Head teachers' conception of gifted students in mathematics in Swedish upper secondary school

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The article presents a study of how Swedish upper secondary head teachers, working within mathematically intensive study programs, conceptualize giftedness in mathematics. The study is based on a survey of 34 randomly selected head teachers, in a population of about 400, who have answered questions about how they characterize and detect gifted mathematics students. The results show that teachers characterize such students as creative, strong in logical ability and keen in their motivation for mathematics. The teachers detect such students by the students' own initiative for engaging in mathematics, their inclination to orally reason about mathematics and their successfulness on tests. The findings, which are in accordance with results from internationally published studies, are of importance to the current discussion on special provision for gifted students in Sweden.

In recent decades, Swedish research as well as national and regional reports have mostly focused on students' difficulties with mathematics and their declining knowledge of the subject (Engström, 2003; Magne, 2001, 2006; Swedish Government Official Report, 2004; Swedish National Agency for Higher Education, 2005). However, it has been noted that the Swedish school system is less well tuned to the needs of gifted students (Persson, Joswig & Balogh, 2000). During the last five years there has been a notable shift in perspectives towards the special needs of these students (Edfeldt & Wistedt, 2009). This has recently been emphasized in the Swedish government's decision to establish special programs (advanced placement tracks) at the upper secondary level in mathematics and science

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as well as in the humanities and social sciences (Ministry of Education and Research, 2008).

It is well known that the detection of and provision for gifted students should build on an explicit definition of giftedness (Renzulli, 2004). However, in Sweden we lack such a commonly acknowledged definition, in fact we know very little about how different parties within the national education system characterize giftedness and what educational provision they think should be offered to these students. In order to offer more solid ground for the discussion on these matters it is, to begin with, urgent to clarify the conceptions of giftedness held by various stakeholders within the Swedish educational system. The overall aim of this study is to promote such a discussion.

The study focuses on how 34 randomly selected head teachers working in mathematically intensive programs in upper secondary school characterize and detect mathematical giftedness in their students. The study aims at clarifying the variation in conceptions of the two aspects of giftedness among these teachers as well as their most frequently expresses views of these students. We hold that knowledge about such conceptions and views is of great importance to the discussion of how mathematics teaching aiming at talent development should be planned and organized in our country.

Theoretical Considerations

Characteristics and detection of giftedness

There are many models of conceptions of giftedness presented in the literature, such as Renzulli's *three ring model* (Renzulli, 2005) and Sternberg's *WICS model of giftedness* (Sternberg, 2005), which are two of the most well known and elaborated ones. These models strongly emphasize the general character traits or attributes that need to be synthesized in order for a person to be considered gifted or to show gifted behavior. Thus, in the present study questions about how teachers characterize gifted students in mathematics will be at central interest.

The models of giftedness may also acknowledge the interactive nature of human development, such as in the *multifactor model* (Mönks, 1992), which is a modification of Renzulli's three-ring-model. However, Mönks' model considers both personality and environmental aspects of giftedness. Personality traits includes creativity, outstanding abilities in specific domains and motivation (e.g. task commitment, risk taking, anticipation, planning, future time perspective and emotional factors). The most important social contexts for the development of the individual such as *family*, *school* and *peer group* constitutes the environmental aspects of the model.

Following Mönks (1992), we consider the development of giftedness to be dependent on the interaction between these personal and social aspects. This means that one of the main actors in facilitating the development of character traits vital to the expression of giftedness is the individual's teacher. However, in this study we do not focus directly on the interaction between the student and the teacher. Instead we investigate a component of the teachers' basis for participating in such an interaction, namely the teachers' ways of detecting giftedness.

Characteristics of mathematical giftedness

When considering conceptions of giftedness in mathematics we have to take into account the particular context of the subject (Mayer, 2005). However, there is a gap between the general models of giftedness and the research about giftedness in mathematics (Leikin, Koichu & Berman, 2009), often expressed in terms of high mathematical thinking and problem solving abilities (Sriraman, 2005). This makes studies of conceptions of giftedness in mathematics hard to fit to both of the branches of the research. In order to connect these braches we interpret our findings in relation to Mönks', well known, multifactor model of giftedness. Thus, important issues are what outstanding abilities as well as creativity in mathematics mean.

Although gifted individuals in mathematics are not a homogenous group (Marjoram, 1992), they seem to have some common characteristics (for an overview see e.g. Sriraman, 2005; Wieczerkowski, Cropley & Prado, 2000). Researchers often describe such characteristics in terms of *specific mathematical abilities*, based on Krutetskiis' (1976) famous study of mathematically capable children (see eg. Chyriwsky & Kennard, 1997). These children were characterized by a set of abilities that made it possible for them to master the subject in a creative way. Among these abilities we find the ability to grasp the formal structure of a problem, to generalize mathematical material, to think logically and flexibly respectively, to reverse and reconstruct mental processes, to curtail mathematical reasoning processes, to strive for clarity and simplicity of solutions and to memorize results in generalized structures. In addition Krutetskii argue that capable students show indefatigability during extended and intensive mathematical activity. However, according to Krutetskii, character traits such as swiftness of mental processes, computational abilities, memory for symbols, numbers and formulas, ability for spatial concepts and ability to visualize abstract mathematical relationships and dependencies are not necessary for mathematical giftedness.

Today there is no commonly acknowledged definition of mathematical creativity (Sriraman, 2004). Nevertheless, creativity is often associated with original approaches and unusual solutions to problem solving, including flexibility of thought, reversibility of thought processes, and strive for elegance and clarity in explaining reasoning as listed by Krutetskii (see e.g. Scheffield, 2009). In addition creative students might construct problems themselves (Kiesswetter, 1985).

Detection of mathematical giftedness

Internationally different procedures such as e.g. IQ-tests, specific mathematical aptitude tests and teacher nomination, often based on forms with check-boxes, are used to identify mathematical giftedness (see e.g. Wertheimer (1999) for an overview of identification procedures). In Sweden the teachers do not have access to such tests or forms, instead each teacher must find their own way to detect mathematical giftedness.

Methodology

In the literature, there is no common use of terminology for terms referring to giftedness and talent (Gagne, 1995). In this article the terminology is principally adjusted to the once used by different researchers. However, in our communication with the head teachers in this study the phrase ”students with specific potential and pronounced mathematical ability” (elever med utpräglad förmåga och fallenhet i matematik) was consistently used. Since this is a very long expression we use ”gifted students” as a generic term in this article.

In this study we consider ”conception” to be a subset of beliefs where the cognitive and conscious component of these beliefs is emphasized (Pehkonnen & Hannula, 2004). Thus, conceptions are principally seen as ”professed beliefs” (Speer, 2005) which means that they may be expressed in writing. Furthermore, ”conceptions of giftedness” are considered to be domain specific as are the characteristics and detection indicators of giftedness in mathematics. Thus, this study presents two aspects of the head teachers' professed conceptions of gifted students in mathematics by investigating the following issues.

R1 What characterizes gifted students in mathematics?

R2 How are gifted students in mathematics detected?

The character traits are seen as fundamental premises in the manifestation of giftedness and which, in the ideal situation, are expressed in the social interaction through teachers' detection indicators.

Method

Sampling

In order to ensure a sample of teachers who had had the chance to repeatedly encounter gifted students in mathematics, we chose participants among teachers at the most mathematically intensive programs in Sweden, which are the Natural science program (NV) and the Technology program (TE). Further, we chose to approach head teachers that were considered the most influential mathematics teachers in schools and most likely responsible for issues concerning gifted students in mathematics. Due to the special mathematical responsibility held by head teachers we cannot assume that they speak for the whole faculty of teachers at their school. Thus, this study only claims to account for the population of head teachers.

Since there is no comprehensive list of head teachers in the target population, we used schools as sampling units. Considering the possibility to generalize the results of the study (Neuendorf, 2002), we chose to perform a random sampling of about 10% of the population among all Swedish upper secondary schools offering at least one of the two programs NV or TE.

In the autumn 2007, 40 Swedish upper secondary schools offering at least one of the two given educational programs were selected by using a random-generator. The sample turned out to consist of 29 municipal schools and 11 independent schools with a wide geographic spread within Sweden. The head teacher at each selected school was asked via e-mail to participate in the study. If no head teacher of mathematics existed at a school, the person among the teachers who taught mathematics the most within the given educational programs was considered to be *head teacher* (which happened in eight out of 40 cases). In the following we sometimes use the word *teacher* instead of head teacher when the meaning of the word ought to be clear to the reader.

Without a deliberate selection of the respondents based on gender, the sample consisted of 28 men and 12 women. Due to heavy workloads five of the men and one woman decided not to participate. Thus, 34 teachers, 23 men and 11 women, participated in the study, that is, there was a response rate of 85 %. Ten of the participants taught only at the NV-program, one taught only at the TE-program and the remaining 23 participants taught at both programs. More than 70 % of the participants had more than ten years teaching experience and only 12 % had less than six years teaching experience. The number of students taught by the respondents was more than 1,100.

Data collection

In order to attend to the national spread of the respondents and to the time constraints for participating head teachers, data were collected using a web questionnaire. The questionnaire consisted of clusters of questions, considering teachers' demographic characteristics, teachers' ordinary teaching, teachers' conceptions of giftedness in mathematics, and what mathematical activities they offered to gifted students in mathematics. Two of the questions form the basis of the results of this study:

21. How would you characterize the (gifted) students you denoted in question number 20?[1]

23. How do you detect these gifted students in mathematics?

The questionnaire was piloted in two steps, a first test on 15 teachers and a second test on five teachers. After the second test no changes were made in the formulation of questions used in this study. However, due to the specific design of the questionnaire, teachers, who indicated that none of their present students were to be considered gifted in mathematics (question number 22), were not asked to answer the questions regarding their detection of gifted students. This means that the results on teachers' detection indicators are based on answers from 30 respondents. This shortcoming of the questionnaire was not apparent in the two pilot studies since all teachers participating in the pilot studies had gifted students in their present classes. It could be the case that the head teachers who took part in the main study and who were not asked to answer the question on how they detected gifted students, had a limited or a different way of detecting these students and thus did not find any such students in their classes. If so, this could have an effect on the reliability of the results. However, the group of teachers that did not respond to the question about detection did not distinguish themselves from the other set of teachers in their characterization of giftedness, as they referred to *creative ability* (n = 2), *problem solving ability* (n = 2), *manage (school) mathematics with great ease* (n = 2), *ability to generalize and see patterns* (n = 1), *ability to grasp the formal structure* (n = 1), *motivation* (n = 1), and *independence* (n = 1).

Since our purpose was to explore head teachers' conceptions of the two aspects of giftedness, we chose to let the participants express themselves unreservedly by allowing them to answer the questions by free description. In order to be able to remind teachers to answer the questionnaire, and consequently lower the non-response rate, we chose not to let them answer anonymously.

Data analysis

Like researchers conducting similar studies (see e.g. Persson, 1998), we chose to process data using *content analysis* (Berg, 2007). This work involves a delicate process of interpretation and splitting data into units in order to correctly elucidate it. For example, data from each head teacher on student characteristics were split up into coherent units, each unit representing a separate characteristic. These units were then sorted into a scheme of categories, with each unit placed in one and only one category (see Result). Which teacher that referred to each category was recorded. Data from one and the same head teacher can be found in several different categories.

Framework of categories

Since we didn't find suitable frameworks in the literature, we developed our own. The framework of character traits was created in relation to previous research and the framework for the identification indicators was developed out of the data (Glaser & Strauss, 1967).

Data of character traits were split into two main categories referring to *cognitive* and *non-cognitive attributes* respectively. Since the work of Krutetskii is considered fundamental in research in gifted education in mathematics, we used his list of abilities as far as possible when creating subcategories. However, considering that creativity often is viewed as an essential part of the characteristic of a gifted individual (see e.g. Sriraman, 2005; Mönks, 1992) we decided to create a subcategory of creativity. Thus, for the purpose of this study the character traits flexibility and reversibility of thought processes, strive for elegance and clarity in solutions, display of original approaches and unique solutions to problem solving and construction of own problems span the category of *creativity*.

As subcategories were defined in order to elucidate the width and breadth of the data in this study, we needed to add an ability that was not included in the list of Krutetskii – the *ability to communicate mathematically*. Moreover, we needed to add four categories that partially may coincide or include one or more of the abilities listed by Krutetskii's – *problem solving ability*, *manage school mathematics with ease*, *easy learning* and *ability to apply knowledge and connect it to other subject areas*. In teachers' answers there were references made to problem solving ability. Depending on the specific interpretation of "problem", this ability could be seen as diverse abilities such as, for example, an overarching ability including all of the categories defined so far, or as a pure computational ability. Since the specific abilities vital in the problem solving situation was not articulated by the teachers, we created a category referring to the broader ability of problem solving. However, when data specifically referred to the creative part of the problem solving process, such as the ability to solve problems of a type that had never been met before, data was put into the category creativity instead of problem solving ability.

Krutetskii's specific abilities are tied to a problem solving situation, which was not necessarily the case with all the data from the teachers in our study. Thus, there was a need to consider other broader aspects of characteristics of giftedness in mathematics. We couldn't assume that these aspects such as manage school mathematics with ease, easy learning and ability to apply knowledge and connect it to other subject areas did not include problem solving abilities, but we still needed to treat them separately in order to clarify the specific view of giftedness that they might indicate.

The choice of non-cognitive subcategories such as *motivation*, *independence*, the negative attitude expressed in *reluctance to standardized teaching*, and the role model articulated by *helpfulness* was influenced by previous research (Persson, 1998), whereas the subcategories of *systematic work* and *thinker* was drawn directly from data.

The units of detection indicators were initially split into two main categories based on their reference to activity aspects or quality aspects of ways of detecting gifted students. The former category, called *students' initiative*, is distinguished from the other category since answers in this category refer to situations where the students are the promoters of an activity and in which no quality aspects are evaluated/mentioned (e.g. students active in posing questions and in discussion), whereas data in the other category refer to quality aspects that are expressed by gifted students in class (e.g. the *type* of questions posed and the *way* students reason mathematically). The five categories *oral mathematical reasoning*, *test results*, *type of questions posed*, *written solutions of problems* and *swiftness* elucidate the different quality aspects that teachers referred to.

The process of categorization

Consider the following expression from teacher nine (T9)

T9: These students often display an ability to draw their own far-reaching conclusions and independently develop their own mathematical ability from the conceptions and methods everyone in the group is learning. Their capacity for solving problems is at a level that exceeds others'. Often, they have a greater interest in mathematics (consequence or cause?) than what is commonly shown.

This expression was split into the units *power of deduction* ("These students often display an ability to draw their own far-reaching conclusions..."), *independent learning* ("...and independently develop their own mathematical ability from the conceptions and methods everyone in the group is learning."), *great ability for solving problems* ("Their capacity for solving problems is at a level that exceeds others'."), and *interest* ("Often, they have a greater interest in mathematics (consequence or cause?) than what is commonly shown."). These were then put into the categories *logical ability*, *independence*, *problem solving ability* and *motivation* respectively.

Another example shows how data regarding detection of gifted students were split into the units *interest in new material* ("They show interest during presentations of new subject matter..."), *pose relevant questions* ("...pose relevant questions."), *work fast and systematically* ("They work fast and systematically.") successively sorted into the categories *students initiative*, *type of questions posed* and *swiftness*.

T5: They show interest during presentations of new subject matter and pose relevant questions. They work fast and systematically.

When using the created frameworks, all data were similarly categorized by two researchers independently, thus indicating a high level of intercoder reliability (Krippendorff, 2004).

Result and discussion

What characterizes gifted students in mathematics?

Teachers' responses gave evidence of a variety of conceptions among Swedish teachers regarding the characteristics of gifted students in mathematics, see table 1.

Table 1. *Characteristics of gifted students in mathematics as stated by the teachers*

Character trait	**Number (percentage) of teachers**
Cognitive attributes	
Creative ability	11 (32 %)
Logical ability	8 (24 %)
Manage (school) mathematics with great ease	7 (21 %)
Ability to generalize and see patterns	6 (18 %)
Problem solving ability	6 (18 %)
Ability to apply knowledge and connect it to other subject areas	5 (15 %)
Ability to grasp the formal structure	5 (15 %)
Easy learning	2 (6 %)
Ability to communicate mathematically	2 (6 %)
Non-cognitive attributes	
Motivation	15 (44 %)
Independence	4 (12 %)
Reluctance to standardized teaching	3 (9 %)
Systematic work	3 (9 %)
Helpfulness	1 (3 %)
Thinker	1 (3 %)

Notes. The number of teachers is 34. Each teacher may have provided more than one character trait.

Cognitive attributes

The cognitive character trait most frequently found (32 %) in the head teachers' responses was *creative ability*. By writing, for example "they can also have an individually pronounced and sometimes unconventional way of thinking that make them find alternative solutions of problems" (T34), "they can think freely and create their own methods of solution" (T2), "They have the ability [...] to solve problem of a type that they have never seen before" (T21), and "Rather often they do things in another way than the teacher does, ie. they try to solve problems in alternative ways" (T35), teachers emphasized the same aspects of creativity as has been stressed in other studies as researchers characterize creative mathematics student by their inclination to approach and solve problems in unusual ways (e.g. Greenes, 1981; Sheffield, 2009; Wolfle, 1986). The teachers in our study also expressed that gifted students solve mathematical problems that they had never seen before. Although not explicitly referring to flexibility and reversibility of thought (Krutetskii, 1976) these are abilities that are useful in those novel problem solving situations. Moreover, even if the students' strive for elegance and economy of solutions was not emphasized by any teachers in this study, it still might be an underlying force for students searching for alternative solutions.

The second most common cognitive attribute stressed by eight teachers (24 %) in this study was gifted students' *logical ability*, for example, "These students often display an ability to draw their own far-reaching conclusions" (T9) and "Logically thinking students" (T26). Since logic sometimes is thought of as equal to mathematics (see e.g. Gardner's (1983) definition of logic-mathematical intelligence) we could expect that more teachers in this study would stress this ability. However, our results show that creativity in mathematics is considered more significant by the teachers when they characterize giftedness.

Without giving any further explanation seven (21 %) head teachers characterized gifted students as students that *manage (school) mathematics with great ease*. Teachers' comments were for example "Keeps up with the ordinary courses easily" (T36), "Manage the courses very well" (T8), and "The subject comes easy" (T19). But, since memorization of facts and procedures based on superficial properties to a large extent defines the school subject of mathematics in Sweden (Palm, Boesen & Lithner, 2005) such characteristics may be inconsistent with the character traits emphasized in research (see eg. Krutetskii, 1976). However, interpreted in another way, results might just show that teachers interpret giftedness in mathematics as an outstanding achievement in the subject, thus easy success in school is a necessary condition for giftedness. It is to be noted that units placed in the category of *manage (school) mathematics with great ease* are distinguished from units placed in the category of *easy learning* since units in the former category refer to performance in school mathematics whereas units in the latter rather refers to swiftness and ease in comprehension.

In line with researchers on mathematical giftedness (see eg. Sriraman, 2003, Krutetskii, 1976) six teachers (18 %) in our study stressed gifted students' *ability to generalize and see patterns*. For instance, teachers wrote "Perhaps what most clearly separates them from other very good students is their ability to generalize" (T30), "They have an ability to see connections and to generalize their solutions" (T6) and "[They are] the ones that sees pattern in mathematics" (T13).

No teacher mentioned gifted students' mathematical memory, although memorization is singled out as an important ability manifested by capable problem solvers (Krutetskii, 1976). However, teachers might connect memorization to procedural reproduction in mathematics, whereas Krutetskii rather meant a memorization of generalizations. Further, no teacher in this study specifically referred to "curtailment" of mathematical processes (Krutetskii, 1976). Even if some teachers detected gifted students by their swiftness of thought (see below), one way in which curtailment may be expressed, it is not evident that teachers thought of curtailment as a specific mathematical characteristic of gifted students.

Six teachers (18 %) characterized gifted students by their *problem solving ability* and wrote for example ”Their capacity for solving problems is at a level that exceeds others” (T9), ”They manage to solve problems of pass-with-special-distinction character with great independence” (T18) and ”They have [...] a natural flair for solving mathematical problems” (T34). However, since there was no further information about what this ability meant to the teachers or what they considered to be a mathematical problem this characterization may refer to diametrically opposed traits such as the ability to reproduce school mathematics or the ability to creatively master mathematical problems, the latter described by Krutetskii (1976).

Earlier research characterizes gifted students by their ability to apply and to transfer mathematical ideas to new situations (Greenes, 1981). This was also emphasized by five teachers (15 %) when they referred to students' *ability to apply knowledge and connect it to other subject areas*. Examples of response given are ”Can connect the subject with knowledge from other subject areas and the everyday life” (T20) and ”They can apply their knowledge 'outside the ordinary frames of references' ” (T21).

Another ability stressed by Krutetskii (1976) is the *ability to grasp the formal structure* of a problem. Five teachers referred to this category, as they wrote ”They have a great ability to see what the problem is all about, they make solutions that exactly contain the relevant facts” (T1), ”Sort out the essential parts” (T20) and ”They have an analytical ability” (T33).

Non-cognitive attributes

More than half of the teachers referred to non-cognitive features, and the character trait most frequently (44 %) found in the head teachers' responses was *motivation*. Into this category answers that have to do with the students' attraction to and desire to work within mathematics are sorted. Thus, motivation includes features such as curiosity, persistence in work and digging beyond the surface of a problem. The teachers in this study stated for example ”They take a great interest in mathematics, curious” (T29) and ”Pose follow-up questions” (T6). Furthermore, as portrayed in previous research (Krutetskii, 1976; Winner, 1996), the teachers described students' keen drive to practice their area of interest and their intense attraction to mathematics, as they wrote ”They work hard in order to get better and better at mathematics” (T21), ”Want to proceed at a faster pace and learn more within the subject than the average student" (T19) and "They seek out challenges spontaneously" (T3). This characterization of giftedness, expressed by the teachers, might indicate an assumption that, no matter what is offered or required by mathematics education in school, gifted students will by themselves further their studies within the subject. Extensive research on gifted underachievers contradicts this assumption (Peters, Grager-Loidl & Supplee, 2000).

This latter characteristic may be closely related to the feature *independence*. However, in this study students' independence was clearly articulated in the teachers' responses, as shown by statements such as "Are independent in their work" (T22) and "[...] independently develop their own mathematical ability from the conceptions and methods everyone in the group is learning" (T9). It is possible that a manifestation of independence is a consequence of the gifted students' striving for freedom in their learning, enabling them to do mathematics as they wish themselves (Heid, 1983). But, it could be the case that gifted students in Sweden are left to their own as a consequence of teachers prioritizing the interaction with students who find mathematics hard to master.

There were three teachers that expressed a more troublesome view of gifted students, such as "Sometimes relatively low motivation for 'standardized teaching' – sometimes a bit 'hard to catch'" (T4) and "They don't want to work with easier routine problems" (T23). However, most character traits stated by teachers were expressed in a positive or neutral spirit which might indicate that head teachers' in general have a positive attitude to these students, and to these students' contribution to the class.

How are gifted students in mathematics detected?

In table 2 detection indicators of giftedness expressed by the teachers are presented. Fourteen head teachers (47 %) referred to *students' initiative*, which has to do with the students desire to finish courses in advance, actively participate in teaching, search for and create mathematical challenges and learn new material. Teachers wrote for example "They can come from compulsory school with grade/grades in one/some of the introductory courses at upper secondary school.[...] Others ask to take an examination of a course in advance in order to proceed at the next course" (T23), "Listen very carefully, discuss, question" (T16), "It can also be shown in that students [...] develop the tasks they are given or find new/seek out harder problems" (T9) and "They show interest during presentations of new subject areas" (T15). According to Krutetskii (1976) motivation for school mathematics is not a sufficient condition for being a capable student in mathematics. However, spontaneous formulation of

Table 2. *Detection indicators of gifted students in mathematics as stated by teachers*

Detection indicators	**Number (percentage) of teachers**
Students' initiative	14 (47 %)
Oral mathematical reasoning	11 (37 %)
Test results	10 (33 %)
Type of questions posed	8 (27 %)
Written solutions of problems	6 (20 %)
Swiftness	6 (20 %)

Notes. The number of teachers is 30. Each teacher may have provided more than one indicator.

problems and finding answers to these self-generated problems (Greenes, 1981), and striving to work intensively within their specific domain of interest, which may not coincide with school mathematics, are often emphasized attributes when characterizing capable or gifted individuals (Krutetskii, 1976; Winner, 1996).

Eleven head teachers (37 %) wrote that they detected giftedness through students' *oral mathematical reasoning*. Teachers stated for example ”How they solve problems orally” (T20), ”through discussions of mathematical problems” (T21) and ”by mathematical reasoning” (T33). It is not always clear what the quality measures are that makes the oral interaction a detection indicator. Nevertheless, teachers often stated that they notice the gifted students during their mathematical discussions. This could be explained by some researchers' description of gifted mathematics students as having a predilection for oral communication, given their speed in switching over from one train of thought to another (Greenes, 1981, Krutetskii, 1976).

High *test results*, on diagnostic tests as well as on ordinary tests, were also mentioned by ten head teachers (33 %) as an indicator of gifted students in mathematics. Teachers wrote for example ”almost maximal score on the diagnostic test” (T19), ”Is often evident from a diagnostic test taken at the beginning of their upper secondary school studies” (T36) and ”Test results show this inclination more clearly [...]” (T4). Internationally, written tests are a common instrument for identifying giftedness, but according to Swedish research, tests constructed by teachers at the Natural science program have a rather low call for *global creative reasoning* (Palm et al., 2005), thus limiting the likelihood of detecting the highly valued creative ability by looking at test scores. Also, it could be the case that diagnostic test results tell us little about a student's ability to profit and develop by future teaching, but reflects the quality of the students' previous learning environment. It is worth noting that six head teachers (19 %) specifically referred to product oriented methods to detect mathematical abilities as when they focused on the students' methods of solution when solving mathematical problems.

Eight head teachers (27 %) in this study detected gifted students' through the *types of questions* they pose. As expressed by teacher T9:

> Often, these students are noticed by the types of questions they pose. For example, they pose theoretical questions intended to explain a definition or to discuss a derivation. It can also be shown in that students pose questions that aim at extending a problem.

The nature of these questions remains to be further studied. However, research emphasizes that asking the right kind of questions is a driving force towards the development of mathematical knowledge (Scheffield, 2009).

Gifted students were also detected by their *swiftness* of thought or actions in the classroom, expressed by head teachers as for example "[...] swiftly incorporate their new knowledge in mathematics [...]" (T16), "Students are quickly done already in the first lesson" (T13) and "Calculate fast and effectively. Do not waste time on trivial tasks" (T8). Some researchers in *gifted education in mathematics* make a difference between working tempo and progress rate, and assert that working quickly is not necessarily a sign of giftedness (Krutetskii, 1976). However, some teachers clearly referred to rapidity in the students' learning processes and their efficiency in choosing which mathematical problems to work on in order not to spend time on material they already know. In other studies this has been considered to be typical of gifted students in mathematics (e.g. Wolfle, 1986).

Conclusion

This study gives evidence of a variety of conceptions of giftedness in mathematics among Swedish head teachers. The three most common character traits of gifted students expressed by the teachers are *creative ability*, *logical ability* and *motivation*. We notice that these traits coincide with the three clusters of personality traits presented by Mönks (1992). Thus the head teachers, as a group, have a "latent" awareness of these three well established and essential dimensions for giftedness to be manifested. The experienced and well educated teachers in this study did not stress all the specific mathematical abilities set up by Krutetskii (1976), as for example curtailment and mathematical memory. However, teachers referred to characteristics closely bound to these abilities, as the ability to generalize which is a premise for curtailment as well as memorization.

Head teachers used various indicators to detect giftedness, but most often students were detected by their own *initiative* for engaging in, or pursuing, mathematics. The teachers also used several different detection indicators involving quality aspects. The three most frequent were *oral mathematical reasoning*, *test results* and *types of questions posed*. Relying heavily on the students own initiative may imply that students with low self esteem or low profile is not identified as gifted students, and teachers leaning heavily on test result when recognizing high ability might not succeed in identifying gifted students who are underachievers. However, it is worth noting that such exclusions might be a consequence of the teachers staying true to their own overarching conception of giftedness.

In order to investigate potential connections between the character traits of giftedness and the ways of detecting those character traits we analyzed the possible relation between each of the certain types of characterizations of gifted students and the detection indicators of gifted students between teachers' answers. Results showed that there was no immediate relation between a particular characteristic and a specific detection indicator in the set of answers from the group of head teachers. For example, teachers that (at least) characterized gifted mathematics students as creative stated detection indicators from all types found in this study. So did teachers that characterized gifted students as (at least) motivated even if the two groups of teachers were not the same.

In Sweden, the results of this study are relevant to our mutual understanding of giftedness in mathematics. They may also contribute to the development of *gifted education* in Sweden, viewed as one small step towards understanding teachers' conceptions about gifted students in our country and how we can develop detection indicators to fit with our conception of what characterizes a gifted student. The study invites future research on how conceptions about giftedness influence teachers' actions, how schools are organized and what teachers do to encourage the manifestation, development and creation of abilities highly valued by the teachers in this study. Such questions are of immediate interest not only to the current discussion on the creation of special educational programs for able students, but also to the establishment of gifted education in Sweden.

Despite the absence of a debate on conceptions of giftedness in Sweden and its long history of egalitarianism in school and society, this study offers findings that are consistent both with international studies of teachers' characteristics of gifted students and with studies focusing directly on gifted students' character traits. These results underline the relevance of international findings in gifted education for the Swedish discussion in the field and vice versa. However, the reliability of the study ultimately rests on the comparison with similar national studies.

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Notes

1 The series of questions of relevance to this study are

20. In the heading the label "gifted student in mathematics" is used, but what wording do you usually use in school when talking about these students?

21. How would you characterize the students you denoted in question 20?

22. Are there any gifted students in mathematics in your classes?

    O Yes O No (Proceed to question number 39)

23. How do you detect these gifted students?"

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