Research on mathematical beliefs

The birth and growth of the MAVI group in 1995–2012

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In the 1980s the meaning of beliefs for teaching and learning aroused also to the consciousness of mathematics educators. Therefore, here it is firstly sketched the research field of mathematical beliefs, in order to understand why belief research has been a topic for an international research group for more than 30 years. The aim of the paper is to have a look at the history of MAVI and to describe its development within the years 1995–2012. A special look is given at the birth of the MAVI group in the middle of the 1990s and then its development through the last 18 years will be described. Also some statistics on the MAVI meetings and their participants are presented. And in the appendix the total list of the MAVI proceedings is documented.

In the industrialized countries, everybody seems to know what mathematics is. But when the question is put forward, one gets different answers depending on the respondent in question. School children understand mathematics differently from their mathematics teachers and teachers of other subjects will explain it again differently. Still another description is received e.g. from a "man-on-street". And mathematics professors have their own view of mathematics.

This big variety of answers to the question "What is mathematics?" hints that there is not only one understanding of mathematics, but several different views of mathematics. And not in the sense that there is only one *right* view of mathematics and the others are *wrong*. Philosophers of mathematics (e.g. Ernest, 1991; Hersh, 1997) have introduced several *right* views of mathematics that are also accepted among mathematicians. This state of art with the constructivist view of learning has led researchers of mathematics education to investigate teachers' and pupils' views of mathematics and their implication for mathematics teaching and learning.

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Pehkonen, E. (2012). Research on mathematical beliefs: the birth and growth of the MAVI group in 1995–2012. *Nordic Studies in Mathematics Education*, 17 (3-4), 7–22.

What are mathematical beliefs?

In the beginning of the 1980s, some researchers (e.g. Schoenfeld, 1983; Silver, 1982) noted that one cannot fully understand an individual's problem solving behavior by only considering cognitive factors, such as his mathematical knowledge or his use of heuristics. Also affective factors should be used as predictors (e.g. Buchanan, 1987; McLeod & Adams, 1989). Some meta-cognitive elements such as an individual's mathematical beliefs have been stressed more than others. Research has been carried out into the quality and meaning of these mathematical beliefs (e.g. Schoenfeld, 1989). With the help of earlier studies, Pehkonen (1991) pointed out in his paper the underlying importance of beliefs for pupils' problem solving behavior.

The affective domain was for a long time a neglected area in research of mathematics education. About three decades ago an individual's attitude towards mathematics was brought up as one of the central research topics in mathematics education; the well-known Fennema-Sherman attitude scale (Fennema & Sherman, 1976) represents this phase. Today the focus of research has changed from broadly defined attitudes to more specific sub-concepts: emotions, narrowly defined attitudes, values, and most commonly beliefs. One may state that McLeod and Adams (1989) initiated a new phase with their famous book and in the research the constructs have been further elaborated. McLeod (1989) used the partition of the affective domain in mathematics into emotions, attitudes and beliefs. But there are also alternative definitions for the affective domain, for example, Goldin adds to the three components (emotions, attitudes, beliefs) a fourth one "values" (Goldin, 2002).

Within belief research in mathematics education, further subcategories have been distinguished. For example, Op't Eynde, DeCorte and Verschaffel (2002) introduced three main domains:

1. beliefs about mathematics education (mathematics as a subject, mathematical learning and problem solving, mathematics teaching in general),
2. beliefs about self (self-efficacy, control, task-value, goal-orientation), and
3. beliefs about the social context (social and socio-mathematical norms in the class).

Another direction of sharpening has been within mathematics itself. Today we are no more considering attitudes or beliefs towards mathematics as an entity, but researchers distinguish e.g. attitudes or beliefs on geometry or problem solving.

Within research in school, the prevalent understanding of learning has emphasized cognitive academic achievements. Affective by-results that are in connection with an individual's meta-cognitions, however, determine, to a large extent, how good a learner of mathematics one has become. Researchers around the world have paid more and more attention to mathematics learning as a process. Such a view highlights the importance of meta-cognition and affect, especially in the form of pupils' and teachers' beliefs. Beliefs seem to be situated in the "twilight zone" between the cognitive and affective domain and thus, have a component in both domains.

Individuals continuously receive perceptions from the world around them. According to their experiences and perceptions, people make conclusions about different phenomena and their nature. The individual's personal knowledge that includes one's beliefs is a compound of these conclusions. Since beliefs seem to form and change in social environment, people compare these beliefs with their new experiences and with the beliefs of other individuals and thus their beliefs are under continuous evaluation and change. When one adopts a new belief, this will automatically form a part of the larger structure of their personal knowledge, of their belief system, since beliefs never appear fully independently. Thus, the individual's belief system is a compound of one's conscious or non-conscious beliefs, hypotheses or expectations and their combinations.

Although beliefs are popular as a topic of study, the theoretical concept of "belief" has not yet been dealt with thoroughly. The main difficulty has been the inability to distinguish beliefs from knowledge and the question seems still to be unclarified (e.g. Abelson, 1979; Thompson, 1992). These problems are discussed e.g. in papers of Furinghetti and Pehkonen (2002) and Pehkonen and Pietilä (2003). Another difficult issue has been the distinction between beliefs and other affective variables such as emotions, attitudes, and values. These affective variables have often been located on a continuum, stretching from intense and fluid emotions to stable and cool beliefs (McLeod, 1992). Such a view, however, has been also criticized, for example, for exclusion of mild or long-lasting emotional states and emotions that arise regularly (Evans, Hannula, Philippou & Zan, 2003).

Our characterization for beliefs

As an implication of this fuzziness in the definition, one might mean different matters with beliefs, depending on the discipline and the researchers who deal with them. For example, beliefs are considered equal to concepts, meanings, propositions, rules, preferences or mental images

(cf. Thompson, 1992). At other occasions, beliefs are seen in a much broader sense as "mental constructs that represent the codifications of people's experiences and understandings" (Schoenfeld, 1998, p. 19) and that shape their perception and cognition in any set of circumstances. In social psychology the impressions of and reactions to other people are typically divided into beliefs, expectations and attitudes. For them, beliefs are statements thought to be true, whether or not they actually are. Expectations are explicit or implicit predictions about people's future behaviors and attitudes are emotional reactions to them (e.g. Brophy & Evertson, 1981). These questions of fuzziness in defining beliefs are dealt with more broadly e.g. in the paper by Furinghetti and Pehkonen (2002).

On one hand, beliefs can be thought to form one part of an individual's meta-cognition (e.g. Schoenfeld, 1987). On the other hand, attempts have made to define beliefs i.a. through attitudes (e.g. Törner & Grigutsch, 1994). In the different definitions, a truth-value is usually attributed to a belief (i.e. beliefs are a kind of knowledge) and they are often seen to include an emotional component. Sometimes the definitions also say something about the stability and intensity of beliefs and about the nature of their origin or warranty. Here an individual's *beliefs* are understood in a rather wide sense as his/her subjective, experience-based, often implicit knowledge and emotions on some matter or state of art. Such a characterization is very near the one given e.g. in the paper by Lester, Garofalo and Kroll (1989).

In the literature, the term conception is often used parallel to beliefs. Conceptions are explained as conscious beliefs, i.e. conceptions are understood as a subset of beliefs. Thus conceptions are higher order beliefs that are based on such reasoning processes for which the premises are conscious. If we want to distinguish unconscious beliefs, we may speak about basic (or primitive) beliefs. One variation of conceptions is *views*. They are more spontaneous than conceptions and the affective component is more emphasized in them. Conceptions are more considered than views and the cognitive component will be more stressed in them (cf. Pehkonen, 1998). But it is clear that there are different interpretations of the words, as shown in the book of Leder, Pehkonen and Törner (2002).

The spectrum of an individual's beliefs is very wide and they are usually grouped into clusters of beliefs. The belief clusters influence each other. Some beliefs depend on others, for the individual more important beliefs. Here Green (1971) uses the term "the quasi-logical structure of beliefs" which means that the individual himself defines the ordering rules. Thus, beliefs form belief systems that have a quasi-logical structure and that might be in connection with other belief systems or might not. Therefore, the term *belief system* is used as a metaphor to represent how the individual's beliefs are structured. The affective dimension of

beliefs influences the role and meaning of each belief in the individual's belief system.

Some publications on belief research

In the literature, one can find several overviews on mathematics-related belief research (e.g. Op't Eynde et al., 2002; Pehkonen, 1994, 2004; Pehkonen & Törner, 1996; Philipp, 2007; Rösken, Törner & Pepin, 2011; Schoenfeld, 1992; Thompson, 1992; Underhill, 1988a, 1988b). About a decade ago the first book on mathematical belief research (Leder et al., 2002) was published, in order to give an overview on different research perspectives and on research done.

The continuing interest on beliefs shows that there are still many unanswered questions. In his review of research on affect and mathematics, McLeod (1992, p. 575) noted: "Although affect is a central concern of students and teachers, research on affect in mathematics education continues to reside on the periphery of the field". Additionally Schoenfeld (1992) states that there is much research done on students' beliefs, but not so much on teachers' beliefs. The situation now appears to be changed and teachers' beliefs are more studied. Furthermore, Pehkonen and Törner (1996) tried to sketch the cover of belief research by plotting a sample of the published papers in a matrix form.

Research done in different areas of beliefs is uneven in the sense that in some areas researchers have not yet been able to produce effective answers, whereas in some areas there is some clear influence of research to be seen. An open issue is, for example, change in teachers' beliefs, whereas an example of a well-documented content area is gender differences (cf. Leder et al., 2002).

A brief history of the MAVI group

The Finnish-German research group MAVI (Mathematical Views) is an initiative of my colleague Günter Törner (University of Duisburg, Germany) and myself, and its aim is to study and examine those questions that arise through research on mathematics-related beliefs. The proceedings of the workshops have been published in the pre-print series of the corresponding institute, or today also in a book form.

The birth of the MAVI group

In the 1990s, we both – my colleague Günter Törner and myself – were interested in mathematics-related belief research and we both had a group of doctoral students working on the topic. Therefore, we decided

in 1995 to apply for money from the Academy of Finland and the German organization DAAD[1] to organize a set of cooperative workshops on belief research. The aim of the workshop series was to offer a forum, especially for junior researchers and doctoral students, where they could practice the skills needed in the international cooperation: to write a paper, to present a paper, to evaluate other persons' papers, and to participate in scientific discussions – all this using a common language, English.

Actually the first MAVI meeting was held in Duisburg 1995 already before the finance resolutions. But we were both successful to get a grant for organizing such workshops for three years, myself from the Academy of Finland and Günter Törner from DAAD.

Implementation of the first years of MAVI

In the application we promised to organize within three years (1996–98) always two meetings (research workshops) per year, one in Duisburg and another one in Helsinki. Therefore, there was an annual meeting both in Finland and in Germany during the first three years of MAVI activities. These meetings were mainly meant to our doctoral students, but we were able to finance the invitation of some foreign specialists, too. Thus in the first meetings, there were senior researchers invited from Cyprus, Italy, Russia, Spain and Ukraine. Additionally, there were some belief researchers from other countries (Canada, Denmark, Hungary, Norway), too, who had heard about the MAVI workshops and wanted to participate.

Enlargement of MAVI at the end of the 1990s

When the financing period of three years was over (1998), we decided to continue as a research group, to meet annually and to enlarge our workshops to a European research group. At the same time, we formulated our focus as a working group, as follows:

> The aim of the MAVI group is to study and examine those mathematical-didactical questions which arise through research on mathematics-related beliefs in school connection.

The MAVI group is meant to be open for everyone interested in mathematics-related beliefs (also outside of Europe). Later on, the focus has been widened to the affective domain, i.a. beliefs, attitudes and emotions, of mathematics teaching and learning. There are many different subgroups within MAVI with their own interest field. For example, Markku Hannula has been drawing the focus of the group to affective domain more generally (cf. Hannula, 2007). Today one might say that the main

themes of MAVI workshops are affective and cognitive factors and their influence on the teaching and learning processes.

The MAVI group is based on a voluntary organization and it tries to be as democratic as possible, e.g. there is no chair and each year the group votes where the meeting of the next year will take place. A leading idea is that MAVI workshops will have an inclusive, open and free atmosphere. In the beginning, there was even no participation fee; now there is collected a small amount in order to cover e.g. coffee breaks. The site and the organizer of the annual meeting will take the responsibility to publish the proceedings that are today peer reviewed. Researchers have an opportunity to improve the text of their manuscript according to the feedback in the meeting and to the reviewers' reports.

Thus, the MAVI group has been active almost twenty years in Europe, having their annual meeting in different universities. During the first years, MAVI workshops took place in spring (i.e. March, April, May), but today the time slot has changed to early autumn (September). All the time, the sites of the workshops have been circulating in the different countries (cf. table 1).

The steps in the development of the MAVI group can be pointed out with some published books or papers in the existing literature: The state-of-art and the list of belief publications at the beginning of the MAVI activities (cf. Pehkonen & Törner, 1996), the early developments and products of the group (Pehkonen & Törner, 1998), the world-wide view on belief research (Leder et al., 2002), the state-of-art in the influence of the work done (Pehkonen, 2004), and the latest overview on belief research (Rösken et al., 2011). In addition, one may mention all the 18 MAVI proceedings (from the years 1995–2012) that are given in the appendix.

Some statistics on MAVI meetings

The MAVI meetings have taken place for 18 years (1995–2012) in altogether seven European countries: Austria, Cyprus, Estonia, Finland, Germany, Italy and Sweden (cf. table 1). As a rule, there has been only one annual meeting, but within the three first years (1996–1998), as mentioned earlier, there were two meetings per year, one in Germany and one in Finland.

The number of participants has varied from 9 to 43 researchers, usually a combination of senior researchers and junior researchers. There have been participants altogether from 27 countries, also outside of Europe (from Africa, Asia, Australia and America): Australia, Austria, Botswana, Canada, Cyprus, Czech Republic, Denmark, Estonia, Finland, Germany, Greece, Hong Kong, Hungary, Israel, Italy, Japan, Latvia, Mexico, Norway,

Table 1. *The statistics of the MAVI meetings: year, site (university), number of participants (part.), number of presentations (pres.), the countries with the number of presenting participants.*

Year	Site	Part.	Pres.	Countries and the presenting participants
1995	Duisburg, GER	19	10	Finland (2), Germany (17)
1996-1	Duisburg, GER	16	9	Finland (4), Germany (11), Spain (1)
1996-2	Helsinki, FIN	15	13	Cyprus (1), Finland (5), Germany (6), Hungary (1), Italy (1), Russia (1)
1997-1	Duisburg, GER	24	16	Cyprus (1), Canada (1), Denmark (1), Finland (7), Germany (13), Norway (1)
1997-2	Helsinki, FIN	17	12	Finland (9), Germany (6), Norway (1), Ukraine (1)
1998-1	Duisburg, GER	16	15	Canada (1), Finland (7), Germany (7), Norway (1)
1998-2	Helsinki, FIN	14	12	Cyprus (3), Denmark (1), Finland (4), Germany (4), Italy (1), Norway (1)
1999	Nicosia, CYP	15	15	Cyprus (3), Finland (4), Germany (4), Greece (1), Italy (2), Norway (1)
2000	Vienna, AUS	32	19	Austria (5), Cyprus (1), Czech Republic (2), Finland (7), Germany (10), Greece (1), Hong Kong (2), Hungary (1), Norway (1), Spain (2)
2001	Kristian-stad, SWE	9	7	Finland (4), Germany (3), Italy (2)
2002	Pisa, ITA	15	13	Cyprus (1), Finland (6), Germany (2), Greece (2), Hungary (1), Italy (3)
2006	Inari, FIN	21	14	Austria (1), Botswana (1), Canada (1), Estonia (1), Finland (13), Germany (2), Sweden (1), UK (1)
2007	Gävle, SWE	27	12	Austria (1), Botswana (1), Canada (1), Finland (4), Germany (1), Norway (1), Sweden (18)
2008	St. Wolf-gang, AUS	15	13	Austria (1), Botswana (1), Canada (1), Estonia (1), Finland (2), Germany (2), Israel (2), Norway (1), Sweden (2), USA (2)
2009	Genoa, ITA	25	20	Austria (2), Botswana (1), Canada (1), Estonia (3), Finland (4), Germany (4), Hong Kong (1), Israel (2), Italy (4), Norway (1), Sweden (2)
2010	Tallinn, EST	26	21	Australia (1), Austria (1), Estonia (3), Finland (4), Germany (2), Israel (2), Italy (5), Norway (2), Sweden (5), UK (1)
2011	Bochum, GER	23	22	Australia (1), Canada (1), Estonia (3), Finland (4), Germany (3), Hungary (1), Israel (4), Italy (1), Norway (1), Portugal (1), Spain (1), Sweden (1), UK (1)
2012	Helsinki, FIN	27	27	Canada (1), Czech Republic (1), Estonia (1), Finland (7), Germany (2), Israel (1), Italy (4), Japan (1), Mexico (1), Norway (1), Spain (1), Sweden (3), Turkey (1), UK (2)

Note. The table is based on the information given in the proceedings. The next MAVI meeting (September 2013) will take place at the University of Freiburg (Schwarzwald, Germany).

Portugal, Russia, Spain, Sweden, Turkey, UK, Ukraine and USA. As a rule, almost every participant has given a presentation, usually describing the design and/or results of his/her own studies.

On contents in the MAVI meetings

The proceedings contain a varying number of research papers, from 7 to 28 chapters and many of them have several authors. The papers could be grouped e.g. according to the following classification: theoretical issues, empirical studies on teachers, and empirical studies on pupils. In the empirical studies concerning teachers, all school levels (nursery, primary, secondary, and university level, pre-service and in-service teachers) are considered with reference to teacher education or teacher practice. Analogously, all school levels have been considered as regards empirical studies with focus on pupils.

In general, the contributions touched different issues related to beliefs: beliefs on mathematics and its teaching and learning, beliefs on oneself in relationship to mathematics, beliefs on specific subject areas such as proof or problem solving, history of mathematics, ICT, attitudes, and identities. Cross cultural and curricular studies were also present. Furthermore, the empirical studies presented a variety of methodologies for data gathering: e.g. questionnaires, interviews, narratives, field notes, video clips. The appendix contains a complete reference list of all MAVI proceedings for the years 1995–2012.

Themes dealt with in the published papers

When looking through the MAVI proceedings, one gets a feeling that at the end of the 1990s there were more empirical studies considering pupils' and teachers' beliefs in general. In the 2000s, the scope of studies in MAVI was enlarged to include the whole affective domain. And the number of more focused studies has increased. When looking at the five latest (2008–12) proceedings, one may spot different topics for which teachers' / pupils' beliefs were charted, as identity, self-efficacy, proof, anxiety, gifted students, ICT, problem solving, problem posing. Additionally, there are about 10–15 % of the papers having a theoretical character. Thus, the published proceedings of the workshops act as a treasure box for new belief researchers in mathematics education.

Now we have been running the MAVI meetings for almost twenty years. Especially in the beginning, we really tried during the meetings to define the concept "belief" in a way that might be satisfactory to the research community (e.g. Pehkonen, 1998). However, there exists until

today no common characterization of "belief", although there are better and better trials. Therefore, almost every researcher will (and should) formulate his/her own definition. About 10 years ago it was brought into light that even the specialists cannot agree with any characterization (cf. Furinghetti & Pehkonen, 2002). Today the situation does not seem to have changed. It might be that the concept "belief" is as many-sided as we have seen in the case of mathematics: For example, in the study of Mura (1993), she could extract 14 features in mathematicians' responses to the question "What is mathematics?"

Evaluation of the international meaning of the MAVI group

In the beginning of MAVI, all presentations were published in the proceedings, in the way the authors offered them. The leading idea, at that time, was that doctoral students should have an opportunity and experience in practice of writing scientific papers and will get a reward in the form of a publication. But in the beginning of the 2000s, the local editors of the proceedings adopted little by little the model from the PME practice and asked the participants to evaluate each other's papers (i.e. peer review) beforehand. Thus, it was also a learning situation for younger researchers. But there existed also a difference to the PME practice: papers were evaluated by two colleagues – usually one senior researcher and one junior researcher – and the author gets an opportunity to improve his/her paper before publishing. This meant that the proceedings were published some months after the meeting. Thus, the scientific level of the proceedings has improved, e.g. some of the proceedings have even been published in an international book series, as in the Sense Publisher (cf. appendix).

One could say that the annual MAVI meetings have brought beliefs into the focus of research in mathematics education. Until the beginning of the 1990s, the main papers in beliefs were generally dealing with education (e.g. Abelson, 1979; Pajares, 1992). There were in fact only the Underhill papers (1988a, 1988b) in mathematics education. Furthermore, in the Handbook of research on mathematics learning and teaching (Grouws, 1992), two papers on mathematics-related beliefs were published; one by Schoenfeld and one by Thompson. But after the middle of the 1990s, the number of belief publications increased clearly.

The fact that the international value of the MAVI meetings is increasing could be also seen in the broader interest of researchers to the MAVI work. For example, in the last five meetings the base of the participants has enlarged within Europe and also outside. There are new countries where researchers are coming from to the MAVI meetings, also even from other continents (Africa, Asia, Australia and America).

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Notes

1 Deutsche Akademische AustauschDienst (German Academic Exchange Service)

Appendix: The list of all MAVI proceedings

1. Törner, G. (Ed.). (1995). *Current state of research on mathematical beliefs* (Proceedings of the MAVI workshop, University of Duisburg, October 4-5, 1995. Preprint nr. 310). Schriftenreihe des Fachbereichs Mathematik, Gerhard-Mercator-Universität Duisburg Gesamthochschule.

2. Törner, G. (Ed.). (1996). *Current state of research on mathematical beliefs II* (Proceedings of the 2nd MAVI workshop, University of Duisburg, March 8–11, 1996. Preprint nr. 340). Schriftenreihe des Fachbereichs Mathematik, Gerhard-Mercator-Universität Duisburg Gesamthochschule.

3. Pehkonen, E. (Ed.). (1996). *Current state of research on mathematical beliefs III* (Proceedings of the MAVI-3 workshop in Helsinki 23–26.08.1996. Research report 170). Department of Teacher Education, University of Helsinki.

4. Törner, G. (Ed.). (1997). *Current state of research on mathematical beliefs IV* (Proceedings of the MAVI workshop, University of Duisburg, April 11–14, 1997. Preprint nr. 383). Schriftenreihe des Fachbereichs Mathematik, Gerhard-Mercator-Universität Duisburg Gesamthochschule.

5. Hannula, M. (Ed.). (1997). *Current state of research on mathematical beliefs V* (Proceedings of the MAVI-5 workshop in Helsinki 22.–25.8.1997. Research report 185). Department of Teacher Education, University of Helsinki.

6. Törner, G. (Ed.). (1998). *Current state of research on mathematical beliefs VI* (Proceedings of the MAVI-6 workshop, University of Duisburg, March 6–9, 1998. Preprint nr. 404). Schriftenreihe des Fachbereichs Mathematik, Gerhard-Mercator-Universität Duisburg Gesamthochschule.

7. Hannula, M. (Ed.). (1998). *Current state of research on mathematical beliefs VII* (Proceedings of the MAVI-7 workshop 2.–5.10.1998. Research report 198). Department of Teacher Education, University of Helsinki.

8. Philippou, G. (Ed.). (1999). *MAVI-8 proceedings. Eight European workshop. Research on mathematical beliefs*. Nicosia: University of Cyprus.

9. Götz, S. & Törner, G. (Eds.). (2000). *Research on mathematical beliefs* (Proceedings of the MAVI-9 European workshop. Preprint nr. 482). Schriftenreihe des Fachbereichs Mathematik, Gerhard-Mercator-Universität Duisburg Gesamthochschule.

10. Soro, R. (Ed.) (2001). *Current state of research on mathematical beliefs* (Proceedings of the MAVI-10 European workshop. Pre-Print Series nr. 1). Department of Teacher Education, University of Turku.

11. Di Martino, P. (Ed.). (2002). *MAVI European workshop, MAVI-XI, research on mathematical beliefs* (Proceedings of the MAVI-XI European workshop, April 4–8, 2002). University of Pisa.

12. Hoskonen, K. & Hannula, M.S . (Eds.). (2007). *Current state of research on mathematical beliefs XII* (Proceedings of the MAVI-12 workshop, May 25–28, 2006. Research report 288). Department of Applied Sciences of Education, University of Helsinki.

13. Attorps, I. & E. Kellner, E. (Eds.). (2008). *Conceptions and beliefs in mathematics and science education including MAVI XIII*. Department of Mathematics Education, University of Gävle.

14. Maass, J. & Schlöglmann, W. (Eds.). (2009). *Beliefs and attitudes in mathematics education*. Rotterdam: Sense Publishers.

15. Furinghetti, F. & Morselli, F. (Eds.). (2010). *Proceedings of the conference MAVI-15: ongoing research on beliefs in mathematics education*. Dipartimento di Matematica, Università di Genova - Provincia di Genova.

16. Kislenko, K. (Ed.). (2011). *Current state of research on mathematical beliefs XVI* (Proceedings of the MAVI-16 conference, June 26–29, 2010 in Tallinn). University of Tallinn.

17. Rösken, B. & Casper, M. (Eds.). (2011). *Current state of research on mathematical beliefs XVII* (Proceedings of the MAVI-17 conference). University of Bochum.

18. Hannula, M. S. (Ed.). (2012). *Current state of research on mathematical beliefs XVIII* (Proceedings of the MAVI-18 conference). Department of Teacher Education, University of Helsinki.

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