Structure of university students' view of mathematics in Estonia

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This study reports on first-year Estonian university students' view of mathematics. The data was collected from 970 university students of different disciplines. The participants filled out a Likert-type questionnaire that was developed using previously published instruments. The study confirmed that several different attitudes, beliefs, and motivational orientations can be identified and validly measured as separate components of Estonian university students' view of mathematics. However, the low reliability of some scales highlights the necessity for careful testing of instruments in any new population.

Mathematics educators agree that mathematics learning is not only about cold reason. As Lester, Garofalo and Kroll (1989) point out: "Any good mathematics teacher would be quick to point out that the students' success or failure in solving a problem often is as much a matter of self-confidence, motivation, perseverance, and many other non-cognitive traits, as the mathematical knowledge they possess" (p. 75). However, most research has attempted to analyse one or a few of these non-cognitive variables at a time. In this article, we aim to handle the system of non-cognitive traits more holistically, as a *view of mathematics*.

Research on mathematics-related attitudes and beliefs has been rather separate from research in motivation, the previous being the main trend among mathematics educators and the latter among educational psychologists (see Hannula, 2011 for elaboration). In this article we attempt to bring together these two traditions, joining together the parts of some previously published instruments on mathematical beliefs/attitudes and motivational orientation to mathematics.

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This new instrument was composed and used to a) confirm its applicability in Estonia at the university level; b) explore the structure of (Estonian) university students' view of mathematics; and c) compare the structure with results found earlier among Finnish, Spanish and English high school students.

Research on mathematics-related affects began with studies regarding mathematics anxiety and attitudes. Thereafter, the focus has been more widely on a variety of mathematics-related beliefs. The studies of attitude were based on two assumptions: attitude toward mathematics is related to achievement, and affective outcomes (such as liking mathematics) are significant *per se*. As with mathematics anxiety, the construct was borrowed from another field; namely, from social psychology. Attitudes were measured by using questionnaires, typically Likert-scale items (e.g. Fennema & Sherman, 1976). However, some of these questionnaires, besides having items about liking/disliking mathematics, included items on mathematics anxiety and beliefs about mathematics and oneself (Zan, Brown, Evans & Hannula, 2006).

McLeod (1992) made an important contribution to organizing the field. He suggested that mathematics related affect should be conceptualised using beliefs, attitudes and emotions. In his framework these range along a dimension of increasing stability and decreasing intensity – with emotions as most intense/least stable, beliefs as most stable/least intense, and attitudes in between. Moreover, motivation was conceptualized as motivational beliefs. McLeod's work in particular ushered in a new period of research on affect in mathematics education. Later, DeBellis and Goldin (1997) added a fourth element, *values*, but argued that the four types could no longer be ordered on a single stability/intensity dimension. More recently, Hannula (2011) suggested that mathematics related affect has an emotional, a motivational, and a cognitive dimension, each of which would have a state aspect and a trait aspect. Empirical studies have provided support for the separate character of these three types of affective traits among Finnish students in comprehensive (Hannula & Laakso, 2011) and secondary school (Roesken, Hannula & Pehkonen, 2011). In this paper, we explore the validity of this approach among Estonian university students.

Defining the view of mathematics

In the expanding research field, several different definitions were given for the central concepts. Furinghetti and Pehkonen (2002) wanted to identify the common ground for discussing the mathematics-related affect and, therefore, asked a virtual panel of mathematics education researchers to evaluate different definitions that these same researchers had suggested for the concepts *attitude*, *belief* and *conception* in their papers. Sadly, the researchers could not agree on any of their definitions. The problem of ambiguous terminology has been raised repeatedly over the years (for elaboration, see Hannula, 2011).

Most studies of mathematics-related affect have been carried out with a separate focus on cognitive, motivational or affective aspects and only a few contributions explicitly address beliefs as a system (Op 't Eynde & De Corte, 2003). In order to emphasize the present focus on studying the structure of students' mathematical beliefs, attitudes and motivation, we use the term *view of mathematics* in this paper. This term was originally introduced by Schoenfeld (1985) and later adapted by others (Pehkonen, 1995; Pehkonen & Törner, 1996). Students' view of mathematics is a result of their experiences as learners of mathematics and as such, it provides an interesting window through which to study mathematics teaching. Moreover, mathematical competence is not only about knowledge and skills, but also about the disposition to act in productive ways. Students' view of mathematics is an indication of this disposition.

Beliefs systems can be seen to be developed from simple perceptual beliefs or beliefs based on authority – via new beliefs, expectations, conceptions, opinions, and convictions – to a general outlook on life (Saari, 1983). Thus, for example, conceptions are higher-order beliefs. They are based on reasoning processes for which the premises are conscious.

Op 't Eynde, De Corte and Verschaffel (2002) used the following definition of the concept of beliefs in their study:

> Students' mathematics-related beliefs are the implicitly or explicitly held subjective conceptions students hold to be true about mathematics education, about themselves as mathematicians, and about the mathematics class context. These beliefs determine, in close interaction with each other and with the students' prior knowledge, their mathematical learning and problem solving in class. (p. 27)

Finally, we refer to the structure of view, which was obtained in a previous study when analysing the data of student elementary teachers (Hannula, Kaasila, Laine & Pehkonen, 2005, 2006). This study led to eight scales describing these students' views of themselves as learners of mathematics, and particularly three dimensions that form a core of these views. In a study by Roesken et al. (2011), the dimensions are described by reliable scales, which allow for the outlining of an average image of Finnish students' views of themselves as learners of mathematics. Moreover, they analysed the relations between the seven dimensions and what kind of structure they generated. Against the aforementioned theoretical background and their subsequent findings, we are now interested in the structure of university students' views of themselves as learners of mathematics at the tertiary level as well as the relations between the dimensions. With regard to this focus, we pay attention to the cognitive component described by beliefs as well as to motivational aspects. The choice of concept draws on the following aspects: first, beliefs are often considered to be on a more cognitive side of the affect (e.g. McLeod, 1992). Using "view" instead of "beliefs", we want to emphasize that not all dimensions we address are cognitive ones. Second, we consider the term "view" more appropriate to capture the structural properties of the affect–cognition interplay in social learning situations. In some sense, the term "beliefs" is separate while "view" is holistic (Roesken et al., 2011).

The dimensions of view of mathematics

For the dimensions of students' view of mathematics Op 't Eynde, De Corte and Verschaffel (2002) identified three main categories of belief-related research which helped to form the development of their instrument. These were beliefs about mathematics education, beliefs about the social context and beliefs about oneself as a learner of mathematics. Starting from existing questionnaires that usually measure only one kind of beliefs (e.g. or beliefs about math, or beliefs about the self), they developed a more integrated instrument that asked students about their beliefs on mathematics education, on the self in relation to mathematics, and on the social context in their specific math classroom (limited here to beliefs about the role and the functioning of their teacher). Op 't Eynde and De Corte (2003) tried to develop, from a warranted theoretical perspective, a comprehensive instrument for the assessment of students' beliefs about mathematics, as well as its teaching and learning. The questionnaire (Op 't Eynde & De Corte, 2003) was developed for and evaluated using Flemish students. An adaptation of the mathematics-related beliefs questionnaire (Diego-Mantecon, Andrews & Op 't Eynde, 2007) reported on an attempt to refine the questionnaire in order to determine empirically the structure of Spanish and English students' mathematics-related beliefs with the result that the questionnaires transfer outside the original Flemish context. In addition, Roesken et al. (2011) had primarily focused on the systematic character of beliefs and they were interested in dimensions describing such a view of mathematics. The study (Roesken et al., 2011) led to eight scales describing these students' view of mathematics, and particularly three dimensions that were closely related. There is considerable overlap between these studies, but also some important differences (see table 1).

Table 1. *Scales from five mathematical beliefs survey instruments organized according to categories identified by a literature review.*

Beliefs about ...	Component				
Mathematics education					
Mathematics as a subject		Difficulty of mathematics			
Mathematical learning and problem solving	**Mathematics as rote-derived knowledge**				
Mathematics teaching in general					
Self					
Self-efficacy beliefs	**Competence**	**Competence**	**Attitudes to mathematics**		
		Confidence	Attitudes to problem solving		
Control beliefs		**Effort**		**Cheating behaviour**	
Task value beliefs	**Relevance**	Enjoyment of mathematics			Enjoyment of mathematics
					Interest in mathematics learning
					General value of mathematics
					Personal value of mathematics
Goal orientation beliefs				**Performance-approach goal orientation**	
				Performance-avoidance goal orientation	
				Mastery goal orientation	
Social context					
Social norms	**Teacher's role**	Teacher quality			
		Family encour-age-ment			
Sociomathematical norms					
Op 't Eynde et al. (2002)	Diego-Mantecon et al. (2007)	Roesken, et al. (2011)	Yusof & Tall (1994)	Midgley et al. (2000)	PISA technical report 2006 (2009)

Note. Scales, where items were selected from for the present research are in boldface.

It should be noted in this table, that while instruments for attitudes and beliefs have a significant overlap, they seem to have little in common with the instrument for motivation. There is a general assumption that there is a relationship between mathematics-related motivation and beliefs, yet the theories of their relationships are new (Hannula, 2006; Op 't Eynde, De Corte & Verschaffel, 2006). Research has identified a positive relation between mastery orientation and attitudes, effort, competence beliefs (Hannula & Laakso, 2011; Seo, 2000) and positive emotions (Kumar, Gheen & Kaplan, 2002; Midgley et al., 1998; Pekrun, Elliot & Maier 2006). Some items on motivation were included in the Finnish questionnaire, but they failed to form a reliable component (Hannula et al., 2006). In order to include motivation in the instrument, we used selected scales from Midgley et al.'s (2000) personal achievement motivation questionnaire. This motivation questionnaire has been developed and refined over time by a group of researchers using goal orientation theory to examine the relation between the learning environment and students' motivation, affect, and behaviour. Student scales in their instrument assess 1) personal achievement goal orientations; 2) perceptions of teachers' goals; 3) perceptions of the goal structures in the classroom; 4) achievement-related beliefs, attitudes, and strategies; and 5) perceptions of parents and home life. Midgley et al. (2000) used 18 students scales. The student scales have been successfully administered at the elementary, middle, and high school levels with approximately equal proportions of male and female participants. Student scales have been used with samples of children from third to ninth grade. For this study, we used personal achievement goal orientation scales: a mastery goal orientation whose alpha is 0.85 and a performance-approach goal orientation whose alpha is 0.89 (Midgley et al., 2000).

In 1987, the California Department of Education labelled cheating an "epidemic" after finding that 75 % of secondary school students reported that they had at some time cheated on school work (Schab, 1991). Most of the research on cheating has been conducted with college students (e.g. Whitley, 1998). However, college students often report that they cheated more in high school than in college (e.g. Baird, 1980). There is also evidence that cheating is more widespread in high school than in middle school (e.g. Brandes, 1986; Evans & Craig, 1990). Anderman and Midgley's (2004) results indicated that self-reported cheating increased more after the transition to high school than before it, and that self-reported cheating was related positively to a perceived classroom emphasis on performance goals and negatively to a perceived classroom emphasis on mastery goals. However, the hypothesized gender differences did not emerge. In order to explore the regularity of cheating, we used a cheating behaviour scale from Midgley et al. (2000). This refers to students' use of cheating in class and included items such as "I sometimes copy answers from other students during tests" and "I sometimes cheat on my class work". The Cronbach alpha for the cheating scale was 0.87.

There are fewer studies about mathematics-related beliefs at the university level (with the exception of elementary education students; e.g. Hannula et al., 2006). Yusof and Tall (1994) developed a questionnaire for attitudes to mathematics and investigated students' attitudes before and after a university course with an emphasis on cooperative problem solving and reflection on the thoughts of the problem-solving process. Our reading of the items specified that the "attitudes" measured in this study were mostly self-efficacy beliefs, according to the Op 't Eynde's classification.

Previous results concerning students' beliefs

Although in other countries, studies on students' mathematical beliefs are numerous, there are fewer studies at the university level. Studies at the university level typically focus on mathematics majors, teacher education students or students of compulsory statistics courses. Among university level mathematics students, studies have indicated the important role of beliefs in mathematical problem solving (Schoenfeld, 1985) and students' reasoning processes (Sumpter, 2009). In a study by Yusof and Tall, before the mathematics course, half of the students responded that university mathematics did not make sense to them. A majority declared negative attitudes such as anxiety, a fear of new problems and a lack of confidence (Yusof & Tall, 1994). Juter (2005) found that university mathematics course students considered mathematics to be about facts and processes to remember and solving problems but also about coming up with new ideas. Most students (1st- and 2nd-year students from Swedish universities) claimed they did not learn by rote. The majority also stated that they could synthesise mathematical ideas that they had learned, but many had to work very hard to understand mathematics at the university level (Juter, 2005). Kadriye (2005) used an exploratory study that examined factors that might be associated with achievement in mathematics and participation in advanced courses in various countries. Confidence in mathematics was the strongest predictor of achievement for students from Canada and Norway; whereas, for the students from the USA, their parents' education level was the main predictor of achievement. In the study of Lazim, Osman and Salihin (2004), they found that students held strong beliefs that their teachers play a major role in contributing to their interest in mathematics. They also found that "drill and practice" was a very important element in learning mathematics. Midgley, Feldlaufer and Eccles (1989), in a longitudinal study of 1301 students and the teachers they had in mathematics before and after the transition to junior high school, they assessed whether changes across the transition in students' perceptions of their teachers' supportiveness were related to changes in their valuing of mathematics. For students' perceptions of the usefulness and importance of mathematics there was an interaction with achievement level. Values for mathematics decreased more sharply during the first year of junior high for low-achieving students who moved from more supportive to less supportive teachers than for high-achieving students who experienced the same change. For students' perceptions of the usefulness and importance of mathematics, there was an interaction with their achievement level. Mathematics values decreased more sharply during the first year of junior high for low-achieving students who moved from more supportive to less supportive teachers than for high-achieving students who experienced the same change.

While there is a substantial amount of research in mathematics education at the school level, the amount at the tertiary level is still modest. Some tertiary studies (e.g., those investigating the effects of gender or the kinds of students who succeed in mathematics) have been conducted by mathematics education researchers without a particularly strong background in tertiary mathematics (Selden & Selden, 2001). However, there has not been an investigation of students' views of mathematics in Estonia at the university level and thus at present this is an unexplored area in Estonia. In the Nordic countries and Baltic States, the field of affect in mathematics at the university level is an almost uncovered theme. In these countries, there have only been a few studies at the university level (Juter, 2005; Sumpter, 2009). Additionally, the special issue of the ICMI Study "The Teaching and Learning of Mathematics at the University Level" does not cover the field of affect.

Previous results concerning students' beliefs in Estonia

While assessing Estonian developments and the results of mathematics education research, one should consider the very small size of the country. It has clearly set the limits for research environments and resources available to carry out scholarly activities in the field. A research topic in mathematics education in Estonia since the 1990s has been the turn towards affective factors of mathematical learning (Lepik, 2009). With the cooperation of Finnish and Norwegian researchers, the focus of studies was raised towards students' and teachers' beliefs and attitudes. There has been an emphasis on research focusing on understanding the students' beliefs about mathematics and its learning (Lepmann & Afanasjev, 2005). In addition, a comparative study between Estonian and Norwegian students' mathematical beliefs has been carried out (Kislenko, 2009). Teachers' and student teachers' beliefs about mathematics, its teaching and learning have been the focus of others as well (Hemmi, Lepik & Vihoalainen, 2010). Some studies about students' and teachers' attitudes in comprehensive schools or in upper-secondary schools have been carried out in Estonia (Kislenko, 2009; Lepmann, 2000; Lepmann & Afanasjev, 2005; Pehkonen & Lepmann, 1994). The study of Lepmann and Afanasjev (2005) revealed that high-attaining pupils have considerably greater faith in achieving success in mathematics learning than low-attaining pupils. Compared to other pupils, high-attaining students are considerably more desirous of each pupil being able to work according to his or her ability. They want to develop their ability and are ready to do more work in the name of success. However, low-attaining pupils are more disposed to giving up than pupils with high attainment. Kislenko's (2009) more recent results indicated that students in comprehensive and secondary schools perceive mathematics to be important, but studying it tends to be difficult and boring. Research into mathematics education at the tertiary level may be itself an interesting field of research and may give rise to useful results for all teachers to apply to their teaching (Alsina, 2001). Alsina (2001) said that at the freshman level, the mature students' myth assumes that during the few weeks between high school and university registration, students have grown in such a way that their integration into the new university atmosphere does not require any special attention. In particular, students going into scientific or technical courses are assumed to be already motivated and aware of the relevance of mathematics to their training, and students going into other studies are assumed to constitute a low-interest class. Clearly, the transition from secondary schools to universities needs special attention.

The purpose of the survey is to explore whether the structure of Estonian university students' view of mathematics is similar to the structure identified among younger populations in Finland, and to record the Estonian university students' view of mathematics. Hence, the study is testing both the applicability of the theoretical framework and the reliability of the instrument in a new population and, in addition, exploring the previously uncharted area of Estonian university students' view of mathematics.

Methodology of research

Instrument

The view of mathematics indicator used in this research (Kaldo, 2011) has combined scales from the six studies identified above (table 1, for sample items, see table 3). The items were selected to measure all main components of mathematics-related affect with a specific focus on the different dimensions of self-beliefs. The composed instrument consisted of the following elements of students' view of mathematics:

Performance-approach goal orientation (Midgley et al., 2000), 4 items

Mastery goal orientation (Midgley et al., 2000), 6 items

Mathematics as a rote-learnt subject (Diego-Mantecon et al., 2007), 4 items

Attitudes to mathematics (Yusof & Tall, 1994), 6 items

Relevance (Diego-Mantecon et al., 2007), 9 items

Personal value of mathematics (PISA, 2006), 3 items

Students competence (Roesken et al., 2011, 3 items; Diego-Mantecon et al., 2007, 3 items;), 6 items

Teacher role (Diego-Mantecon et al., 2007), 5 items

Cheating behaviour (Midgley et al., 2000), 3 items

Effort (Roesken et al., 2011), 4 items

The questionnaire was translated into Estonian and after a back translation by an independent translator minor reformulations were made. A pilot study was carried out in Estonia in spring 2009 with 93 students. Since the aim of the study was to confirm the earlier scales on beliefs (Diego-Mantecon et al., 2007; Rösken et al., 2007), attitudes (Yusof & Tall, 1994) and motivation (Midgley et al., 2000), a confirmatory factor analysis was performed. The pilot study gave a positive signal about the usefulness of the instrument (Kaldo, 2011).

The main survey was carried out in Estonia in the autumn of 2009. Participants were informed that the aim of the research was to study their attitudes, beliefs and motivation regarding mathematics. The students were given the questionnaire on paper and they were asked to respond anonymously on a Likert scale (strongly disagree to strongly agree). We used a 4-point scale because, because we feared that students might be too eager to choose the neutral position. The administration time was 45 minutes.

Sample

There are five universities in Estonia. The University of Tartu (hereinafter UT) was founded in 1632 and it is the largest university in Estonia. There are a total of 17,643 students studying at the university's nine faculties and five colleges. The Tallinn University of Technology (hereinafter TUT) was founded as an engineering college in 1918, acquiring university status in 1936. TUT is one of the largest universities in Estonia, providing an interdisciplinary higher education and technological advancement. TUT has 13,430 students. Tallinn University (hereinafter TLU) is a public institution of higher education. Its main strengths lie in the fields of humanities and social sciences, but it also has a strong and constantly growing component of natural and exact sciences, as well as a notable tradition of teacher training and educational research. Tallinn University is the third largest university in Estonia, consisting of 20 institutes and 5 colleges. It has 8692 students. The Estonian University of Life Sciences (hereinafter ULS) is the only university in Estonia whose priorities in academic and research activities are to provide for the sustainable development of natural resources necessary for the existence of man as well as the preservation of heritage and habitat. There are five institutes and 4898 students there. The Estonian Business School (hereinafter EBS) is a private business school of university standing, offering programs in the fields of business administration, public administration and information technology management. It has 1553 students.

The total number of bachelor students in Estonia is 31,691 and first-year bachelor students' number 8770 (Estonian Ministry of Education and Research, 2011). To provide valid estimates of student achievement, the sample of students had to be selected in a way that ensured sufficient representation of the full target population. The target population is students who study at least one course of mathematics as part of their studies. The target sample group for the current study was a purposeful sample of 970 students from first-year mathematics course students from one private and four public universities: the Estonian Business School, Tallinn University, Tallinn Technical University, Tartu University and the University of Life Sciences. The participants were volunteer bachelor students taking at least one first-year compulsory mathematics course at the university level, from all over Estonia. The survey was completed during lectures of mathematics courses that were compulsory for them. Almost all the students filled in the questionnaire (less than 1 % refused to respond). It is common for students to be absent during lectures and responding was voluntary. The response rate was 69 % (out of a total of 1406 enrolled students), which is high. The average age of the responding students was 19.8. There were 508 males and 462 females from 49 curriculums (study programmes) (table 2).

Table 2. *Sample description*

University	No. of curriculums represented in the sample	No. of students in the sample	Percentage of students in the sample	Male	Female
EBS	3	91	9.4 %	43	48
ULS	10	228	23.5 %	130	98
TLU	8	103	10.6 %	52	51
TUT	12	314	32.4 %	185	129
UT	16	234	24.1 %	98	136
Total	49	970	100.0 %	508	462

Validity and reliability

In the current study, a questionnaire was used as a measuring device. Thus, the validity and reliability of the instrument and methodology ware carefully checked. The current study emphasises the importance of internal validity, which shows the extent to which side effects that can affect the phenomena being researched have been taken into account. In compiling the selection, the following aspects were taken into account:

- all the students in our study are students at universities, not for example students at colleges;
- we covered all the universities in Estonia;
- all the students involved in the study are bachelor students taking at least one first-year compulsory mathematics course; thus, all of them have at least one mathematics course in their curriculums;
- the participants are from several different curriculums.

The requirement of external validity, related to the generalisation of the outcomes to the entire student community who takes mathematics courses is not a limiting factor in the study at hand because in Estonia in the autumn of 2009, there were a total of 5090 first-year bachelor students in the various universities and not all of them were studying mathematics during that time (Estonian Ministry of Education and Research, 2011). For the whole population, the sample size required to get an expected prevalence rate of 0.5 % is 598, but we collected 970 questionnaires, which means that our sample size is representative. However, we are aware that because we collected our data from lectures, our sample is biased towards students who attend lectures more regularly. This is not likely to be a problem with regard to analysis of the structure of students' view of mathematics. However, it is likely to produce some bias in the mean values measured for each scale and it may compromise the validity of results considering gender differences. Content validity shows the extent to which the content corresponds to the meaning to which it is ascribed. In other words, how well single questions reflect the measurements of students' achievement and skills. In the current study, an expert opinion method was used.

Reliability characterises the stability, consistency and suitability of the methodology used. Reliability shows how well the results of repeated measurements (by either the same researcher or different researchers) carried out in the same circumstances coincide. Reliability also indicates whether a certain indicator measures consistently and continuously. In other words, how reliable the result of the measurement is. In our study, the Cronbach's alpha was used as a measure of the internal consistency reliability of the questionnaire. If the reliability coefficient is 0.70 or higher, it is considered "acceptable" in most social science research situations.

Data analysis

We used the statistical program SPSS Statistics for the data analysis. Since the aim of the study was to confirm the earlier scales on beliefs (Diego-Mantecon et al., 2007, PISA, 2006; Roesken et al., 2011), attitudes (Yusof & Tall, 1994) and motivation (Midgley et al., 2000), we used items from the earlier research and computed the reliabilities of our modified scales. In addition, we made an exploratory factor analysis which revealed a similar factor structure as found in earlier studies. For the exploratory factor analysis, we used the maximum likelihood method with direct oblique rotation to determine useful and statistically robust dimensions regarding this construct. This method of factor analysis allows making inferences from sample to population; the sample of 970 students is, therefore, large and adequate enough. The choice of oblique rotation was used to establish the factorial validity of the scales. The dimensions of the construct we are dealing with cannot be regarded independently of each other; therefore, we allow correlations among factors. In that case an oblique rotation will lead to a better estimation of factors since it derives factor loadings based on the assumption that they are correlated (Fabrigar, Wegener, MacCallum & Strahan, 1999). In the program SPSS with the Kaiser criteria "eigenvalue > 1", we got a suggestion to use 13 factors. According to Cattell's scree–test, with an inspection of the scree plot, the proper number of factors appeared to be between 5 and 7. We decided to use a 7-factor solution for the whole survey because a 7-factor solution corresponded with the expected number of factors. Another reason was that some factors contained only two items factor solutions or their Cronbach alphas were low. Items, which had communalities less than 0.3, were removed. Moreover, we explored the structure of the view of mathematics through calculating correlations between the reliable components in SPSS. In addition, we also calculated the mean scores and standard deviations for the whole sample (n = 970) on each of the components. We also analysed gender differences in the different dimensions of the view using a t-test for the equality of means (independent samples).

Results of research

In the following section, we will present several results of our data analysis. A summary of the identified dimensions of the students' view of mathematics is presented in table 3. Table 3 shows the factors and their related items as well. The Cronbach's alpha demonstrates the reliability of the components. After examining the reliability coefficients, it can be said that three of the factors were not reliable, whilst the others can be considerate moderately to highly reliable.

The reliability analysis confirmed 7 factors: Performance-Approach Goal Orientation, Mastery Goal Orientation, Relevance, Personal Value of Mathematics, Student Competence, Teacher Role and Cheating Behaviour. The factors Relevance and Personal Value of Mathematics were combined, because they were highly correlated (the correlation coefficient was 0.723, $p < .001$), their items were similar, and their correlation with other measured factors were comparable. The reliability of the combined variable was on an equal level to the reliability of the original scales. The factors Mathematics as a Rote-Learnt Subject, Attitudes to Mathematics and Effort were not confirmed in this sample. We also analysed the possibility of removing items to gain a higher alpha, but the reliability did not get any higher. However, we decided to include the factor Attitudes to Mathematics in further analyses, because its reliability was close to 0.70 as well as the factor Effort, because the scale had been confirmed in earlier studies.

Table 3. *Factors: sample items, reliability, mean and standard deviation.*

Factors	Sample items	Number of items	Cronbach's alpha	Mean	Std. deviation
F1. Performance-Approach Goal Orientation	It's important to me that other students in my class think I am good at my class work.	4	0.78	1.99	0.65
F2. Mastery Goal Orientation	It's important to me that I improve my skills this year in mathematics.	6	0.74	3.51	0.65
F3. Mathematics as A Rote-Learnt Subject	Learning mathematics is mainly about having a good memory.	5	0.15	2.92	0.14
F4. Attitudes to Mathematics	Mathematics is about solving problems.	6	0.63	3.11	0.38
F5. Relevance	Some knowledge of mathematics helps me to understand other subjects.	9	0.82	2.56	0.64
F6. Personal Value of Mathematics	Mathematics is useful for our society.	3	0.70	2.35	0.62
F5 & F6 combined: Value of Mathematics	Knowing mathematics will help me earn a living.	12	0.80	2.49	0.52
F7. Student Competence	I am good at mathematics.	6	0.82	2.36	0.33
F8. Teacher Role	My lecturer explains why mathematics is important.	5	0.72	2.71	0.44
F9. Cheating Behaviour	I sometimes copy answers from other students during tests.	3	0.82	1.86	0.83
F10. Effort	I have worked hard to understand mathematics.	4	0.52	2.91	0.55

Conclusion and discussion

In this study, our aim was to construct an instrument to explore the structural properties of students' view of mathematics. We used a number of instruments and combined a number of scales that were found reliable in earlier studies. However, all the scales from the earlier studies were not confirmed in our study. Seven factors had high Cronbach's alpha and their reliability for Estonian university students was confirmed. Due to high correlation between two of the scales and high similarity in their content, we decided to combine two of the scales (Relevance and Personal Value), constructing a new scale Value of Mathematics that had high reliability. The reliability of two scales used in the study was not confirmed. The original scale for "Mathematics as a Rote-Learnt Subject" had 13 items and its reliability in previous studies was 0.76. We chose only five of the items for our study and the reliability was not confirmed. The original scale for "Effort" had six items and the reliability in the Roesken et al. (2011) study was 0.83, while in this study it was only 0.52. The reliability was also low for the scale "Attitudes to Mathematics", for which the original study did not report the reliability of the scale. We used six out of the ten original items in our study. Decreasing the number of items typically decreases the reliability of the scale. However, the reliability of the scale "Mathematics as a Rote-Learnt Subject" was so low that there must be some serious deficiencies. This was one of the dimensions that was not found reliable in Op 't Eynde's original instrument, and the revised version of the instrument seemed to have slightly different meaning (Diego-Mantecón, Andrews & Op 't Eynde, 2007). The standard deviation of the item responses was very low, which may have contributed to the low reliability of the scale in this sample. We note that the scales had been previously tested on high school students outside of Estonia and suggest that the differences in reliability can be related to differences between samples (age, level of study, field of study, culture) or the translation of the items. Moreover, we are well aware that quantitative surveys such as this one will identify only those psychological constructs that are measured by the specific instrument in use. Hence, it is possible that our study has missed some important dimensions of Estonian university students' view of mathematics.

These results regarding the structure of affective components share only some components with previous studies. In our study, the confirmed structure relates to structures in Diego-Mantecon et al. (2007) regarding the factors Relevance, Student Competence and Teacher Role, but our study did not confirm the factor Mathematics as a Rote Learnt Subject. Compared to the Roesken et al. (2011) study, our structure confirmed the factor Student Competence, but not the factor Effort. The different dimensions found in previous studies and in our study may indicate that Effort and View of Mathematics as a Rote Learnt Subject are not valid dimensions in an Estonian context. However, a different instrument did identify a reliable scale (Cronbach's alpha = .71) for Effort ("Hard-working in mathematics") among Estonian grade 7-11 students (Kislenko, 2009). Alternatively, the differences might indicate that these dimensions are not valid among university students, perhaps due to lack of variation.

This survey was the first one which investigated the students' views of mathematics in Estonia at the university level. The instrument was built on previous works in this area by synthesising conceptual frameworks and the new instrument combines scales and items from previous studies of students of different educational levels and in different countries. For identical items in both populations, we found the same factor structure and reliability analysis confirmed the internal consistency of seven factors. Based on the reliability analysis, the first-year Estonian baccalaureate students' views of mathematics are formed mainly by similar constructs as in earlier research (Diego-Mantecon et al., 2007; Midgley et al., 2000; PISA, 2006; Roesken et al., 2011; Yusof & Tall, 1994). This gives a positive signal about the usefulness of the selected scales in other populations too. However, some robust differences in scale reliabilities ask for caution when importing questionnaires to new cultural contexts.

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References

Alsina, C. (2001). Why the professor must be a stimulating teacher. In D. A. Holton (Ed.), *The teaching and learning of mathematics at the university level: An ICMI Study* (pp. 3–13). Dordrecht: Kluwer Academic Publishers.

Anderman, E. M. & Midgley, C. (2004). Changes in self-reported academic cheating across the transition from middle school to high school. *Contemporary Educational Psychology*, 29, 499–517.

Brandes, B. (1986). *Academic honesty: a special study of California students*. Scaramento: California State Department of Education, Bureau of Publications.

DeBellis, V. A. & Goldin, G. (1997). The affective domain in mathematical problem solving. In E. Pehkonen (Ed.), *Proceedings of the 21st PME Conference*, Vol. 2 (pp. 209–216). Lahti: PME.

Diego-Mantecón, J., Andrews, P. & Op 't Eynde, P. (2007, February). *Refining the mathematics-related beliefs questionnaire*. Paper presented at the working group 2 on affect and mathematical thinking of the fifth European Conference on Mathematics Education, Cyprus.

Estonian Ministry of Education and Research. (2011). *The number of students studying according to higher education curricula*. Retrieved from http://www.hm.ee/index.php?048055.

Evans, E. D. & Craig, D. (1990). Teacher and student perceptions of academic cheating in middle and senior high schools. *The Journal of Educational Research*, 84, 44–52.

Fabrigar, L. R., Wegener, D. T., MacCallum, R. C. & Strahan, E. J. (1999). Evaluating the use of exploratory factor analysis in psychological research. *Psychological Methods*, 4 (3), 272–299.

Fennema, E. & Adajian L. B. (Eds.). *New directions for equity in mathematics education*. Cambridge University Press.

Fennema, E. & Sherman, J. A. (1976). Fennema-Sherman mathematics attitude scales. *JSAS Catalog of Selected Documents in Psychology*, 6, 31.

Furinghetti, F. & Pehkonen, E. (2002). Rethinking characterizations of belief. In G. Leder, E. Pehkonen & G. Törner (Eds.), *Beliefs: a hidden variable in mathematics education?* (pp. 39–57). Dordrecht: Kluwer Academic Publishers.

Hannula, M. (2006). Motivation in mathematics: goals reflected in emotions. *Educational Studies in Mathematics*, 63 (1), 165–178.

Hannula, M. S. (2011). The structure and dynamics of affect in mathematical thinking and learning. In M. Pytlak, E. Swoboda and T. Rowland (Eds.). *Proceedings of the seventh Congress of the European Society for Research in Mathematics Education*, 34–60. University of Rzeszów.

Hannula, M., Kaasila, R., Laine, A. & Pehkonen, E. (2005). Structure and typical profiles of elementary teacher students' view of mathematics. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 89–96). Melbourne: PME.

Hannula, M. S., Kaasila, R., Laine, A. & Pehkonen, E. (2006). The structure of student teacher's view of mathematics at the beginning of their studies. In M. Bosch (Ed.), *Proceedings of the fourth congress of the European Society for Research in Mathematics Education* (pp. 205–214). Fundemi IQS: Universitat Ramon Llull.

Hannula, M. S. & Laakso, J. (2011). The structure of mathematics related beliefs, attitudes and motivation among Finnish grade 4 and grade 8 students. In B. Ubuz (Ed.) *The Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education*. Ankara, Turkey: PME.

Hemmi, K., Lepik, M. & Viholainen, A. (2010). Upper-secondary school teachers' views of proof and proving – an explorative cross-cultural study. In K. Kislenko (Ed.), *Proceedings of MAVI 16, The 16th conference on Mathematical Views* (pp. 137–156). Institute of Mathematics and Natural Sciences, Tallinn University.

Juter, K. (2005). Students' attitudes to mathematics and performance in limits of function. *Mathematics Education Research Journal*, 17 (2), 91–110.

Kadriye, E. (2005). Factors associated with mathematics achievement and participation in advanced mathematics courses: An examination of gender differences from an international perspective. *School Science and Mathematics*, 105 (1), 5–14.

Kaldo, I. (2011). Structure of students' view of mathematics in an Estonian business school. *Nordic Studies in Mathematics Education*, 16 (1–2), 77–94.

Kislenko, K. (2009). Mathematics is a bit difficult but you need it a lot: Estonian pupils' beliefs about mathematics. In J. Maaß & W. Schlöglmann (Eds.), *Beliefs and attitudes in mathematics education: new research results* (pp. 143-164). Rotterdam: Sense Publishers.

Kumar, R., Gheen, M. H. & Kaplan, A. (2002). Goal structures in the learning environment and students' disaffection from learning and schooling. In C. Midgley (Ed.), *Goals, goal structures, and patterns of adaptive learning* (pp. 143–173). Mahwah: Lawrence Erlbaum.

Lazim, M. A., Abu Osman, M. T. & Wan Salihin, W. A. (2004). The statistical evidence in describing the students' beliefs about mathematics. *International Journal for Mathematics Teaching and Learning*, 6 (1), 1–12.

Lepik, M. (2009). Mathematics education research in Estonia. In M. Lepik (Ed.), *Teaching mathematics: Retrospective and perspectives* (pp. 57–70). Tallinn University.

Lepmann L. (2000). Eesti ja vene õpilaste arusaamad matemaatikaõpetusest [Estonian and Russian students' perceptions of mathematics teaching]. *Koolimatemaatika*, XXVI, 33–37.

Lepmann, L. & Afanasjev, J. (2005). Conceptions of mathematics in different ability and achievement groups among 7th grade students. In C. Bergsten & B. Grevholm (Eds.), *Proceedings of NORMA 01, the 3rd Nordic Conference on Mathematics Education* (pp. 185–194). Linköping. SMDF.

Lester, F. K., Garofalo, J. & Kroll, D. L. (1989). Self-confidence, interest, beliefs, and metacognition: key influences on problem-solving behavior. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 75–88). New York: Springer-Verlag.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. G. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575–596). New York: McMillan.

McLeod, D. B. & McLeod S. H. (2002). Synthesis – beliefs and mathematics education: implications for learning, teaching, and research. In G. C. Leder, E. Pehkonen & G. Törner (Eds.), *Beliefs: a hidden variable in mathematics education?* (pp. 115–127). Dordrecht: Kluwer Academic Publishers.

Midgley, C., Feldlaufer, H. & Eccles, J. S. (1989). Student/teacher relations and attitudes toward mathematics before and after the transition to junior high school. *Child Development*, 60, 981–992.

Midgley, C., Kaplan, A., Middleton, M., Maehr, M. L., Urdan, T. et al. (1998). The development and validation of scales assessing students' achievement goal orientations. *Contemporary Educational Psychology*, 23, 113–131.

Midgley, C., Maehr, M. L., Hruda, L., Anderman, E. M., Anderman, L. et al (2000). *Manual for the patterns of adaptive learning scales (PALS)*. Ann Arbor, MI: University of Michigan.

Op 't Eynde, P. & De Corte, E. (2003, April). *Students' mathematics-related belief systems: design and analysis of a questionnaire.* Paper presented at the annual meeting of the American Educational Research Association, Chicago.

Op 't Eynde P., De Corte, E. & Verschaffel, L. (2002). Framing students' mathematics-related beliefs: A quest for conceptual clarity and a comprehensive categorization. In. G. C. Leder, E. Pehkonen & G. Törner (Eds.), *Beliefs: a hidden hariable in mathematics Education?* (pp. 13–37). Dordrecht: Kluwer Academic Publishers.

Op 't Eynde, P., De Corte, E. & Verschaffel, L. (2006). Epistemic dimensions of students' mathematics–related belief systems. *International Journal of Educational Research*, 45, 57–70.

Pehkonen, E. (1995). *Pupils' view of mathematics. Initial report for an international comparison project.* Research Report 152. Department of Teacher Education, University of Helsinki.

Pehkonen, E. & Lepmann, L. (1994). Teachers' conceptions about mathematics teaching in comparison (Estonia – Finland). In M. Ahtee & E. Pehkonen (Eds.), *Constructivist viewpoints for school teaching and learning in mathematics and science* (pp. 105-110). Department of Teacher Education, University of Helsinki.

Pehkonen, E. & Törner, G. (1996). Mathematical beliefs and different aspects of their meaning. *ZDM*, 28, 101–108.

Pekrun, R., Elliot, A. J. & Maier, M. A. (2006). Achievement goals and discrete achievement emotions: a theoretical model and prospective test. *Journal of Educational Psychology*, 98 (3), 583–597.

OECD. (2009). *PISA 2006 Technical Report.* Paris: OECD.

Rösken, B., Hannula, M. S., Pehkonen, E., Kaasila, R. & Laine, A. (2007). Identifying dimensions of students' view of mathematics. In D. Pitta-Pantazi & G. Philippou (Eds.), *European Research in Mathematics Education V. Proceedings of the fifth congress of the European Society for Research in Mathematics Education* (pp. 349–358). Nicosia: University of Cyprus.

Roesken, B., Hannula, M. S. & Pehkonen, E. (2011). Dimensions of students' views of themselves as learners of mathematics. *ZDM*, 43 (4), 497–506.

Saari, H. (1983). *Koulusaavutusten affektiiviset oheissaavutukset* [Affective consequences of school achievement] (Publication No. 38). Institute for Educational Research, University of Jyväskyla.

Schab, F. (1991). Schooling without learning: thirty years of cheating in high school. *Adolescence*, 26, 839–847.

Schoenfeld, A. (1985). *Mathematical problem solving.* Orlando: Academic Press.

Selden, A. & Selden, J. (2001). Tertiary mathematics education research and its future. In D. Holton (Ed.), *The teaching and learning of mathematics at the university level: An ICMI Study* (pp. 237–254). Dordrecht: Kluwer Academic Publishers.

Seo, D. (2000, June/July). *The motivational construct in mathematics learning using structural equation modeling: the Korean elementary school math class.* Paper presented at the Head Start National Research Conference, Washington.

Whitley, B. E. (1998). Factors associated with cheating among college students: a review. *Research in Higher Education*, 39, 235–274.

Yusof, Y. B. M. & Tall, D. (1994). Changing attitudes to mathematics through problem-solving. In J. P. da Ponte & J. F. Matos (Eds.), *Proceeding of the eighteenth international conference for the Psychology of Mathematics Education* (Vol. 4, pp. 401–408). Lisbon: PME.

Zan, R., Brown, L., Evans, J. & Hannula, M. S. (2006). Affect in mathematics education: an introduction. *Educational Studies in Mathematics*, 63, 113–121.

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