

Orchestrating mathematical activities in the kindergarten: the role of inquiry

MARTIN CARLSEN

The aim of my study is to address the role played by inquiry in orchestrating a mathematical activity in the kindergarten. The study takes the inquiry cycle as a methodological departure, i.e. the whole process of designing, acting, observing, reflecting, and feeding back. The subtleties of how the kindergarten teachers and a didactician reason in order to orchestrate the mathematical activity and the role played by inquiry in this orchestration are hence analysed. The analyses show how inquiry in every phase of the inquiry cycle, plays a significant role in the orchestration of the mathematical activity. The participants become co-learners in these processes, and are involved in appropriating the mathematical tools and concepts as well as didactical issues involved in orchestrating a mathematical activity in the kindergarten.

In 2006, mathematics for the first time was mentioned as an explicit domain within the curriculum for kindergarten (Kunnskapsdepartementet, 2006). Number, space, and shape, as the domain concerning mathematics is called, puts particular emphasis on the role of the kindergarten teacher, in interaction with the children in general and more specifically as regards the topics in which the children should be engaged. In the curriculum it is said, among others, that the children should be engaged in experiencing, exploring and playing with numbers, shapes, and patterns, and that the kindergarten teacher has to support and encourage the children in such activities featured by curiosity, mathematical enjoyment, and eagerness to explore mathematical relationships. Moreover, the kindergarten teacher, in joint activity with the children, has to be engaged in reasoning and wondering concerning mathematics.

Martin Carlsen
Agder University

To describe and encapsulate terms such as exploring, playing, and wondering, I will use the term *inquiry*. The term will be described further later, but in short it labels a process of approaching mathematical challenges, both as a problem-solving tool in particular and as a way of being in general (Jaworski, 2005; Wells, 1999).

The study reported here springs out from a research project called Teaching Better Mathematics (TBM)¹ that is initiated and managed by didacticists² at University of Agder (UiA). The TBM project is focusing on collaboration between the didacticists, teachers at several levels in school (grade 1–11), kindergarten teachers and their leaders/tutors in the region where the University is situated. The TBM project is aiming for promoting development and improvement of mathematics teaching both in kindergartens and in schools. The following is said in the project proposal:

Our first aim is to develop knowledge and practice in the teaching and learning of mathematics, in order to provide learning experiences for pupils to achieve better conceptual understandings of mathematics. Pupils should demonstrate a *fluency* with mathematics, based on deep understandings, and *competence* for independent use of mathematics in different contexts.

(TBM, 2006, p.1, emphasis in original).

To possibly promote such improvement, the teachers and kindergarten teachers are participating in workshops arranged by the didacticists at UiA and visits are made in schools and kindergartens to discuss, carry out, and reflect on the mathematical activities promoted. Research is then conducted into the learning and developmental processes involved. For the study reported here, from Bambi pre-school centre, there are three kindergarten teachers, Bente, Elida, and Aina, and a didacticist, Ove, involved³. It was Elida who particularly carried out the mathematical activity.

Theoretical framework

For this study I have adopted a sociocultural perspective on learning and development. That is, I view learning as foremost a social and situated process of appropriation in which individuals make concepts, tools, and actions their own by way of collaboration and communication with others (Rogoff, 1990; Säljö, 2005; Wertsch, 1998). Significant in the process of appropriation is the role played by tools, in particular language in interaction with both physical and other psychological tools (Vygotsky, 1978, 1986). The main reason for adopting this theoretical stance is my aim of

analysing interaction among three kindergarten teachers and a didactician in context (cf. Wertsch, 1998). Moreover, this perspective encapsulate in several aspects, the study's emphasis on the process of orchestration of participation in a social and mathematical activity and context (Kennell, 2001). Before going into my use of the term orchestration, a more thorough discussion of the term appropriation is needed.

Appropriation – an individual process of learning

In a sociocultural perspective the learning process is viewed through the metaphor of appropriation. According to Wertsch (1998), appropriation is understood as to denote the individual process of "taking something that belongs to others and making it one's own" (p.53). The term appropriation is a translation of the Bakhtinian term *prisvoenie* which he used to label the process of integrating others' words in one's own discourse (Bakhtin, 1981, 1986). By this integration of words it is not meant imitation. It is rather a process of ventriloquating the words and at the same time taking over ideas, values and connotations associated with the words, applying them in various contexts and hence transforming them to fit with the individual's needs and purposes (cf. Wells, 1999). The term appropriation is thus particularly helpful when analysing interaction and collaboration between kindergarten teachers and a didactician. Appropriating mathematical tools in the kindergarten context is by sake also a process of appropriating particular words as well as language to describe and communicate experience and observations made. According to Carlsen (2010), the constituent parts of appropriation are: (1) involvement in joint productive activity, (2) having a shared focus of attention, (3) developing shared meanings, (4) transforming the insights taken over from others, and (5) attend to the relationships between sense and meaning (Moschkovich, 2004; Rogoff, 1990). Thus, according to Rogoff (1995), "Appropriation occurs in the process of participation, as the individual changes through involvement in the situation at hand" (p.153). Appropriation is hence a process of changed participation in and contributions to collaborative, interactive practices (Vianna & Stetsenko, 2006). This emphasis is further explicated by Wells (1999), who views appropriation as

the taking over and mastering of cultural artifacts and practices in the course of engaging in joint activities, in which the functional significance of these artifacts and practices is modelled and the learner receives assistance in their use. (p.155)

Appropriation may thus be seen as a collaborative endeavour, firstly taking place at a social, or interpsychological, level and secondly at the

individual, intrapsychological level (cf. Vygotsky, 1978). Appropriation is in this way a process involved in the interaction among the kindergarten teachers and the didactician. With respect to this setting, Wagner (1997) coins the term *co-learning*. This term is meant to describe the mutual relationship between, in this case, the kindergarten teachers (practitioners) and the didactician (researcher):

In a co-learning agreement, researchers and practitioners are both participants in processes of education and systems of schooling. Both are engaged in action and reflection. By working together, each might learn something about the world of the other. Of equal importance, however, each may learn something more about his or her own world and its connections to institutions and schooling.

(Wagner, 1997, p.16).

An important point here is that when viewing the collaboration between the kindergarten teachers and the didactician as a co-learning agreement, it is acknowledged that all participants bring and contribute with different expertise and engage collaboratively together. With the words of Johnsen-Høines (2009), we joined in "exchanging and developing insights, in expounding perspectives, and in turning and twisting issues" (p. 44), in order to develop and improve our own practice respectively. Another way of phrasing this is to say that each of the participants assists each of the others in their learning process as mentioned by Wells (1999) above. At the same time assistance is a significant element of what is called orchestration.

Orchestration

The term orchestration, as coined by Kennewell (2001), is used to label the kindergarten teacher's process of planning and carrying out of actions as regards a mathematical activity. In my perspective and in accordance with Kennewell (2001, p.106), one of the kindergarten teacher's roles is to

orchestrate the supporting features – the visual cues, the prompts, the questions, the instructions, the demonstrations, the collaborations, the tools, the information sources available, and so forth [...].

Orchestration thus includes what the teacher is thinking, saying and doing with respect to the activity and in interaction with the children. In this term, and of major significance in this study, I also include the preparations made ahead of the session as being part of the orchestration, i.e. planning of the activity, mathematically inquiring into the fairytale, bringing forth mathematical learning goals for the session, what and how

equipments could be used as tools in the interaction, and contextual reasoning with respect to the activity and framing of the learning environment. Such planning, arrangements done, and the actions made during the mathematical activity are hence included in the supporting features as denoted by Kennewell.

Inquiry – a tool and a way of being

In the TBM project, a key issue is to promote inquiry in teaching and learning of mathematics (Jaworski, 2005, 2007a). It is thus an aim to include the process of inquiry in the orchestration made by kindergarten teachers. Inquiry is a main theoretical notion within the project in general and in this study in particular. According to Wells (1999), *dialogic inquiry* is a process denoted as "a willingness to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to make answers to them" (p.121). Wells thus emphasises, in the inquiry process, the essential role played by dialogue, through which questioning, investigating, and exploring may be made explicit. Moreover, these activities may be discerned in analysing dialogues in interaction. Hence, inquiry is by no means a method, procedure or a set of rules, but it is a process an individual, in my case a kindergarten teacher, in collaborative endeavour with a colleague and a didactician, might be involved in featured by wondering and exploring when facing new situations and challenges. The purpose of inquiry is not primarily the development of knowing, but how the individual becomes disposed to and develops ability to apply the appropriated knowing in future situations. Inquiry is thus both a process used as a tool in concrete settings as well as a way of being – a profound and basic attitude when encountering challenges and problems (Jaworski, 2005, 2007a; Wells, 1999). Moreover, with the words of Cochran-Smith and Lytle (1999), the essence of inquiry may be described as an *inquiry stance*, i.e. a position where one as a teacher/researcher looks critically, in a positive way, on the practice in which one is engaged and asks critical questions in order to enter deeper into the substance of this activity.

The link between appropriation, co-learning and inquiry is established when in co-learning the participants agree to collaborate and shared ideas and arguments. This constitutes a significant context in which the process of appropriation occurs. The ultimate essence of appropriation is to make something ones own (cf. Wertsch, 1998), and this process is energised through how inquiry initially is used as a tool, both in co-learning and in establishing shared foci of attention and meaning, which gradually evolves into a way of being (cf. Jaworski, 2005, 2007a). In taking these

theoretical constructs as a background, the following research question has been formulated for the study:

What role does inquiry play in orchestrating a mathematical activity in the kindergarten?

Methodology

A fundamental assumption in the TBM project is that in order for there to be any change and development in the mathematics activities held in the kindergarten (and schools), it is the kindergarten teacher (and teachers) that has to bring this change about. The research conducted, in the TBM project in general and this study in particular, bears thus characteristics of what is called developmental research in the literature (Freudenthal, 1991; Goodchild, 2008; Gravemeijer, 1994). The most characteristic feature of this kind of research is the cyclical relationship between development and research. Development nurtures research, and research guides development (Goodchild, 2008). According to Freudenthal (1991, p. 161), developmental research means

experiencing the cyclic process of development and research so consciously, and reporting on it so candidly that it justifies itself, and that this experience can be transmitted to others to become like their own experience.

If research is done in accordance with Freudenthal's view, developmental research is able to deal with the complexity of educational settings such as orchestration of mathematical activities in the kindergarten, and it bears a possibility to make an impact upon mathematics education in the kindergarten. Through the co-learning agreement described above (cf. Wagner, 1997), developmental research also acknowledges the importance of viewing the kindergarten teacher(s) as a co-researcher(s), collaborator(s) and participant(s) *in* the developmental process and the research process (Goodchild, 2008). In such a way, developmental research aims at making contributions to both the field of practitioners, i.e. in orchestrating mathematical activities in the kindergarten, and to the field of scientists, i.e. the field of mathematics education research. When conducting developmental research, the problems at stake as well as features of their plausible solutions are clarified. Following van den Akker (1999), it is desirable to promote an "iterative process of 'successive approximation' or 'evolutionary prototyping' of the 'ideal' intervention" (p. 8–9). Developmental research is thus a type of research in which both the developmental process and the promotion of development through

engagement and questioning are studied simultaneously (Jaworski & Goodchild, 2006).

The inquiry cycle

In this study the inquiry cycle is used as a methodological approach. The inquiry cycle is closely related to action research, as described by Carr and Kemmis (1986) and Wellington (2000). Action research aims at changing and improving the rationality and justice behind practice, the understanding of the practice, and the situations where practice is carried out (Carr & Kemmis, 1986). According to Wellington (2000, p.21), action research is a spiral of cycles involving five steps: *planning – acting – observing/evaluating – reflecting – re-planning*. In the inquiry cycle, as described by Jaworski (2007b), the step called planning is replaced by designing and re-planning is replaced by feedback. Despite these terminological differences, it is basically the same kind of thinking that lags behind both action research and the inquiry cycle. However, in the latter cycle the process of inquiry is focused at every phase. Inquiry is hence used as a tool aiming at changing and improving the practice of mathematics teaching. In the study reported here, the kindergarten teachers and the didactician have all been involved in the orchestration of a mathematical activity in the kindergarten. Even though these different participants are considered to be co-learners (cf. Wagner, 1997), it is worth mentioning the possible strong role the didactician might take in the inquiry cycle. As will be seen in the analyses, the didactician takes a leading role in the discussions, at least in the first excerpt below. The possible consequences of that are discussed in the analyses.

All participants have been involved at every step in the inquiry cycle process, i.e. in *designing, acting, observing, reflecting, and feedback* (cf. Jaworski, 2007b). The *design* phase includes didactical considerations concerning objectives, mathematical focus, tasks, and so on. The phases of *acting* and *observing* concern the carrying out of the plan, i.e. a transformation of plans into practice, and observations of these actions in the practical setting. After what has been called the orchestration of the mathematical activity above, the kindergarten teacher, together with colleagues and the didactician, will inevitably be *reflecting* upon the actions and the observations made during the acting. Such reflections may be explicitly communicated and shared with others. *Feedback* is thus an important phase, where didacticians and kindergarten teachers discuss the issues that emerged from the orchestration of the mathematical activity. Reasoning on such issues might have as an outcome a modification of how to orchestrate a mathematical activity in the future.

Data material, participants and context

Data have been collected following the inquiry cycle, i.e. video recordings have been made at three different occasions to document the participants' reasoning with respect to the phases of designing, acting, observing, and reflecting during one cycle period. The collected data consist of video recordings, field notes, and reflection notes, the latter made after the first occasion of data collection. The context of the study is the participants' collaboration concerning orchestration of a mathematical activity of using a fairytale to promote mathematical learning. In this first data collection, the design phase, Bente, Elida and Ove were present to plan and discuss how to orchestrate this mathematical activity. Both Bente and Elida are well experienced in their profession. As concerning mathematical learning, they agreed to use the fairytale called "Goldilocks and the three bears", with a mathematical focus on number (the number of legs on the chairs, separately and the three chairs all together) comparisons and contrasts (size, softness, temperature). This is the contextual background for the discussions presented below. In the two last occasions of data collection, in the four other phases, our group was joined by the third kindergarten teacher, Aina, now working as an advisor in a council administration. Within the TBM project her role is to be the other two kindergarten teachers' leader and tutor.

The data material has been looked at several times and transcribed⁴ in detail to serve an in-depth analysis of the involved processes of appropriation and orchestration with respect to the application of inquiry in mathematics teaching in the kindergarten.

Results

The data analysis is structured with respect to the inquiry cycle (figure 1) and with regard to the three data collection periods. Hence, the following excerpts are analysed to possibly answer the research question formulated for this study.



Figure 1. *Inquiry cycle* (Inspired by Jaworski, 2007b)

Excerpt 1. Discussing the inquiry cycle

In advance of the first period of data collection, the kindergarten teachers uttered a wish for discussing the inquiry cycle (figure 1). This phase is regarded important in our developing awareness of the cycle and the role of inquiry within the cycle. Thus, this phase might be considered as a necessary prerequisite of the design phase. This is the background when the following discussion took place.

1. Ove: I wrote in the e-mail that we might talk about the so called inquiry circle or cycle
2. Bente: Yes, I feel very insecure with respect to that
3. Ove: Yes (...) it is possible that (.) it might sound a bit more, what should I say (.) advanced than it actually might be. This is what I used at another place which I just printed for us on the run. But actually it is the last (.) the last picture
4. Bente: Mm
5. Elida: Mm
6. Ove: So actually it is about (.) those five key words there. Plan, Act, Observe, Reflect, Feedback
7. Bente: Mm
8. Elida: Mm
9. Ove: And then it is in a way, then it goes in cycles, right?
10. Bente: Mm
11. Ove: And then it is inquiry, sort of, at every stage, in the planning of an activity, and in(.) when you sort of carry out the plan one has prepared.
12. Elida: Mm
13. Bente: Mm
14. Ove: It's about keeping your eyes and ears open,=
15. Elida: Mm
16. Bente: Mm
17. Ove: = to observe while acting. That's not easy- if you Elida are busy acting, it might be easier for you Bente, to observe what's going on amongst the children- if you are collaborating?
18. Elida: Mm
19. Bente: Mm
20. Ove: And then it's about reflecting on ones observations. Undoubtly, it's an advantage to be able to write down some key words, both during and after the activity- to think through: What did I do today? What happened? How did it work? and so on.

21. Bente: Mm (.) 'Cause we probably are(.) we probably are more like very engaged in it, right here and now
22. Elida: ☺
23. Ove: Mm
24. Bente: We are heavily engaged both of us when we have those math groups, and then we don't get the opportunity to talk afterwards

The picture of the inquire cycle (figure 1) ought to be commented on. This picture was shown to the kindergarten teachers by the didactician, and it constitutes his recollection of the phases involved in the inquiry cycle. These do not fit with the terms used by Jaworski (2007b), since Plan (Planlegge in Norwegian) is used instead of Designing as the first phase. Moreover, Feedback is translated into Norwegian as Implementere (Implementation). To me as researcher, this shows some of the close relationships between action research and inquiry cycle. Furthermore, I believe implementation is meant to cover the meaning of feedback, because the idea is to reason and reflect upon observations and the possibilities for such experience to feed back into or to be implemented in the mathematical practice in the future.

In this discussion several interesting and important aspects emerge. From what the kindergarten teachers say, it is concluded that they are in doubt of the meaning of the inquiry cycle. The emphasis of the words *very* and *that* supports this interpretation. Moreover, they have, at least so far, not been able to make sense of the various elements included in the cycle, what the phases are called as well as their meaning. In the excerpt, Ove takes a leading role in introducing the ideas behind the phases in the inquiry cycle, and Elida and Bente are more like receivers here. In developmental research it is important to be aware of the different roles the participants have and the possible asymmetrical communication resulting from that. However, the participants are seen as co-learners without neglecting the fact that they bring different experience and knowing to the scene. Here the didactician takes a dominant role, only because that was, as I see it, part of the didactician's role in this particular case, as well as something that was expected from him requested by the kindergarten teachers in advance of this meeting.

An additional interesting aspect of this dialogue is the kindergarten teachers' honesty as regards how they are able to (or in fact unable to) take an inquiry approach and use the inquiry cycle actively in their day-to-day practice. As Bente (21, 24) says, she and Elida are heavily engaged in the activities as these unfold together with the children. Moreover, the demands on them as adults and caretaking persons for the children often

result in lack of time and space to explicitly reflect together afterwards in more systematic ways.

I interpret this utterance by Bente as significantly describing some of the conditions under which she and Elida as kindergarten teachers are working. Their role in the kindergarten setting is apparently filled with several obligations and requirements. One of the aims of TBM is to use inquiry as a tool at every phase of the inquiry cycle (cf. Jaworski, 2005, 2007a). This is apparently not an easy matter for them to incorporate in their practice.

I would further argue that the above discussion and initial, plausible establishment of shared meaning (cf. Moschkovich, 2004; Rogoff, 1990) of the inquiry cycle is an important fundament to agree and build on in the design phase, with all its aspects and issues.

Excerpt 2. The design phase

The kindergarten teachers, in spite of the situation described in Excerpt 1, seem both willing to change their practice and start to use the inquiry cycle in their orchestration of mathematical activities in the kindergarten. This is more evident in the following dialogue, taking place short after the previous one, consisting of four parts:

Part 1. Choice of fairytale

100. Bente I was thinking that I on one occasion should try a fairytale, with Goldilocks and the three bears
101. Ove: Mm
102. Elida: My favourite of the fairytales
103. Bente: Yes ☺ Then you can do it, right?
104. Ove: Mm
105. Elida: Yes ☺
106. Bente: It was another person that told me, 'cause she had used it in a so-called mathematics group
107. Ove: Mm
108. Bente: And then she just told that, and then(.) 'cause there you got, right?, there you got, you have to count to three, you have the smallest one, the middle-sized one and the biggest one
109. Elida: Mm
110. Ove: Mm

This dialogue brings about a thought that Bente had been thinking of for some time (100), and she suggests to use the fairytale called Goldilocks and the three bears. Both Ove (101) and Elida (102) seem to support the idea of using this fairytale in order to mediate mathematical ideas to the children. This particular fairytale was even the favourite of Elida. Bente (103) thus suggests that Elida should be the person doing the particular orchestration of the fairytale in interaction with the children, and Elida (105) obviously agrees and are willing to do it. Then, in (106, 108), Bente gives reasons for why she thinks that this particular fairytale is suitable and useful in addressing mathematical aims in a kindergarten activity. She tells that she has spoken with another person who has experience from using the fairytale. In the dialogue Bente does not tell explicitly what she is referring to, but since all participants have collaborated and discussed in previous work shops at the university, it is reasonable to believe that she is referring to a conversation with one of the other kindergarten teachers participating in the project. Bente uses the word mathematics group, a label one of the other participating kindergartens consistently used with respect to their inclusion of mathematical activities in their kindergarten work.

This small incident is interpreted as showing some evidence that the participants in the project are learning from each other, they share ideas and tell each other about the experiences they do and have done with orchestrating different mathematical activities in their own kindergarten. Moreover, this excerpt may also point to how these two kindergarten teachers collaboratively are trying to transform the ideas of others in making inquiry as a tool their own, that is inquiry as a way of being (cf. Jaworski, 2005, 2007a; Moschkovich, 2004; Wells, 1999). The utterances of Bente (106, 108) also to some extent shows her "willingness to wonder" and she seeks "to understand by collaborating with others" (Wells, 1999, p. 121). She is ready to try out a mathematical activity some of her colleagues in the TBM project has used. She has wondered to use the fairytale for some time and now she shows willingness to explore for herself. This exemplifies the exchanging of insights and expounding of perspectives as described by Johnsen-Høines (2009), with the purpose of developing and improving practice. Furthermore, Bente here (108) initiates a mathematical inquiry into the fairytale, by explicitly pinpointing some of the mathematical issues and concepts involved. She argues that issues related to number, counting, comparing and contrasting size could be addressed.

This mathematical inquiry of the fairytale is continued by a passage focusing more generally on the orchestration level (cf. Kennewell, 2001), addressing the issue of what is called mathematical coherence:

Part 2. Mathematical coherence

151. Ove: Is it possible to find, I don't know if there are any, but are there any such songs that we may use, which carry some of the same aspects in a way(.) the mathematics in Goldilocks and the three bears?
152. Elida: Yes, to have one of the same kind
153. Ove: I thought(.) it would have been interesting to be able to make=
154. Elida: Mm
155. Ove: =a thirty minutes session in a way, all the different elements=
156. Elida: I understand what you mean
157. Ove: =that we include in the activity, all support the same [goal]
158. Elida: Mm
159. Bente: Mm

In this passage, Ove (151) makes a contribution to the orchestration of the mathematical activity through inquiring into whether the kindergarten teachers know of "such songs that we may use", that concern some of the same mathematical aspects that are included in the fairytale. This contribution is interpreted as an attempt to build mathematical coherence in the orchestration of the activity. Elida (152) seems to follow the idea to include a song that is "of the same kind". This expression is interpreted as another way of communicating that the fairytale and possible songs used should all contribute to achieving the mathematical goals of the activity. Ove's reasoning (153, 155, 157) is explicitly addressing this issue. If the mathematical aspects of songs used during the activity and the mathematical aspects of the fairytale coincide, they "support the same" mathematical learning goal(s). Elida (156) and Bente (159) seem to share this view. By inquiring into and seeking to make all the different elements share mathematical foci, these three co-learners (cf. Wagner, 1997) seek to achieve mathematical coherence within the planned activity. Inquiry is used as a tool (Jaworski, 2005) in their collaborating efforts to achieve a shared focus of mathematical attention in the activity (Moschkovich, 2004; Rogoff, 1990).

A couple of minutes later, the following discussion took place concerning the use of questions:

Part 3. The use of questions

253. Ove: When you have done it before, did you have a kind of list of relevant questions to ask, or did you do it on the spot there and then?
254. Elida: No, I didn't. Yea, yea. I haven't really considered that. I haven't done it in such as setting before, right? No, I haven't

255. Ove: 'Cause it might be that (.) maybe wise to think, that we think these things through in advance. Are there any such particular questions we in any case want to ask?
256. Elida: Mm. Mm. Mm
257. Bente: Mm
258. Ove: And then there might be others which may be included on basis on what the kids responds and or do
259. Elida: Mm
260. Bente: Mm

The reason for including this excerpt in the analysis is that here we can see how the issue of asking questions as part of orchestrating a mathematical activity emerges in the discussion. The dialogue starts with a conversation between Ove and Elida concerning whether she had explicitly prepared any questions when engaging with the fairytale on an earlier occasion. The answer is addressing the fact that when Elida has used the fairytale before she had not taken a mathematical focus. My interpretation, based on this response, is then that she has used the fairytale as a nice story to tell the children. The mathematical potential of the fairytale has apparently not been previously attended to, neither by Elida nor Bente. However, Ove's suggestion to really consider what questions might be relevant and purposeful to ask, is well received by the kindergarten teachers. It seems that they think it is a good idea both to prepare some basic questions in advance as well as being concentrated and open during the activity and attend to the responses from the children and ask questions and comment accordingly. In this way, the future orchestration (cf. Kennewell, 2001) of the mathematical activity has been enriched and changed as a result of the collaboration between the participants. This may also be seen as an incident where the participants are involved in establishing shared meaning (cf. Moschkovich, 2004; Rogoff, 1990) with respect to using inquiry as a mathematical tool in kindergarten activities (Jaworski, 2007a).

Four weeks later, the two kindergarten teachers and the didactician was joined by the teacher's tutor Aina, and they met to reflect on and discuss the orchestration of the mathematical activity, both what had happened in the design phase and issues emerging from the phases of *acting* and *observing*.

Excerpt 3. The reflection phase (based on actions and observations)

This excerpt, and the next (excerpt 4), took place during the third data collection period, taking place just after Elida had orchestrated

and carried through the activity of mathematically inquiring into the fairytale of Goldilocks and the three bears.

346. Ove: And it is in a way as we talked about previously, those more open questions that potentially stimulates more genuine inquiry Those are not easy to find. It isn't=
 347. Bente: Mm
 348. Elida: Mm
 349. Ove: =easy to find those questions
 350. Elida: No, I agree with you. And if one tries to help in a way (.) how should I, what should one say in order not to tell the answer. It isn't easy
 351. [short discussion of a different issue]
 352. Elida: 'Cause it isn't easy to give cues or hints. I find this difficult
 353. [short discussion of a different issue]
 354. Aina: And I don't have any easy answers me neither about how to possibly change the questions. But I think that is a challenge=
 355. Elida: Yes, absolutely
 356. Ove: Mm
 357. Aina: =to detect those open questions in such a way that you yourself don't even know how to answer them
 358. Elida: Yes, right. Mm mm

This dialogue concerns the issue of using questions as part of orchestrating mathematical inquiry in the kindergarten, as the participants are reflecting upon the mathematical activity recently carried through. Both Ove and Aina address the (difficult) issue of using open questions in such activities, upon which they both agree that it is a challenging issue. Elida, who carried through the activity, certainly also find this difficult. Elida's utterances in (350, 352) are interpreted as communicating aspects of her view upon doing inquiry in her practice. She is very concrete when explicating that she finds it difficult to ask questions, to give cues and to come up with hints that do not in the same time spoil the inquiry by telling the answer.

At the same time, this excerpt is interpreted as showing how Elida, in her struggle to incorporate an inquiry approach in her orchestration of mathematical activities in the kindergarten, is honest about issues she finds difficult and challenging. At the same time she shows eagerness in order to develop her expertise in practicing mathematical inquiry. Elida thus may be seen to take an inquiry stance (cf. Cochran-Smith & Lytle, 1999) towards her own way of practicing her craft and interacting with the children. This also shows that Elida is involved in a process of

appropriating the concept of inquiry – what it might mean and include in various contexts (cf. Carlsen, 2010; Rogoff, 1990; Säljö, 2005).

Excerpt 4. Feeding back into practice (based on reflections)

Some minutes later the discussion turned into the fifth phase of the inquiry cycle – the phase of *feedback* (cf. Jaworski, 2007b).

518. Aina: But then I think, one has possibilities later, even during the day, maybe some [children]=
519. Elida: Yes, certainly
520. Aina: =want to play that fairytale. And then it is possible to spontaneously sit down [with the children] and develop it further
521. Elida: And we can actually use it one more time, and then we can include those=
522. Aina: Ye, right
523. Elida: =things [the issue of using open questions that has been discussed previously]. We don't have to use it [the fairytale] only one time.
524. Aina: No, no no
525. Elida: And then they may act as the different characters [bear, Goldilocks]
526. Aina: Actually, they may take those roles if they are well familiar with the fairytale

In this excerpt the dialogue turns into discussing possibilities for feeding back to the kindergarten teachers' practice. The conversation between Elida and Aina points to two different things that might be fed back; to include more open questions in the orchestration and to let the children themselves be actors in the fairytale. Both these things come to the floor due to reflecting on the activity observed, and they are both possible improvements of the orchestration of such an activity. At the same time, if the activity is changed by including more open questions or making the children become actors, improved possibilities may be created for the children to appropriate the mathematics implicitly included in the fairytale.

This excerpt also shows the plausible benefits of explicitly reflecting on the kindergarten teachers' practice and what may be improved by taking an inquiry stance towards the practice (cf. Cochran-Smith & Lytle, 1999). However, it is also important to emphasise how this critical view is emerging here. I claim that it is resulting from the co-learning agreement (cf. Wagner, 1997) established between the kindergarten teachers and the didactician. They are all engaged in reflecting upon the practice. They

thus come further in their process of appropriating inquiry as a tool in mathematics in the kindergarten (cf. Jaworski, 2005, 2007a).

Discussion

In this study I have aimed at bringing forth the role of inquiry (cf. Jaworski, 2007a) in orchestrating a mathematical activity in a kindergarten. The analyses show how the inquiry approach is used in the phases of designing, reflecting and feedback, as regards engaging with a fairytale. Therefore, inquiry as a tool (Jaworski, 2005) is a significant element of orchestrating this mathematical activity (Kennewell, 2001). The analyses show how the participants are able to collaborate in different phases of the inquiry cycle, in reasoning about the mathematical focus of the activity, what questions to ask, and what learning goals to emphasise, even though the didactician initially takes a dominant role in the co-learning process (cf. Wagner, 1997). They all make experience that assumingly will change and improve their practice in the future. They have all made something new their own to some extent (cf. Wertsch, 1998), i.e. they are involved in a process of appropriating how to orchestrate a mathematical activity in the kindergarten, the mathematical concepts and tools and didactical issues involved.

Moreover, by approaching orchestration of a mathematical activity in the kindergarten through inquiry, I want to argue that the possibility increases that the learning goals set for the activity will be achieved. Inquiry is about investigating different possibilities, wondering about various approaches, asking questions in order to come to solutions and decisions, relative to orchestrating the mathematical activity. Furthermore, inquiry is about being open and critical towards the various elements that might be incorporated in the activity, with an aim of orchestrate the activity in a manner beneficial for the participants. From the analyses made in this study, I want to argue that using inquiry as a tool at every phase of the inquiry cycle (even though the actual carrying out of the activity is left out here), significantly supported the orchestration of the activity. However, the analyses also reveal that the kindergarten teachers experience difficulties in their orchestrations of mathematical activities. They claim it is a challenge as regards the inquiry cycle, to be able to systematically co-plan and co-reflect upon their experience due to issues of time, working conditions, obligations, and requirements. Another challenge as regards inquiry as a tool is the timing and phrasing of questions, hints and cues when discussing mathematical issues with the children. Nevertheless, they and the didactician show a willingness to wonder and they seek to understand how to orchestrate mathematical

activities in the kindergarten. From the analyses I conclude that they have all started their process of entering deeper into the substance of this part of their practice. The didactician got the opportunity to discuss theoretical as well as practical issues involved in orchestrating mathematical activities in the kindergarten. Moreover, he experienced how to cope with affordances and constraints involved, through taking an inquiry stance (cf. Cochran-Smith & Lytle, 1999).

From the analyses made in this study, it is difficult to argue strongly that these individuals have changed their participation in the inquiry cycle process (cf. Rogoff, 1995; Vianna & Stetsenko, 2006). However, some indications are observable looking through the excerpts. The kindergarten teachers tend to increase the amount of making their ideas and arguments explicit. In the first dialogue the didactician takes a dominant role, but makes no explicit contribution to the last dialogue. The tendency seems to be that the kindergarten teachers steadily take more ownership of the orchestration and the role of inquiry within, applying and transforming inquiry to fit their purposes (cf. Moschkovich, 2004; Wells, 1999). Additionally, this to some degree accomplishes the aim of the TBM project, that is to raise the participants' insight and awareness as regards the teaching and learning of mathematics.

By taking an inquiry stance (cf. Cochran-Smith & Lytle, 1999), bridges may be built between kindergarten teachers and didacticians, among colleagues at one kindergarten, and between kindergarten teachers and the children. In this study the bridge between kindergarten teachers and a didactician has been elucidated and analysed. In institutional settings, such as the kindergarten, in collaboration with didacticians at university level, the shared focus of attention on inquiry in my opinion has initiated development of the local practice. The framework created for this study has led to transformation of the practice and individual development. This has happened due to the participants' collaboration, interaction, and interthinking (cf. Mercer, 2000). Furthermore, in discussing and reflecting on issues emerging from practice the participants are at the same time "focused on understanding and improving practice" (Wells, 1999, p. 122). Hence, co-learning illuminates the collaborative process among the participants, in accordance with Wagner's (1997) use of the term.

Conclusively, I want to emphasise the following results from this particular study. First of all, collaboration and interaction in the designing phase reveal that inquiry is used as a tool in order to establish mathematical coherence in the designed activity, what questions to ask and what mathematical issues to focus on. Secondly, the kindergarten teachers experience difficulties in using the inquiry cycle as a tool in their orchestration of mathematical activities. They want to develop inquiry as a way

of being, but working conditions expose constraints. Thirdly, the reflecting phase raises the participants' consciousness as regards important and critical aspects of orchestrating mathematical activities in the kindergarten. Shared meaning is established with respect to the challenge of asking open questions, when and how to hint and give cues. Finally, inquiry as a tool in the feedback phase evidently discloses thoughts and ideas of how to implement and improve this activity further, to include more open questions when discussing the fairytale with the children as well as letting the children become actors. Inquiry is thus seen to play a significant role in the orchestration of the mathematical activity. These ideas might also be considered to be of a more general value of how to orchestrate inquiries in the kindergarten in order for the children, and the adults, to appropriate mathematical concepts and tools.

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Notes

- 1 The TBM project is supported by the Research Council of Norway (NFR no. 176442/S20) and the Competence Development Fund of Southern Norway.
- 2 The term didacticians is used to label the people at University of Agder engaging in mathematics education research.
- 3 All the names used in this paper, Bambi pre-school centre, Aina, Bente, Elida, and Ove are pseudonyms.
- 4 In the transcribed material, the following codes have been used:
= continued utterances, (.) short pause, ☺ laughter in voice, [] author's comments, word emphasised word.

Martin Carlsen

The author's research interests are within the scope of this paper, mathematical inquiry in the kindergarten. He is also particularly interested in mathematical appropriation through problem solving in collaborative small groups. He identifies with a sociocultural perspective on learning and development, and his research is taking place within this theoretical stance.

martin.carlsen@uia.no