Comparing perceptions of mathematics: Norwegian and Finnish university students' definitions of mathematics

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The article presents a comparison between Norwegian and Finnish university students' perceptions of what mathematics is. To carry out the comparison, a mix of qualitative - the creation of abstract and concrete categories for mathematics representations - and quantitative (regression modeling) methods was used in the study. The main result of the study is that Norwegian students were more homogenous in their responses and the vast majority perceived mathematics in concrete terms. The Finnish students, on the contrary, showed greater variety in their responses. There are not many comparative studies among Nordic countries regarding students' perceptions of mathematics. Therefore this study contributes to improving our knowledge about the possible differences and similarities on students' perceptions of mathematics among Nordic students. A total of 239 students were asked how they perceive mathematics, numbers and personal applicability of mathematics via an open questionnaire. We propose that the divergent perceptions of mathematics stem from different types of communication cultures that surround mathematics. The argument is made that perceptions of mathematics should be treated as a type of mathematical knowledge that is valuable whenever mathematics is communicated.

Recently The Wall Street Journal published an article that rated the worst and the best jobs in the United States (Needleman, 2009). The job of a mathematician was rated to be the number one job that one could wish for. One mathematician, Jennifer Courter, was asked to define mathematics in the interview. She gave the following description:

> "It's a lot more than just some boring subject that everybody has to take in school," says Ms. Courter, a research mathematician at mental images Inc., a maker of 3D-visualization software in San Francisco. "It's the science of problem-solving."

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For Ms. Courter, mathematics, in essence, is the science of problem-solving. But does Ms. Courter's definition really exhaust the essence of mathematics, if there even is an ultimate essence to it in a philosophical sense? Consulting the literature in the history of mathematics proves to be only of little help in order to answer this question. The mathematics historian Carl. B. Boyer starts his history of calculus concluding the following:

> Mathematics has been an integral part of man's intellectual training and heritage for at least twenty-five hundred years. During this long period of time, however, no general agreement has been reached as to the nature of the subject, nor has any universally acceptable definition been given for it. (Boyer, 1949, p. 1)

Public perceptions of mathematics, then, perhaps reflecting the lack of scholarly unison, vary greatly in society. Mathematics educators, on their part, widely accept the notion that in the classrooms one encounters various attitudes and perceptions regarding the nature of this much loved and hated subject (Devlin, 2000; Leder, Pehkonen & Törner, 2002; Lim & Ernest, 1999). Perception itself is a concept widely used in science. We situate our work as a study of social cognitions and thus define *perceptions as linguistic constructs, communicable to others, that individuals hold about objects both of the social and inanimate world* (Smith & Mackie, 2007). Based on this definition we are not focusing on sensory perceptions, but rather on communicable ideas that people have about mathematics. However, it is noteworthy that sensory perceptions are always required in the process of forming social perceptions.

In order to add a Nordic perspective to the discourse, we collected qualitative and quantitative data from Finland and Norway. We wanted to know if our respondents show differences in the ways they describe their mathematics and number perceptions. We asked a sample of university students – a total of 239 students – in Norway and Finland to define mathematics in their own words, to rate their attitude toward the subject, and to describe how they thought mathematics was useful in their everyday lives.

The results of this survey were coded into categories and further analyzed via regression modeling. In a recent study Urhahne, Kremer, and Mayer (2011) treat certain perceptions of science to be more or less correct. They treat some perceptions as false and other perceptions as correct based on a given scientific criteria. Contrary to this approach we do not treat any perception of mathematics as "right" or "wrong", but rather as a variety of ways of communicating mathematical objects. Picker and Berry (2000) proceed in a similar vein in their study of pupils' perceptions of mathematicians. ”Mathematician” was found to be a perception that was somewhat similar in five different countries. However, pupils did not seem to have very developed ideas of what mathematicians really do. Researchers were able to report on some intercultural differences in perceptions about mathematicians, but they relied on comparisons of a predestined image of a mathematician.

In our view, information regarding the mental mathematics representations contributes both to the enrichment and development of the public perceptions of mathematics per se. This is important, because it makes mathematics more accessible and communicable to people. Our analysis adds to the body of belief research in mathematics with a perspective of international comparisons. At the end of the paper we discuss the social ontological function of perceptions of mathematics. By social ontological we mean the everyday understandings and socially shared definitions of what mathematics is. In this way our contribution is both philosophical and empirical.

Some previous studies in perceptions of mathematics

When perceptions of mathematics are studied and discussed, they become shared information. There is no direct access to the perceptions as such. One needs to ask questions to attain knowledge of those perceptions. Thus, there is an interesting social epistemological aspect to our inquiry. The study of mathematics perceptions helps to see what we regard as ”mathematics” – in other words – what kind of things we know as mathematics. Social perceptions are shared knowledge and thus a crucial link between thought and communication (Gelman, 2003).

Thus, in order to interpret the results from our open questionnaire we reflect on the ways in which a given mathematics perception may function as a type of heuristic method of thought. Recent empirical studies show that heuristic methods of thought play an important role in the formation of social representations on matters that are abstract, as mathematics is, and which the individual learns or adapts from other people, not from direct sensory observation (Carpini, 2004; Dalton, 2006;). In the discussion we also propose, following Gelman (2003), that perceptions of mathematics may have psychologically essentializing properties, being types of homemade ontological theories of mathematics. Such ontologies then inform the divergent activities that a person views as mathematics or mathematical.

In the majority of previous studies the students’ and pupils’ lay theories of mathematics have been studied as ”beliefs” (Leder et al., 2002). Since previous research has operationalized lay theories of mathematics merely as ”beliefs” (Aunola & Nurmi, 2004; Aunola, Nurmi, Lerkkanen & Rasku-Puttonen, 2003; Chen & Pajares, 2010), it is reasonable to use the terms ”belief” and ”perception” synonymously. In both cases the activity of perceiving is social, something that can be shared with others.

Since perceptions of mathematics are social, a survey approach makes it possible to sample students´ descriptions of mathematics on a given moment from the top of their heads. Thus a survey sample functions at least as a raw road map of perceptions that the students presumably apply in their thinking in real life and real social situations.

To what extent is mathematics a social construct?

Mathematics, as argued, is a hard subject to catch in one phrase and it seems an unreasonable task to pursue an exhaustive philosophical depiction of mathematics. Although it seems to be a fact that there is no academically universal definition for mathematics – as is probably the case for many academic disciplines as well – people apparently, as in the case of Ms. Courter (Needleman, 2009), do hold personal perceptions about the nature of mathematics. This observable variety of mathematics perceptions does not mean directly that we are promoting purely a constructionist view on mathematics. Instead, we believe that developments that stem e.g. from the propositional calculus give a crucial potential to make mathematics better and more applicable.

On the contrary, we do not argue that the epistemic value of mathematics could be summed up as the various linguistic constructs available. This means that the differing perceptions of mathematics are ”true” knowledge of mathematics to the individual, although the subject matter of ”mathematics” evidently has characteristics that rise above any given personal perception. We adhere to the viewpoint provided by Keith Devlin (2000) that there are unifying, even if socially constructed, aspects to mathematics. Thus, there is substantial agreement within the scientific community about the principal premises and objects of mathematics. As Devlin concludes:

> All of mathematics, however, consists of variations on the same theme: the identification, abstraction, study, and application of patterns, using the mental tools of logical reasoning.
>
> (Devlin, 2000, p. 26)

To some extent, then, we would expect these general characteristics – the identification, abstraction, study, and application of patterns, using the mental tools of logical reasoning – to be present at least in some form in the students’ descriptions. In any case, it is fruitful to mirror the lay representations with Devlin’s depiction, as elaborated in the further section. We assume, thus, that mathematics or mathematical objects may have characteristics beyond the social world, but that the perceptions of mathematics are products of social construction. Mathematics, itself, is an abstract and logical discipline which has no immanent physical existence beyond the human mind, although some researchers suggest the contrary. For a detailed discussion, see (Bigalow, 1988).

Beliefs and perceptions in previous studies

Before turning to report on the characteristics of Norwegian and Finnish university students' perceptions of mathematics, we give a brief overview on some recent studies that have had similar aims. The question of both students' and teachers' ideas regarding mathematics has caught attention in both education and psychology, although the focus has usually been on the beliefs regarding activities related to mathematics and behaviors per se, and less on mathematics perceptions themselves. For examples see (Picker & Berry, 2000; Urhahne et al., 2011).

Both Leder and her colleagues, as well as Keith Devlin argue that beliefs about the nature of mathematics have often been forgotten variables in mathematics education (Devlin, 2000; Leder et al., 2002). In many cases, mathematics education researchers take the understanding of what mathematics is as given or merely operationalize research treating mathematics as a school subject that goes without any substantial ontological reflection (di Martino, 2002; Kaasila, 2000).

The line of studies of perceptions of mathematics is typically done in the context of education. For instance, in one study it was found that mathematics teachers' teaching practice was more closely related to the teacher's beliefs about mathematics content than to his or her beliefs about mathematics pedagogy. Teachers' beliefs, in turn, about mathematics content were highly influenced by their own experiences as a student. On the contrary, teacher's beliefs about mathematics pedagogy were primarily influenced by his or her own teaching practice (Raymond, 1997).

In a similar vein, also pupils' beliefs regarding both the nature of a given school subject and the nature of the learning process have been shown to play an important part in the creation of motivation and as facilitators of learning. Chen and Pajares (2010) proved that an incremental view of ability had direct and indirect effects on adaptive motivational factors. On the contrary, the fixed entity views had direct and indirect effects on maladaptive factors of learning. Interestingly, students' epistemological beliefs – beliefs about the nature of knowledge about school subjects – mediated the influence of implicit theories of ability on achievement goal orientations, self-efficacy, and science achievement.

Previously, Pajares and Miller (1994) have shown that the perceived self-efficacy in mathematics was more predictive of problem solving in mathematics than many other factors, such as self-concept in mathematics, perceived usefulness of mathematics or the individuals' belief about her own capabilities to solve specific problems in mathematics.[1]

Based on these previous findings we are prone to believe the perceptions of mathematics per se are related to academic achievement. Hemmings, Grootenboer and Kay (2011) show that previous academic success in mathematics predicts future success. But does academic achievement depend upon mathematics perception? Within the boundaries of our sample we can address this question.

Researchers have found that beliefs relating to the learning of mathematics influence the learning itself. An academic "self-concept", an understanding of self as a learner of a given subject, has proven to be subject-specific. This means that students tend to have specific self-concepts for reading, science, and mathematics (Marsh & Yeung, 1996; Mui, Yeung, Low & Jin, 2000). Interestingly, House (2001, 2004) found that adolescent students in Japan who tended to show high mathematics test scores reported that their teachers more frequently incorporated the use of active learning strategies – such as using things from everyday life when solving problems – during mathematics lessons. Perceptions of mathematics´ applicability, thus, seem to be important facilitators in learning mathematics.

Thus, beliefs regarding own ability and beliefs about the nature of school subjects both play an important part in the learning process. There is even evidence that also parents' beliefs about their children's mathematical abilities play an important role in the child's mathematics performance and development of mathematical skills (Aunola & Nurmi, 2004; Aunola et al., 2003).

Four faces of mathematics

Keith Devlin (2000) argues that mathematics – both in the context of mathematics education and in general – should be viewed to have *four different faces*. For Devlin, these faces are:

- Mathematics as computation, formal reasoning, and problem solving (concrete perception)
- Mathematics as a way of knowing (abstract perception)
- Mathematics as a creative medium (abstract perception)

– Applications of mathematics (applicability perceptions)

Based on Devlin's categorical "faces" of mathematics, we decided to create two categories, abstract and concrete, for perceptions of mathematics, numbers and imaginable applications of mathematics. This decision reflects Devlin's dividing line between "mathematics as a way of knowing" and "mathematics as a creative medium" and "mathematics as computation, formal reasoning, and problem solving". The latter is tied to concrete perceptions whereas Devlin's first two key features of mathematics are decidedly abstract by nature. Since abstract and concrete features were the most striking dividing elements in students' descriptions we decided to use abstract and concrete perceptions as categories for further analysis.

Additional support to this dualistic division comes from Al-Balushi's (2011) study that showed how abstractness and concreteness of perceptions of scientific objects both potentially increases pupils' doubts about the correctness of information and increases its credibility. Al-Balushi encourages further studies in this dualism.

All the responses were coded into "abstract" and "concrete" categories by two coders. Inter-coder reliability was measured by percentage of agreement. Less than five percent of the responses required discussion to gain resolution. Disagreements between coders were resolved through discussion.

It is important to note that we do not see "abstract" and "concrete" as value labels of perceptions. High achieving students may have "concrete" perceptions and many students who report low success in mathematics may have "abstract" perceptions. The relationship between the type of perception and academic achievement is later explored in the regression model (table 5) that uses academic achievement – school grades – as an independent variable.

Our study adds to the body of perception research pursuing the following research questions:

RQ1: What are, in our sample, the typical perceptions of mathematics like?

RQ2: To what extent do these perceptions differ in Norway and Finland?

RQ3: To what extent do perceptions of mathematics and its applicability in life predict attitude towards mathematics when controlled for other influential independent variables?

Methods

Data collection and participants

The Finnish sample (n = 158) of perceptions of mathematics was collected in April of 2006 at a statistics course targeted at the first year social science students at the University of Helsinki. All of the Norwegian students majored in social sciences. In both countries the participants were relatively similar regarding age and gender distribution as seen in tables 1 and 2. In addition to social science majors there was a subgroup of students with majors in natural or engineering sciences.

Table 1. *Age*

Sample	Mean	n	Std. deviation
Finland	22.65	133	3.499
Norway	20.56	66	2.494
Total	21.96	199	3.344

Table 2. *Gender*

Sample		Male	Female	Total
Finland	Count	44	114	158
	% within country	27.8	72.2	100.0
Norway	Count	28	46	74
	% within country	37.8	62.2	100.0
Total	Count	72	160	232

The Norwegian sample (n = 81) was gathered more than one year later, in October 2007 from a philosophy of science course, likewise targeted towards first year social science students at the University of Bergen. It is important to note that in tables 1 and 2 the analyzed sample sizes differ from the total sample sizes. This is due to the fact that not every student responded to every question.

The reason we focused on first year social science students was the availability. Both samples were from statistics classes that gather a critical mass of students with different majors in social sciences. In addition, social scientists do not generally study a lot of mathematics as part of their training. This brings them closer to the general public than, say, a sample of mathematics majors would. Naturally our sample has its limits and larger and purely random samples would bring the research closer to the general public. However, the similarities of the national samples allow for meaningful comparisons.

The Norwegian sample (n = 81) is about half the size of the Finnish sample (n = 158). It is important to note that not every student responded to every question and therefore the amount of total respondents in the following tables may slightly deviate from these figures.

Analysis

The survey results were categorized by the authors to allow for quantitative comparisons (Peräkylä, 2000). Since perceptions of mathematics are based in language, the perceptions are limited in their variety. This became clear when the students' descriptions were read carefully through. As Peräkylä (2000, p. 870) argues, the reason for this is that what we think and say ultimately stems from our surrounding culture. As researchers of public images of mathematics Lim and Ernest (1999) argue: "It is interesting to find the variety and diversity of these metaphors, besides the commonalities that they shared."

The possible bias that could arise with diverging majors within the Finnish sample was controlled by cross-tabulations by major and the responses; there was no statistically significant pattern that any responses would co-vary with major when tested with the Chi-square test. These results are interesting, because traditionally it has been assumed that individuals with an interest in social sciences show poor performance and dislike toward mathematics (Bush, Madow, Raiffa & Thrall, 1954, p. 553).

Importantly, the past secondary school performance did not diverge substantially by country. The Finnish grading system runs from 4 to 10 and the Norwegian grading system from 1 to 6. Students in both countries had relatively good school grades in secondary school mathematics; the averages in an equivalent 6-point grade scale were identical.

In addition, the attitudes as measured on a 5-point Likert scale towards mathematics when asked "How much do you like or dislike mathematics?" were rather identical in both countries; Norwegian and Finnish students reported quite similar and fairly neutral attitudes (neither special liking nor disliking) towards mathematics. Interestingly, in neither country there were no statistically significant gender based differences in the attitudes toward mathematics.

Open questions and examples of abstract and concrete perceptions

In addition to demographic and other background variables reported above, the respondents were asked in the open questionnaire to describe either in Finnish or Norwegian their own view in their own words to the following questions:

- What is mathematics?
- What are numbers?
- What use you think you have for mathematics skills?

The content analysis began by identifying the key elements in the written samples that kept repeating themselves and appeared as the most salient. The next step of the analysis was to group the responses according to their similarity and to form and name categories of similarly formulated responses. We then created groups of answers that had striking similarity to them and which had similar "core" in capturing the perception of mathematics, numbers or the perceived personal applicability of mathematics.

We asked about the students' perception of numbers just for the sake of eliciting perceptions of some well-known parts of mathematics. Needless to say, numbers are only a small part of mathematics and mathematics education. However, at the elementary school level numbers are a vivid part of mathematics training. Although there are a multitude of important concepts in mathematics beyond numbers, we assumed that as a well-known part of mathematics the students' perceptions of numbers matter. In short, perceptions of numbers tell something about the ways in which students see mathematical concepts. In future studies the focus could be on some other widely spread mathematical concepts such as measurement, statistical reasoning and categorization.

All responses were coded into categories by the two authors. After several readings of the responses the coding categories were created to reflect what was salient in the subject's description. We constantly compared the students' descriptions of what mathematics is to a set of categories, the four faces of mathematics, given by Devlin (2000). Such a comparison to a pre-existing categorical structure was made in order to increase the validity of the coding categories from a mere level of face validity (van de Vijver & Leung, 1997; Weber, 1990).

In a majority of the cases the responses were short and typically written in order to capture the salient feature of mathematics. Indeed, a single salient feature was identifiable per response. In the unclear cases the two coders decided, after a discussion, on the placement of the response into a category.

Results

Abstract and concrete perceptions of mathematics

To answer our first research question (RQ1), the following paragraphs describe the typical features of mathematics perceptions, number perceptions, and the perceived applicability of mathematics found in the data.

A close inductive reading of the students' descriptions showed that responses had striking similarities with all of the four faces presented by Devlin (2000). As explained, in every response one of these faces was the most salient in a single response. Thus, every response was coded either as abstract or concrete reflecting the most salient features of the student's description. The descriptions of applicability of mathematics, however, did not fit neatly into a single category. A single answer regarding the applicability of mathematics was therefore coded into two categories – abstract and concrete.

The concrete category reflects the students' answers that emphasized and depicted mathematics as an exact, typically numeric system, entirely aimed at generating calculations and formal operations. These descriptions are quite similar to Devlin's first face of mathematics: Mathematics as computation, formal reasoning, and problem solving. Examples of the concrete category are in table 3.

The abstract category contains the responses in which mathematics was seen as a means to achieve knowledge. Mathematics was not seen as an end in itself. This category reflects Devlin's second face of mathematics: Mathematics as a way of knowing. Examples of the abstract category are in table 3.

The abstract category also encapsulates perceptions in which the essential feature was the mathematics as an end in itself. Responses in this category described mathematics as "a way of thinking" or as "a way of conceptualizing the world". Thus, these descriptions come very close to Devlin's third face of mathematics: "Mathematics as a creative medium". Translated examples for abstract perceptions are presented in table 3.

Similarly with the perceptions of mathematics, in the number descriptions students tended to emphasize one core aspect over others of what numbers are. Perceptions were either abstract or concrete. Responses in the concrete category saw numbers as special types of labels or names. The concrete category also contains perceptions that understood numbers through their instrumental value; numbers exist for the sole purpose that they are useful in a myriad of calculations. This category contains also tautological descriptions. Surprisingly, many students were truly brief in their description of numbers and described them merely as "numbers" or wrote on the answer sheet the numbers from 0–9.

Table 3. *Example responses of abstract and concrete perceptions*

Concrete perceptions of mathematics	Counting with numbers or letters.
	A practical way of solving problems that have to do with numbers.
	Numbers and formulas to find answers.
Abstract perceptions of mathematics	The examination of different quantities, continuity and logic of phenomena, a way of structuring and predicting the world.
	A tool with which one understands the world through patterns.
	Calculations. Describing phenomena logically, using number systems.
Concrete perceptions of numbers	They stand for quantity.
	Something that has something to do with money.
	Umm...numbers.
Abstract perception of numbers	Instruments, units, abstract proportioning.
	Symbols, the meaning of which is dependent on the context and order.
	Systemizing of understood qualitative differences.
Concrete perceptions of the applicability of mathematics	Useful primarily through basic math skills. In other mathematics, such forms of deduction and philosophies are used, that I find hard to understand (so why is this done?).
	One needs some simple calculating in everyday life. Mathematics skills come in handy, for example, when interpreting data tables in newspapers.
	Getting by in everyday life and work.
Abstract perceptions of the applicability of mathematics	I don't feel like mathematics is of any great use to me, but that's probably just an attitude problem and a lack of interest in mathematics. In reality mathematics probably helps logical thinking.
	Basic skills are useful in everyday life. I don't see taking advanced mathematics in secondary school as very useful or meaningful for my part. Maybe it helps develop my perception in different situations?
	Better sense of logic and some everyday knowledge.

The abstract category, on the other hand, contains perceptions that see numbers as a way of conceptual thinking. These descriptions did not emphasize the instrumental value of numbers in the process of gaining numeric or other information; rather, numbers were seen as a type of creative medium for conceptualization. Thus, the characteristics of abstract descriptions for number perceptions are quite similar to key characteristics of abstract mathematics perceptions. Translated examples for concrete and abstract number perceptions are presented in table 3.

Applicability perceptions

As for mathematics and numbers, the coding categories of perceptions of personal applicability of mathematics were abstract and concrete. Although the applicability perceptions are more practical by definition compared to more philosophical mathematics and number perceptions, we wanted to include this dimension in order to bridge this study with the existing belief literature. The majority of the existing research on beliefs in mathematics is coupled with the concern about mathematics performance and mathematical activities. Thus, it is important to study the perceptions of applicability together with the perceptions of mathematics and numbers. Also, the applications of mathematics form Devlin's (2000) fourth face of mathematics.

We found three dominant traits in the perceived usefulness of mathematics in the students´ descriptions: emphasis on everyday practical skills, emphasis on conceptual skills, and emphasis on thinking and creating meaning.

Again, the first key feature was coded as a concrete perception and the latter two as abstract perceptions. Those who emphasized everyday practical skills saw mathematics useful merely in everyday context and in routine tasks such as counting money in the supermarket or inspecting yearly personal income taxes. Those who emphasized conceptual skills thought that mathematics can be applied in a variety of conceptual tasks that transcend the everyday routines. Here, for instance, students saw that mathematics helps to analyze quantitative information and proves to be useful in research projects.

Those who put the emphasis on thinking and creating meaning saw that the applicability of mathematics lies in mathematical thinking and meaning itself. In these descriptions the key feature of mathematics is in its capacity to enrich and enable thought. Examples of both concrete and abstract applicability perceptions are presented in table 3.

Interestingly, our data show that Norwegian students were more homogenous in their responses throughout the questionnaire items. The vast majority of Norwegian students perceived mathematics in concrete terms. The Finnish students, on the contrary, showed greater variety in their responses. To respond to our second research question (RQ2) we examined through cross-tabulations Norwegian and Finnish students regarding the abstractness and concreteness of their given perceptions. Table 4 shows the sample distributions. The following sections further examine the differences between Norwegian and Finnish responses.

Mathematics, number, and applicability perceptions

To clarify the answer to our research question (RQ2) and tackle the third research question (RQ3), this section discusses the differences in the Norwegian and Finnish mathematics and number perceptions, as well as differences in the perceived personal applicability of mathematics. First,

Table 4. *Distribution of mathematics related perceptions*

Sample		Perception of mathematics		Perception of numbers		Perception of mathematics applicability	
		Concrete	Abstract	Concrete	Abstract	Concrete	Abstract
Finland	Count	101	50	97	57	61	95
	% within country	66.9	33.1	63.0	37.0	39.1	60.9
Norway	Count	67	5	61	11	48	15
	% within country	93.1	6.9	84.7	15.3	76.2	23.8
Total	Count	168	55	158	68	109	110

we modeled the predictors of attitude (like or dislike) towards mathematics via multiple regression where age, gender, nationality, primary school high achievers in mathematics, secondary school high achievers in mathematics, mathematics perceptions and applicability perceptions were controlled.

Attitude towards mathematics was measured in the questionnaire on a 5-point Likert scale. High achievers were categorized as those who typically got the two highest grades on a 6-point grading scale in primary or secondary school.

In Norway these grades were 5 or 6 (scale from 1–6) and in Finland 9 and 10 (scale 4–10). Table 4 shows that statistically significant predictors of liking of mathematics (those who scored 4 or 5 on the Likert scale) were nationality ($p<.05$), primary school high achievement ($p<.001$), secondary school high achievement ($p<.05$) and abstract perception

Table 5. *Regression analysis of factors predicting respondents' attitude towards mathematics*

Factors	B	SE B	β
Nationality: *R* is Norwegian	.398	.159	.183*
Gender: *R* is female	-101	.138	-.049
Primary school high achievers	.507	.139	.265***
Secondary school high achievers	.373	.148	.182*
Perception of mathematics: *R* has abstract perception	.013	.152	.006
Perception of numbers: *R* has abstract perception	.115	.138	.057
Perception of mathematics applicability: *R* has abstract perception	.419	.133	.219**

Note. $n = 200$, $F(7,192) = 6.536$, $p = .000$, $R^2 = .192$ (.163), $^{*}p < .05$, $^{**}p < .005$, $^{***}p < .001$

of applicability of mathematics ($p < .001$). Interestingly, perceptions of mathematics or numbers did not predict liking or disliking mathematics. Beta values for these perceptions were decidedly close to zero.

As seen in table 5, the model turned out to be reasonably explanatory – some 19.2 percent of the variance in attitudes is explained by the modeled variables. Considering the abstract nature of the dependent variable its explanatory power is really good. Quite interesting was the fact that the primary school performance together with the perception of the applicability of mathematics were the most significant predictors of liking mathematics in the adult age.

It seems that early positive experiences may build a lasting positive ground for the positive attitude. This finding on its part corroborates the findings of Skaalvik and Valås (1999) who found that students´ elementary and middle-school achievement in mathematics was significantly related to subsequent mathematics self-concept.

As defined, those who held abstract applicability perceptions were the ones who saw the usefulness of mathematics in conceptual skills in thinking and creating meaning. Such abstract view of applicability of mathematics seems to feed strongly to the liking of mathematics. Norwegian students liked mathematics more than Finnish students when controlled for other variables.

The characteristics of the model that are predictors of dislike of mathematics in the Finnish sample do not predict dislike in Norway. In Norway students like mathematics more evenly across the modeled characteristics. On the average, however, that liking of mathematics is similar. The results of one-way ANOVA results $F(0{,}051, p < 0{,}82)$ show the means of attitudes towards mathematics between Norwegians and Finnish students to be almost exactly same. Interestingly, gender did not predict liking of mathematics in a statistically significant way.

In order to find out which factors help to explain students' different views on the applicability, we set up a logistic regression model. Via a standard regression modeling we created dichotomies (e.g. to be Norwegian or not) of respondents and searched for factors that rise above other dichotomies.

Table 6 shows factors that predict students´ abstract or concrete perception of the applicability of mathematics. Out of the controlled variables only nationality of the student has statistically significant ($p < .001$) predictive power over the nature of applicability perceptions.

This finding suggest that when other influential factors are acknowledged in the measure model there may be something in the teaching or communication culture around mathematics that contributes to the unexpectedly high ratio of Norwegians having a concrete perception

Table 6. *Logistics regression analysis of factors predicting respondents' abstract or concrete perception of the applicability of mathematics*

Factors	B	SE B	β
Nationality: *R* is Norwegian	-1.474***	.389	.229
Gender: *R* is female	-.394	.338	.674
Primary school high achievers	-.070	.336	.933
Secondary school high achievers	.405	.354	1.500
Perception of mathematics: *R* has abstract perception	.357	.358	1.428
Perception of numbers: *R* has abstract perception	.086	.331	1.090
Constant	.471		

Note. $n = 201$, $X^2 = 24.999$, $p = .000$, $^{***}p < .001$

of mathematics´ applicability. Finnish students, it appears, have much greater likelihood to perceive applications of mathematics in abstract terms.

Table 7 shows that Finnish students tend to have generally and statistically significantly (p < .05) more abstract perceptions of mathematics than Norwegian students. In addition to students´ nationality, holding an abstract perception of mathematics is also predicted by students´ abstract perception of numbers.

Prior research shows that the features of a given perception of mathematics could be expected to reflect features of a communication culture surrounding mathematics (Sfard, 2001). In the Norwegian case such a communication culture, within our sample, would reflect rather narrow views on mathematics, since a majority of the students' thought of mathematics as an exact system, definable as a concrete discipline.

Table 7. Logistic regression analysis of factors predicting respondents' abstract or concrete perception of mathematics

Factors	B	SE B	e^B
Nationality: *R* is Norwegian	-1.519*	.585	.219
Gender: *R* is female	-.249	.379	.779
Primary school high achievers	.349	.382	1.418
Secondary school high achievers	-.181	.395	.834
Perception of numbers: *R* has abstract perception	.996**	.347	2.706
Perception of mathematics applicability: *R* has abstract perception	.341	.361	1.406
Constant	-1.292		

Note. $n = 201$, $X^2 = 26.923$, $p = .000$, $^{*}p < .05$, $^{**}p < .005$

A given perception of an abstract matter such as mathematics would, then, be used to explain the observable properties of mathematics, in real life and in classroom situations. For the individual, a given perception of mathematics is functional as a heuristic tool of thought when performing mathematical tasks, communicating about mathematics, and observing what is mathematical in the world.

Discussion

In line with the above results and reflections, Keith Devlin (2000, p. 16) notes that mathematics education should show more sensitivity to teaching styles and communications of mathematics. Our results support Devlin's view. More variety of mathematics perceptions should be displayed when mathematics is being communicated.

> Grades K-12 mathematics should therefore be taught much more like history or geography or English literature – not as a utilitarian toolbox but as a part of human culture. Remember that the goal I am advocating is to produce an educated citizen, not a poor imitation of a twenty-dollar calculator. An educated citizen should be able to answer the following two questions about mathematics: What is mathematics? Where and how is mathematics used?
>
> (Devlin, 2000, p. 16)

Such an educated citizen, as Devlin envisions, would be able to confront the various features of the mathematical universe. Such a viewpoint is also a call for checking the epistemic standards that are applied in teaching and evaluating mathematical performance. This would mean, that perceptions related to mathematics could be treated as a type of mathematical knowledge.

Our data show that differing perceptions relating to mathematics do exist within and across cultures. These perceptions have significance in the mathematical thinking process. Perceptions, then, deserve to be discussed at all levels of mathematics education. As Sverre Wide has recently proposed, in any education the mere classical and logical falsifiability does not capture all the meaningful ”errors” that students may make in their thinking (Wide, 2009, p. 574). In fact, many times the error itself should be viewed as ”right” or ”correct”. Some perceptions may be too rigid, or too muddled, although by some standards ”correct”. Yet, more abstract and reflective perceptions may not easily meet textbook definitions, but may be powerful tools for thinking about and learning mathematics.

In any case, students' perceptions have an important meaning for themselves. Within the thought processing of the student the idea of mathematics serves as a type of heuristic method. A heuristic method is simple an easily accessible idea of what something is and how relate that object. With such mental tools individuals can find the "essence" of mathematics and use this essence when using mathematics (Gelman, 2003, p. 9). A given perception would categorize and frame activities that a person would think as mathematical by nature.

Conclusion

The beginning of this article showed that public representations of mathematics are divergent and that there exists no scholarly unison in defining the essence of mathematics. Informed by our data, we proceeded to explore the lay perceptions of mathematics, numbers, and mathematics´ applicability – the salient features of Norwegian and Finnish university students' perceptions – of mathematics, numbers and perceived personal applicability of mathematics.

Our data showed interesting cross-cultural differences in mathematics perceptions. This finding was interpreted to indicate that there may be important differences in the communication cultures surrounding mathematics. Our findings suggest that future research should further examine the link between lay perceptions of mathematics and culture. Another important direction would be to explore the interdependence of math self-efficacy and the function of mathematics perceptions.

After all, students of mathematics are able to generate beliefs of mathematics with their peers and other socially significant others. Students, naturally, influence their instructors and teachers as well. Feelings, attitudes and knowledge of mathematics is an important and fascinating topic. It is our social groups and classrooms that we generate and communicate our beliefs. (Vähämaa, in press.) Ideas from a social group to another may be surprisingly different. Occasionally, these different beliefs of what mathematics may lead to conflicts between a pupil and an instructor. It is only through an open debate that we come to accept errors as social mirrors and hopefully we will gain a new learning experience.

In the context of mathematics education, it is worthwhile to talk about differing perceptions and the possible causes of such differences in the classroom. Ideally such communication would be commonplace in different stages of education. Perceptions of mathematics are accrued and built throughout the schooling years and, eventually, are carried into adulthood.

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Notes

1 Math self-concept differs from self-efficacy in that while self-efficacy is a context-specific assessment of competence to perform a specific task, self-concept is not measured at that level of specificity and beliefs of self-worth associated with one's perceived competence. Although sometimes confounded by imprecise definitions and varying measurements, findings consistently show that self-concept is related to math performance (Pajares & Miller, 1994).

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