subject specific (mathematics) language and forms of expression. Other factors used to evaluate the discussion included the plausibility of any arguments presented, the nature and the relevance of the evidence used in the creation of justification as well as the quality of the reasoning on which they are based (cf. Gal, 1998).

## The study

In this section the results from the PISA survey as well as from the observation study are presented. These appear as Study I (PISA survey) and Study II (observation study) respectively.

### *Study I – PISA results*

Item M179Q01 Robberies, required the students to make a case for or against the claims made by the journalist, in interpreting the graph (figure 2). For a convincing case, a student needed to use optimal arguments and possibly accompanied by appropriate use of mathematics operators and forms of expression. Given that the average success rate for the participating countries for this item was low (OECD average 30 %), this was taken to indicate that a considerable number of students would have difficulties providing acceptable solutions to the task. The student success rate on this item for Sweden was 45.64 % which is among the top OECD country success rates (see OECD, 2010).

Since the aggregate national success rate scores given above do not give much information about the nature of students' responses, we turn to the double-digit rubrics. The double-digit rubrics provide students' solution categories and thus, the quality of the responses given. According to Allerup, et al., (2006) the double-digit rubrics enriches international surveys making them meaningful and useful for teachers in supporting learning experience of students.

The scores for full credit i.e. code 21, 22 and 23 are considered here as indicating some level of critical-analytical approach. This is supported

Table 1. *Characteristics of responses for the item M179Q01 (OECD, 2009)*

| Full credit | Details |
|---|---|
| Code 21: | Solution states that the statement is NOT reasonable based on the observation that only a small portion of the graph is shown. |
| Code 22: | Solution states that the statement is NOT reasonable and uses arguments based on ratio or percentage increase. |
| Code 23: | Solution states that trend data is required before judgement can be made |
| Partial credit | |
| Code 11: | Solution states that the statement is NOT reasonable but lacks supporting explanation. It focuses ONLY on an increase given by an exact numerical figure but does not compare with the total. |
| Code 12: | Solution states that the statement is NOT reasonable with correct method but with minor computational errors. |
| No Credit | |
| Code 01: | The solution contains a NO with insufficient or incorrect explanation. |
| Code 02: | The solution contains a YES, focuses on the appearance of the graph and mentions that the number of robberies has doubled. |
| Code 03: | The solution contains a YES with no explanation, or explanations other than those in Code 02. |
| Code 04: | Other responses. |
| Code 99: | Missing. |

by the nature of the solution provided which indicates aspects of effective communication of solution, use of subject specific tools and forms of expression (using ratio, mentioning trend etc.), whereas the scores code 11 and code 12 are perceived as involving an identification approach. Whereas the ”minor computational errors” are not explicitly mentioned, it is possible that these might involve ratio or percentages.

Figure 3 shows the percentage distribution of the solutions by codes from Sweden (the horizontal axis represents the code).

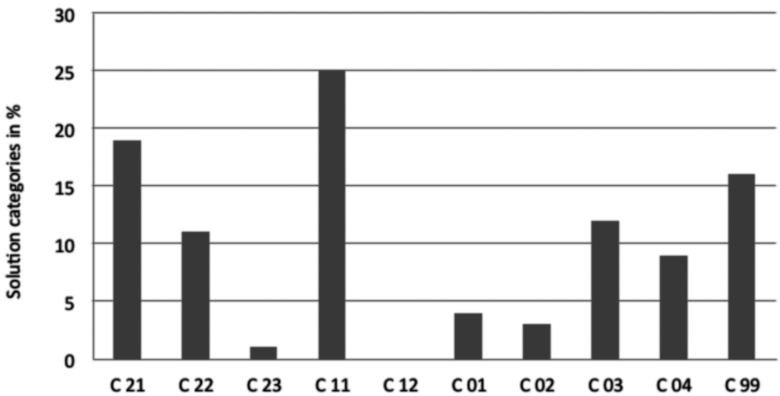



Figure 3. *Distribution of student responses from Sweden using the double-digit rubrics*

From figure 3 it is observed that comparatively higher numbers of students provided solutions that were scored for code 11. The characteristic feature of the solutions given for code 11 was the lack of explanation attached to the provided solution; an observation perceived as indicating an identification approach to the task. The second most frequent solution for Sweden is code 21 (19 %).

Though yielding full credit, the solutions on code 21 seem to be devoid of subject specific operators and forms of expression and are to a large extent also inclined towards an identification approach. These solutions question the production or presentation of the graphical artefact (data integrity) as seen in highlighting the structural aspects of the graphical artefact. Significantly, solutions scored for code 11, though perceived as based on an identification approach, indicate an appreciation of number sense through the use of elementary arithmetic (subtraction).

The general success rate for solutions obtaining full credit (codes 21, 22 and 23) showed that a majority of students gave explanations that were not accompanied by use of subject specific operators and forms of expression as observed in the low response rate scored for solutions on code 22 and code 23. These codes implied the use of mathematical operators and knowledge of statistical concepts respectively.

### *Study II – Video observation*

Whereas the rubrics provide information about students' solution compared to the total scores, the dynamics involved in providing the solutions are still missing. This has the effect of limiting the kind of analysis that can be done on the nature of the students' responses. Thus the video observations were meant to augment the findings from the double-digit rubrics. Three student groups of Swedish pupils were included in this study. The first group comprised of girls from the *Child care recreational programme* while the second and third groups both comprised of students from the *Natural science programme*.

Each member of the group was given a copy of the question and a blank worksheet in case there was need within the group to make mathematical computations. Since the emphasis of the study was placed on group collaboration, there was no time limit placed on the students as they solved the problem items; they were encouraged to work at their own pace. Of the three groups the students in discussion group I chose to discuss the question without independently writing down the answers, while the students in groups II and III discussed the task and independently made some notation on the worksheet provided.

### *Discussion group I*

This group comprised of three girls from the Child care recreation programme. At the start of the video observation, they unanimously agreed to tackle the task as a group. It is observed that one of the girls seemed to have a considerable influence on the group.

The girls began by looking at the question individually and then decided to share their thoughts with each other. However, it was the conclusion that they came to, namely that there was a huge increase in robberies, that set this group apart from the other groups under observation. Indeed based on the PISA double-digit rubrics the conclusion for this group might not have received any credit. However, analysing the discussion shows that the students put forth some valid observations and reasonable arguments as they engaged in the task.

In analysing this transcript the two-way classification mentioned earlier is used and contrasted with the aspects of Curcio (1987). At the very beginning of the transcription it is evident that the students are guided mainly by perceptual concerns lifting out values from the graphical artefacts. This is akin to reading the graph and is typical of what might also be considered as an identification approach.

| | | |
|---|---|---|
| 1 | Karolin | here it is 507 and up here it is about 516 [pointing at the bars], that is a lot of robberies ... in a year, it is quite a lot |

Their concern with the identification approach becomes even more pronounced in turns 12–17 when they literally try to outshine or dominate each other while dealing with structural aspects of reading the graph. At this point it is appreciated that the difference in the number of robberies has been established (by simple arithmetic) to be 8. There is also some form of proportional comparison made in the sense that 500 robberies are considered to be a lot in one year.

| | | |
|---|---|---|
| 12 | Karolin | but it is about eight ... eight |
| 13 | Helen | but even this bit here ... [aligning the apex of the bars with axis values using a pencil] this one is to here |
| 14 | Linn | no ... ! |
| 15 | Helen | yes! it is this ... that there is up to here ... [showing the units alignment on the y-axis as if demonstrating how the graph should be read] |
| 16 | Karolin | yes ... eight, sixteen |
| 17 | Linn | ... and it is at five hundred five hundred |

However, in turns 22–27 Karolin seems to bring about another comparison dimension dealing with the nature of the robberies. This dimension is not related to the graph and can be considered as reflecting a reality outside the task at hand. Indeed this is the intention of the PISA survey; to assess the students' ability to apply the mathematics learned in school in realistic situations. Given that the students seem to make a shift from focusing on factors specific to the graphical artefact in question, to robbery as a societal phenomenon. It can be claimed that the students are reading (reasoning) beyond the graph (graphical artefact).

| | | |
|---|---|---|
| 22 | Helen | we don't know really |
| 23 | Karolin | but if one reflects on this [pointing at the axis] it is quite a lot in a year that is five hundred robberies, it can be big as well as small |
| 24 | Helen | I think the increment is about eight or nine in a year [pause] |
| 25 | Karolin | yes ... but then it depends on the kind of robbery. It doesn't have to be a bank robbery, but it can be like a person is robbed or something. |
| 26 | Helen | I agree with you it can be both big and small |
| 27 | Karolin | robbing a small person or something |

Part of the struggle for this group is in dealing with the number of robberies in a year per se and the differences in the number of robberies (the individual increases). Although these students show some number sense, they do not seem able to effectively apply specific mathematical or statistical tools and forms of expressions to clarify their dilemma. Thus it is not entirely unexpected of them to turn to personal-emotional (*value-laden*) arguments. Just as in turns 25–27, where there are contrasting types of robberies directed to a "small person" as opposed to a material thing such as a bank.

Though Karolin's personal-emotional argument in turn 25 seems to be acceptable to the group, Helen still insists on looking at the differences between the data sets pointing to the fact that the data available is just for two consecutive years. However, it is when Karolin insinuates that previous robberies (from preceding years) were not as many that Helen situates the task at a point that may be perceived as invoking some mathematical/statistical reasoning, thus vindicating their final conclusion (turns 31–35 below). In these turns it appears that an assumption is made that the number of robberies before 1998 was normal or "constant". At this point the students' interaction with the graphical artefact moves a notch higher and serves to justify their conclusion.

| | | |
|---|---|---|
| 31 | Helen | these are values from 1998 and 1999 |
| 32 | Karolin | yes it is quite a lot! |
| 33 | Observer | okey |
| 34 | Helen | the question is if it was the same ... it was probably the same then |
| 35 | Karolin | I don't know how much it was before ... it was probably less |
| 36 | Helen | no ... I don't think that ... this is what is normal. I find it hard to think that it was less than 1999 |
| 37 | Karolin | it seems like ... [mutter] |
| 38 | Helen | yes |

The discussion point for this group was based at an everyday level of interaction with minimal application of mathematical methods and forms of expression. The overriding argument seems to be the personal perspective of robbery in relation to the number and nature of robberies in a given period. However, an observation made by Helen gave room for an assumption that is statistical in nature and which made the group's conclusion reasonable and thus feasible. This assumption though not explicitly expressed, was characteristic of higher level statistical reasoning; demonstrating consciousness of ideas of trend analysis.

The overall discussion was dynamic, successively moving from reading the data to reading between the data and moving toward reading beyond the data- they read values from the graph, made comparisons and interrogated the nature of robberies and appreciated the need to situate the data set in relation to previous years. In terms of critical-analytical approach vis-à-vis identification approach, the group's discussion was at the interface with a strong focus on identification approach. Whereas they showed number sense and appreciation of statistical reasoning, it seems that their grasp on these forms of expression was not firmly grounded.

### *Discussion group II*

In this group there were three boys from the Natural science programme. While the boys got along quite well in the discussions, they decided to submit their written solution separately.

In this discussion group the case seems to be settled when Johan says that the case is classical of misleading diagrams in turns 4 and 5. This is an observation that is probably based on experience with similar diagrams. It ought to be noted that at the same time Johan points out the ”proportional” difference in robberies for the two years, thereby pointing out what might be perceived as a doubling effect of the graph (15 and 7) and then declaring that the actual difference is just 7.

| | | |
|---|---|---|
| 4 | Johan | watch this! [aligning the apex of the bar with the axis using a pencil to highlight the gap between them] I think this is say 15, this is 7, there are 7 more robberies in a year [pause] |
| 5 | Johan | it is this kind of ... it is a classical case of misleading diagram |

However, in the succeeding turns Alex seems to make a comment that seems like a reaction to Johan's use of the term misleading diagram and he insists that that the diagram is correct but there is something more to it.

| | | |
|---|---|---|
| 6 | Alex | yes it is true! ... ah |
| 7 | Johan | it is not really true |
| 8 | Alex | yes, the diagram is correct ... |
| 9 | Amis | wait! [a moment!] |
| 10 | Alex | [completing previous statement from turn 8] ... but there is something about it |
| 11 | Johan | yes |

The group seems to arrive at a consensus that the conclusion by the reporter is incorrect, with Johan providing an explanation in turn 16 which has a remarkable similarity with the example from BBC (see figure 1).

| | | |
|---|---|---|
| 15 | Amis | what did you say it shows? what do you mean? |

| | | |
|---|---|---|
| 16 | Johan | it shows that this 515 is twice as much as 507 or 508 or whatever it is |
| 17 | Amis | aha! 515 is twice as much as much [chuckles] |
| 18 | Johan | ... 15 is meant to be twice ... as ... much...as robberies for ... year 1999, twice as much as about 507 robberies in 1998 although it is ... |

The students written solutions at the end of the discussion were largely similar save for the varying levels of clarity. The written answers were provided as shown below:

| | |
|---|---|
| Amis: | NO ... because the diagram shows that 525 should be twice as much as from the year 1999 |
| Johan: | Answer: No because the diagram shows that 515 robberies from the year 1999 should be twice as much as 507 robberies for the year 1998 although it is just an increase of 8 robberies/year |
| Alex: | No, the robberies increase by just 7–8 every year but the presentation makes you think that there were twice as many robberies in 1999 |

This group showed some sensitivity to mathematics in their expressions as observed when Amis commented on Johan's statement which he immediately corrects (turns 17–18 above). However, they did not use mathematical and statistical methods to bolster the solution. They seem to focus on the visual perception aspects of the graph (i.e. the initial appearance). This is best captured in Alex' written response "No, the robberies increased by just 7–8 every year but the presentation makes one think that there were twice as many robberies in 1999".

The approach taken by this group's interaction with the graphical artefact falls into an identification approach in as much as the students seem to relate this graph to a class of graphs that are "classically misleading". The students interrogated the structural presentation of the data concluding that it was not a viable diagram and therefore this solution would yield a score for code 21. The solution from this group might also highlight some of the drawbacks of testing. In the absence of further information, written solutions do not always provide a clear picture of students' comprehension of the concepts.

### *Discussion group III*

This group comprised three girls from the Natural science programme. Here the discussion was not as vibrant as in the other groups save for one girl, Susanne who led the discussions. Towards the end, she invited reactions from the other as a way of seeking consensus. One of the girls, Petra remains silent throughout the session. However, in the end she provides written solutions that indicate depth in understanding (see account of written answers below).

| | | |
|---|---|---|
| 4 | Susanne | I think this is not ... reasonable, that is the axis is broken, actually if it started at zero [making marks on the bars as if extending then to the x-axis] they'd be the same, so it is not a huge increase at all as they claim. Here it appears as if there is a double increment and that is not the case [pause] |
| 5 | Susanne | that is what I'd say is correct |
| 6 | Susanne | [while addressing Maria] ... do you share the same opinion? |
| 7 | Maria | yes I think it is true [pause] |
| 8 | Observer | what is it that is true? |
| 9 | Maria | yeah ... that it is not certain that ... they ... I mean if they begun from below ... if they begun from zero maybe it would be completely something else [laughs] |
| 10 | Susanne | at least the same ... they appear a little odd ... [referring to the bars] |
| 11 | Maria | yes |
| 12 | Susanne | maybe not the big picture [in audible] either [probably referring to the highlighted parts] |

At the end of the discussion the students provided written answers to the discussion with accompanying inscriptions indicating the suggested changes. The general corrective aspect of the inscriptions was to extend the bars probably to diminish the visual proportional increase created. However, there were some slight but significant differences in the inscription: Susanne extended the bars but also appeared to focus on "streamlining" the broken line, Maria just extended the bars to make them longer while Petra just extended the bars leaving them hanging (both Susanne and Maria provided a type of base or pseudo axis for their bar). The written answers are provided below:

| | |
|---|---|
| Petra: | – Misleading ...<br>It appears as though it is twice as much but actually it is not. |
| Susanne: | Quite unreasonable if one draws the diagram from 0 one observes that the increment is not that huge |
| Maria: | Unreasonable that one draws the diagram from point 0 |

It seems as though the students in this group were generally in agreement that the diagrams had been truncated to make the difference appear larger, as captured in turns 4 and 9. However, Maria's comments in turn 9 indicate a degree of insecurity in respect to the topic under discussion; a fact that is confirmed from her written response (see also here written answer provided above).

The students then proceeded to suggest ways of correcting the graph by providing corrective inscriptions on the task graph. This approach to "correct" the problem with the diagram also shares some similarity with the approach used in the case of the BBC illustration in figure 1: Susanne and Maria include the x-axis (to highlight the origin) in their diagram, without paying specific attention to the scale, while Petra just extends the bars. Clearly the correction is done on the graphs to diminish the visual effects. However, technically, it can be claimed that Petra adheres strictly to the convention of graph construction as regards axis intervals. By starting from the origin as the others had done and with the given scales, the graph would yield much longer bars than those they reproduced.

Whereas the students interrogate the production-presentation of the graph, thus indicating reading between the data and some elements of reading beyond the data, two of the students seem to miss the basic elements of reading the data (making appropriate graph construction). Their final solution is probably influenced by prior experience with similar illustrations, thus clouding their critical stance. Hence, the solution provided is perceived as based on an identification approach emanating from a reproduction of previously encountered cases.

## Discussion

The aim of the present study was to explore students' interaction with a graphical artefact. The contents of the graph under investigation touched on aspects of everyday life as well as mirroring a case from the mass media. This was also typical of items considered to exemplify "misleading statistics". Since this item was embedded in the topic strand statistics, it was expected of the students that they should employ some statistical forms of expressions or mathematical language and processes in solving the task. The solutions were analyzed partly using the categories outlined by graphical comprehension (Friel et al., 2001) as well as a construct based on critical-analytical approach to graphicacy.

Analysis of the PISA results involved reorganizing the double-digit rubrics to suitable solution categories i.e. determining if solutions scored for a given code are typical of an identification approach. It was observed (see figure 3) that the percentage of students providing solutions indicating content or subject knowing (codes 21, 22 and 23) and where there is an indication of the use of mathematical concepts or language were considerably low. In the present study, the use of mathematics and statistics operators and concepts as well as interrogating the factors associated with the graph was taken as an indication of a critical-analytical approach. These solutions can to some extent be categorized as reading beyond the data. The construct critical-analytical approach is consistent with the notion of statistical literacy perceived as constituting critical questioning, knowledge of some statistical and mathematical concepts and terminology, as well as basic communication skills (Gal, 2002; Rumsey, 2002). Thus, the low response score on codes 22 and 23 was probably an indication of the difficulties the students had in applying a critical-analytical approach in determining solutions.

According to Friel et al. (2001), mathematical knowledge and experience are among the qualities necessary for graph comprehension. In the present study mathematical knowledge is perceived as encompassing the identification, selection and appropriate use of suitable subject operators. In discussion group I, using simple arithmetic, the students determined the difference in robberies to be about eight. However, from the transcript it seem uncertain as to whether the numerical value of robberies is considered *per se* (number sense) or it is being related to the total number of robberies for a specific year (proportional reasoning).

Thus, there was no evidence on the part of the students to relate their mathematical and statistical knowledge to the task appropriately. Notions of trend and important assumptions made from the data are put forward but not really developed and used effectively to guide the group's discussion. With uncertainty in the application of subject specific operators and forms of expression, the personal-emotional dimension dominates the discussion and appears to form the basis for the final conclusion.

Students in discussion groups II and III were able to express their solutions explicitly. However, the explanations seem to portray a reproduction of facts rather than a critical-analytical approach. The ”innovativeness” in the discussion in group 1 was in many ways missing. A parallel (for groups II and III) could be drawn to figure 1, where the general approach seemed to be focused on data integrity, that is correcting the graph to conform to conventional presentation methods. The focus on data integrity is not entirely unexpected if the students are exposed to these methods in teaching and learning environments. According to Roth, Pozzer-Ardenghi and Han (2005), students generally adapt to certain ways of doing things by developing structured dispositions. These dispositions generate patterned perceptions and with it the field of possible patterned actions and this leads to indoctrination. Whereas developing structured disposition may be desirable, in the absence of critical sense, indoctrination may lead to acquired blind spots and prejudices. Alex (turns 6–8) in discussion group II seemed to question the purported misleading nature of the graph, unfortunately this discussion point was not picked up by the others in the group. When the observer posed a question in discussion group III (turns 8–9), it is observed that Maria lacked words to support her theory or solution (turn 7). These occurrences are seen as illustrating that students have acquired blind spots or prejudices that hinder them from meeting divergent thoughts or defending their solutions away from standard forms of expressions

Indoctrination occurs not only in teacher led teaching and learning environments but also through instructional materials such as textbooks and internet resources. From the illustrations in figure 1, and with the results from both the PISA survey and video observations, it seems that students have been structured into particular ways of engaging graphical artefacts – focusing on the structural features. Thus the ”misleading” dimension of the graphical artefact used in this study is to a large extent dependent on the indoctrination process – if it encourages a critical analytical stance or an identification approach to the graph. For students focusing on the initial impression from the graph, the misleading aspect was the data presentation, while for those interrogating the production of the graph, the information presented in the graph was insufficient to make a clear judgement. In this study it was apparent that discussion groups II and III focused on the presentation of the data, that is, on data integrity. Thus, it is claimed that the students' graphicacy showed general awareness of graph production and the presentation facet of graphicacy. While the use of everyday forms of expression is expected and not entirely undesirable, it was observed that students, to a limited extent, incorporated mathematical methods and forms of expressions in their discussions

In exploring the visibility of Curcio's (1987) framework for graphical comprehension, it was difficult to pin the students' discussion to one particular level. This would be attributed to the context of solving the task where the students have the opportunity to readjust their thoughts through what can be considered as peer scaffolding. This might also point to the drawbacks of a three level classification of graphical comprehension.

Based on the observations of this study, it is important to emphasise the significance of a critical-analytical approach with respect to graphicacy. Given that students are able to access diverse graphical artefacts from different sources, it is important that teaching and learning are designed to foster a critical-analytical disposition. This can be achieved by constructing appropriate tasks that foster inquisitive and reflective disposition, as well as encouraging the effective communication of mathematical methods and solutions.

Another observation which can be considered a bi-product of the present study is the notion of ”personal-emotional dimension”. It seems that placing mathematical tasks in context might initiate the students' sympathetic insight with the potential of leading them away from the mathematical task they are intended to cope with. This may especially be expected in non-mathematical study programmes as in the case of the students in the Child care recreational programme. Thus, teachers with these students might need to exercise special care and skill to lead their students' attention to the mathematical/statistical content when they have already gained insight into the context of the task. In ordinary teaching, time is restricted and too much distraction must be avoided if the goal is to promote students' understanding of mathematical concepts.

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