

# News from Nordic mathematics education

Also in this issue of *NOMAD* there will be a few pages with news from the Nordic Mathematics Education Community. I intend to keep this column alive and in order to fill it with content I depend on getting information from colleagues in the various countries. Therefore I will encourage all of you having information that might be relevant for this column, and/or also relevant for the *NORME* web page ([www.norme.me](http://www.norme.me)), to send me a mail ([frode.ronning@hist.no](mailto:frode.ronning@hist.no)). I will take this opportunity to thank everybody that has contributed with information leading to what is presented below.

## Norma11 in Iceland

As mentioned also in the "Nordic News" in the previous issue of *Nomad* the next *Nordic Conference on Mathematics Education*, *NORMA11* will be held in Iceland from 11 to 14 May 2011. Please note that the deadline for submitting papers (eight pages in "CERME-style") is 15 December 2010. More information can be found at the website <http://www.vefsetur.hi.is/norma11/>. This has recently been updated and names of plenary speakers are now official. Bharath Sriraman from the University of Montana will give a talk with the title *Polarities in (Nordic) mathematics education research*. Another speaker from outside of the Nordic community is Núria Planas from Universitat Autònoma de Barcelona. Her talk is entitled *The use of languages in multicultural classrooms*. The other two plenary speakers are from the Nordic countries: Roger Säljö from University of Gothenburg will speak about *Learning to model: lessons from word problems*, and Marit Johnsen-Høines from Bergen University College will give a talk with the title *Learning conversation in the context of teacheracy*.

In addition to plenary lectures and paper presentations there will also be possibilities to give short communications. Working groups and

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### **Frode Rønning**

*Sør-Trøndelag University College*

a plenary panel are also planned. For further details and updates, please see the web site of the conference.

### New doctoral dissertations

The Nordic community of researchers in mathematics education is steadily increasing. This is clearly documented by the fact that this issue of *Nomad* can report a total of eight recently completed PhD dissertations. Some of the dissertations have been submitted to departments of education but from their content I think that they clearly should be characterised as being within mathematics education. Out of the eight, three are from Sweden, two from Finland, one from Denmark, one from Norway, and one from Estonia. I choose to start with the dissertation from Estonia, by Ana Palu, since this is the first PhD thesis in 10 years that is defended in this country. I also know that more is on the way, which is very promising for the future of mathematics education in Estonia. After describing Palu's work, the other candidates are presented in chronological order according to the date of the defense.

Anu Palu defended her thesis at the University of Tartu on 6 October 2010. The title of her thesis is *Mathematical knowledge of primary school pupils, its development and related factors*. The thesis rests on three published articles (referred to in the thesis as articles I, II and III) written with one or two co-authors, one of which (article I) has been published in *Nomad*. The objective of the dissertation is formulated as to assess the mathematical knowledge of primary school students and its development in terms of cognitive competences required for solving mathematical problems. Recognising both the teacher's and the students' influence on the learning process an aim of the research is to investigate how the teacher's beliefs and teaching methods as well as students' verbal abilities influence the learning outcome in mathematics. Article I is a study of 103 in-service and 26 pre-service teachers where the aim is to investigate the beliefs about the purposes and methods of teaching mathematics in primary school, and to see the results in relation to the difference in teaching experience among the participating teachers. The study is based on a questionnaire and the results have been analysed using statistical methods. Article II is based on a study of 269 Estonian primary school pupils that have been followed over a period of three years, grades 1 through 3. The aim is to study the development of mathematical skills, and towards the end of the period the pupils' verbal skills and motivation are also investigated. In the three year period the pupils have been tested three times per year. The tests that were used were taken from the International Project on Mathematical Attainment in which 17

countries, including Estonia, participated (<http://www.cimt.plymouth.ac.uk/projects/ipma/ipmaexp.pdf>). The tasks can be classified into four types by content: whole numbers; fractions and decimals; number sentences; and data reading and interpreting, and they cover three cognitive domains: knowing, applying, and reasoning. The results are analysed using statistical methods and correlation between various factors is found. Article III also deals with the teaching of numbers and operations, or what could be called arithmetic, but here connected to word problems. In previous studies it has been found that Estonian third grade children have acquired good calculation skills but they are not able to implement them in solving word problems. This study therefore seeks to go deeper into the case of word problems, and it does so by testing 494 pupils in 3<sup>rd</sup> grade and then later when they were in 4<sup>th</sup> grade. It has been found that the wrong solutions to the text problems mostly fell into two categories, firstly a relatively large group of students whose answer was based only on a partial solution, and secondly a large group of students who randomly combined the numbers they found in the word problem.

Linda Mannila (Grandell) defended her thesis *Teaching mathematics and programming – new approaches with empirical evaluation* at Åbo Akademi University on 27 November 2009. This thesis is concerned with both the teaching of mathematics and of programming, and it is based on three approaches to teaching these topics. The first approach is referred to as *structured derivations* – a logic-based approach to teaching mathematics, offering a systematic presentation of mathematical solutions and proofs. Here logic becomes a tool for doing mathematics rather than an object of mathematical study. The second approach deals with the teaching of a particular programming language, *Python*. Python is originally designed for education and the aim of using Python in instruction is to address algorithms and their implementation in a way that allows the main focus to be put on algorithmic thinking and programming concepts rather than on sorting out and memorising a complex syntax. The third approach is called *invariant based programming* and is an approach to developing programmes that are correct by construction. The approach is based on elementary propositional and predicate calculus, and does not require prior knowledge in advanced mathematics or logic. Based on these three approaches Mannila formulates her research questions in the following way: "How can the approaches be introduced in education?", "How are the approaches experienced by the students?" and "What are the potential difficulties and how can they be avoided/eased?" The study is based on empirical studies in upper secondary school and at the university. The thesis is based on nine articles, out of which two has been

written with Mannila as the sole author. The other articles have from two to four authors.

Staffan Stenhag defended his thesis at the University of Uppsala on 21 May 2010. The title of the thesis is *Betyget i matematik: Vad ger grundskolans matematikbetyg för information?* (The grade in mathematics: What information can be obtained from the grades in mathematics in compulsory school?). The aim of Stenhag's study is to investigate what the grade in mathematics tells us about the pupils' general academic success in other school subjects in Sweden's compulsory school. This aim can be linked to the fundamental question about why do pupils study mathematics to the extent that they do in compulsory school. Is it just a tradition, or does studying mathematics give some benefits for the pupils to cope with their life in society in general? Stenhag has made a literature review to document the official arguments for the school mathematics of today. He classifies the arguments in four categories – utilitarian, cultural, transfer, and selection arguments. This constitutes the first phase of the thesis. In the second phase Stenhag has collected data for the approximately 124 000 pupils who completed compulsory school in Sweden in 2006, and he correlates their grades in mathematics with grades in other subjects. It appears that the pupils who obtain high grades in mathematics also obtain high grades in other subjects, both in theoretically and in more practically oriented subjects. In the third phase of the thesis Stenhag looks in particular at a possible connection between grades in mathematics and the pupils' scores on reading comprehension tests. This part of the study is motivated by a hypothesis that pupils who are successful in mathematics in their ninth year of compulsory school also have good reading comprehension. This hypothesis is tested with data from the pupils' results in the reading comprehension test that was included in the national exam in Swedish in 2006. The results provide strong support for the statement that pupils with high final grades in mathematics also have good reading comprehension. The opposite seems, however, not to be the case. A good result in the reading comprehension test is not a reliable predictor for a good grade in mathematics.

Andrew Binde defended his thesis *Conceptions of mathematics teacher education: thoughts among teacher educators in Tanzania* at Åbo Akademi University on 11 June 2010. This is a study about the education of mathematics teachers in Tanzania. After a reform of the teacher education in this country in 2000 there has been divergent views on the consequences of the reform. The discussion has mostly dealt with the question whether the reform has placed too much emphasis on teaching methods at the expense of the mathematical content. The main questions in the study are "What are the beliefs of the teacher educators about the teacher

education in mathematics?”, and “What kind of development do they see as desirable?” The results indicate clear differences in the beliefs about the education of mathematics teachers. The most prevalent belief about the teacher education is that it is an integration of subject knowledge and pedagogical knowledge but there are also those who see teacher education as having a clear emphasis on mathematical investigations, inspiring methods and problem solving.

Anna Palmer defended her thesis at Stockholm University on 11 June 2010. Her thesis has the title *Att bli matematisk. Matematisk subjektivitet och genus i lärarutbildningen för de yngre åldrarna* (Becoming mathematical. Mathematical subjectivity and gender in early childhood teacher education). The thesis is built on three articles – Article 1, Article 2, and Article 3. Palmer is the only author of all three articles. The aim of the thesis is described as to investigate the processes through which mathematical and gendered subjectivity is constituted, reconstituted and maintained in different situations during early childhood teacher education. The empirical data was collected from three cohorts (2005–2007) of a ten-week long mathematics course included in a one-year pre-service teacher education course called *Investigative pedagogy – Dialogue Reggio Emilia*. In Article 1 Palmer describes how the participants in the course change their conceptions about mathematics and their mathematical subjectivity during the ten weeks that the course lasts. The article is based on a number of different writings that the students have produced during the course from which their experiences with and conceptions of mathematics can be derived. In Article 2 Palmer investigates how different ways of teaching influence both children’s and student teachers’ development of mathematical subjectivity related to gender. The article is connected to a student project where children’s interest for break dance and choreography is connected to mathematical thinking. Article 3 is a study of what happens with the understanding of mathematical and gender related subjectivity in the shift from a discursive and performative way of understanding mathematical subjectivity to an agentic realistic one. This shift implies a decisive meaning for how pedagogical practices can be viewed and, in the long run, how mathematics didactics can be approached for student teachers and young children alike.

Odd Tore Kaufmann defended his thesis at the University of Agder on 24 August 2010. The thesis has the title *Elevenes møte med multiplikasjon på småskoletrinnet* (The pupils’ encounter with multiplication in the primary years). This is a qualitative study based on observations in 3<sup>rd</sup> grade classrooms in Norway. Seven 3<sup>rd</sup> grade teachers from five different schools participated in the project. The first three lessons dealing with the topic multiplication in each class were observed. The data consists of

video and audio recordings as well as field notes, and also written work produced by the pupils during the lessons. The study is based on a socio-cultural perspective on learning and therefore the interaction between the teacher and the pupils, and between the pupils, is important. The same can be said about the various artefacts that are used in the teaching of multiplication. The development that can be observed is characterised as different phases of appropriation towards an increased mastering of the multiplication activity. Based on the data the different phases are categorised, and seven categories have been identified: Counting one by one, addition, repeated addition, skip counting (rekketelling), doubling, multiplying, and conversations about different properties of multiplication. The study also looks at the role of the artefacts and demonstrates that one artefact may mediate quite different meanings for teacher and pupils. As an example can be mentioned a situation where the teacher first illustrates  $4 \times 2$  by holding four pencils in each hand. Afterwards she removes one pencil from each hand, and asks the pupils for a calculation problem (regnestykke) for this situation. A pupil answers  $3 + 3$ , and when the teacher asks what it will be if we should turn it into multiplication, the same pupil answers "three times three". This, and other examples, show that there is a challenge involved in using the same artefacts in different contexts. This phenomenon is characterised as resistance in the artefacts.

Angelika Kullberg defended her thesis entitled *What is taught and what is learned. Professional insights gained and shared by teachers of mathematics*, on 27 August 2010 at the University of Gothenburg. The main interest in Kullberg's research is in the relationship between teaching and learning in classrooms. Through a framework called variation theory she analyses teaching and learning in ways that make it possible to connect "what the teacher intends the students to learn", "what is made possible to learn", and "what students actually learn". Variation theory is based on the idea that every concept, situation or phenomenon has particular features and that, if one feature is changed or varied and another remains unchanged, the altered feature is likely to be noticed. What the students are able to discern is dependent on the variation that is created or present in a learning situation. A central concept is what is referred to as *critical features*. These are features that the students must be able to discern in order to experience a specific object of learning in a certain way. The thesis is based on two learning studies. A learning study is a collaborative and iterative process of planning, conducting, analysing and revising lessons. The first learning study is about rational numbers in grade 6 and is based on eight lessons. In connection with this, certain critical features with rational numbers are identified, such as decimal numbers

as points on a line, different representations of rational numbers, rational numbers as parts of a whole, and divisibility of rational numbers. The analysis shows that, when all critical features were not enacted in full the effect on students' learning concerning density of rational numbers was small. However, when all critical features were enacted in the classroom the effect on student learning was significant. It is shown that the divisibility of numbers had a decisive difference for student learning about density of rational numbers. The second learning study is about addition and subtraction of negative numbers in grade 7. This study was implemented in eight classes with a total of 134 students. Critical features here are "subtraction as a difference", "commutative law does not apply in subtraction", "the sign", and "the number system". As in the previous study two lesson designs were constructed, involving different selections of the critical features. Also here the results show that when it was possible for students to experience more features of the object of learning simultaneously, there was a noticeable difference in students' learning outcomes compared to when it was possible to experience fewer features of the object of learning.

Mario Sánchez Aguilar defended his thesis *How to stimulate rich interactions and reflections in online mathematics teacher education?* at Roskilde University on 27 August 2010. This dissertation reports on a theoretical and empirical study of the emergence of mathematics teachers' reflections in online in-service teacher education. The study begins by presenting two reviews of the research literature on mathematics teacher education. Taking into consideration the information obtained through the literature reviews, but also drawing on the author's practical experiences in the design and implementation of online in-service courses for mathematics teachers, two research questions are formulated: "What are the characteristics of the online interactions that promote emergence of mathematics teachers' reflections?" and "Which non-human elements of an online course promote the emergence of mathematics teachers' reflections?" These two questions are investigated through the design, implementation and analysis of the outcomes of two online in-service courses for mathematics teachers. To answer the first research question a characterisation of the communicative acts present in online interactions where teachers' reflections appear, is carried out. Then, the common characteristics that are considered as key to the emergence of teachers' reflections are located. The results indicate that the evaluative acts and the challenging acts are crucial for the emergence of mathematics teachers' reflections. To answer the second research question a connection between the resources that are part of an online course and the reflections that emerge within the online course are established.

Through such connection the resources that influence the formation of teachers' reflections are located. It is found that theoretical concepts from mathematics education research are resources that help to trigger the emergence of mathematics teachers' reflections.

### Network for PhD students in mathematics education

The idea of a network for Swedish PhD students in mathematics education has been discussed during two of the summer schools that have been held in 2009 and 2010. Now this network is a reality and the first meeting was held at the University of Karlstad on 19 October 2010. The purpose of the network is that PhD students shall get to know each other, so that they can keep in touch during the studies but also keep contact in the future, when hopefully they will become colleagues, working at different universities. A page on Facebook has been established where information of common interest can be published. A new meeting is planned for the autumn of 2011.

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