

Developing mathematics teaching through inquiry

A response to Skovsmose and Säljö

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This paper constitutes a response to the article by Skovsmose and Säljö (2008) in *NOMAD*. We focus on the concept of *inquiry* as used in the KUL projects at the University of Agder, Norway, 2004–2007, from which Skovsmose and Säljö offered an evaluation and critique. We begin by clarifying certain aspects of the two KUL projects, *Learning communities in mathematics* and *ICT in mathematics learning*. In doing so, we agree substantially with several of the points made by Skovsmose and Säljö. We go on to address their two main criticisms: that research in the KUL projects shows little documentation of inquiry processes or patterns of classroom interaction between teachers and students, or among students; and that the KUL projects demonstrate few attempts to use real life environments as a basis for establishing inquiry processes. Finally we come back to significant issues related to inquiry and the main focus of the two projects, further research questions and relations between the micro and the macro in mathematics education research.

The notion of inquiry has been central to our activity as mathematics educators separately and jointly for around 20 years (Fuglestad, 1992, 1999; Jaworski, 1992, 1994; Jaworski et al., 2007). In the case of Fuglestad, early activity involved exploration of the contribution of technology in various forms to the learning and teaching of mathematics. In the case of Jaworski, early activity included the roles of investigational activity in mathematics learning and teaching in classrooms and the development of teaching through collaborative inquiry between teachers and university academics. Our recent work together in the *KUL projects*² has involved a combination of all of these. In this paper, we are responding to an article by Ole Skovsmose and Roger Säljö in *NOMAD* (2008), entitled *Learning*

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mathematics through inquiry. In their article, Skovsmose and Säljö write from their position as invitees to provide a formal evaluation of the KUL projects at the University of Agder, Norway. We should like to emphasise, before going further, our sincere appreciation of their evaluatory research and the subsequent report that they produced (Skovsmose & Säljö, 2007). Their article to which we refer was also invited as a consequence of their appreciation of the substance of the KUL projects and we were able to respond to an early draft. In this paper, we make a three-fold response to the published version.

1. We clarify from our own perspectives some of the substantive issues from the KUL projects, to which Skovsmose and Säljö (2007, 2008) refer.
2. We respond to their two major criticisms of the nature of inquiry within the KUL projects, and more generally.
3. We take further the ideas about inquiry which these considerations have raised.

In the case of (3), we wish to thank Skovsmose and Säljö for giving us this opportunity to extend our own knowledge and visions.

Inquiry in the KUL projects

The main focuses of the KUL projects

The KUL projects were entitled *Learning communities in mathematics* (LCM) and *Information and communications technology in mathematics learning* (ICTML). The former set up the basic philosophy for the two projects which was to create a learning community, between teachers of mathematics in schools and didacticians of mathematics in the university, to develop mathematics teaching in school classrooms. Fundamental to creating such a learning community were notions of *developmental research* and *inquiry*. Briefly, developmental research is research which not only studies and documents development, but which contributes fundamentally to development through the research activity (Goodchild, 2008; Jaworski, 2003). *Inquiry* is seen as asking questions and seeking answers, recognizing problems and seeking solutions, wondering, exploring, investigating and looking critically at what we do and what we find (Jaworski, 1994). It builds extensively in mathematics education on the work of Polya (1945) and the problem-solving movement of the 1980s (Mason, Burton & Stacey, 1982; Schoenfeld, 1985). Inquiry could be seen

to engage learners with mathematics, creating interest and enjoyment, and motivating conceptual understanding (Collins, 1988).

Important for our response to Skovsmose and Säljö is that the KUL projects focused primarily on the teaching of mathematics and its development rather than on mathematics learning per se (Jaworski, 2005). The ICTML project focused particularly on the use of technology in the classroom teaching of mathematics and inquired into how teachers could use technology as part of their design of mathematical activity for their students (Erfjord, 2008; Fuglestad, 2007). Thus, although the creation of better learning environments for students to learn mathematics in classrooms was central to both projects, the main focus was on the teaching of mathematics and its development and on the use of technology as a part of this teaching.

Inquiry in the KUL projects

The KUL projects focused on inquiry in three layers, or levels.

1. Inquiry in mathematical tasks for pupils' mathematical learning in classrooms.
2. Inquiry in the developmental process of planning for the classroom and exploring how to create better learning environments for pupils in mathematics.
3. Inquiry in the research process of systematically exploring the developmental processes involved in (1) and (2) above.

(Jaworski, 2007a, p.15)

Central to project activity was the design of tasks for engaging in mathematics, firstly for teachers and didacticians in project workshops and secondly for pupils in classrooms. Regular workshops, attended by all teachers and didacticians in the projects, included mathematical tasks on which didacticians and teachers worked together and which promoted further discussion on the use of inquiry-based tasks in classrooms. Teachers often took tasks from project workshops and adapted them for their pupils (Jaworski, 2007a). This adaptive process was central to both developmental research and the inquiry which permeated KUL activity. Project workshops were mainly designed by didacticians and involved participation and interaction between teachers and didacticians in an inquiry mode. Didacticians' design of tasks for workshops could be seen to parallel teachers' design of tasks for classrooms. Both groups, didacticians and teachers, engaged in inquiry within their respective

design processes through an "inquiry cycle" of design, action, observation, reflection and feedback (Jaworski, 2007b; Skovsmose & Säljö, 2008). It was central to development activity in the projects that both groups were involved in inquiry and design. We talked about using inquiry "as a tool" to promote "inquiry as a way of being" in our learning and teaching (Jaworski, 2007b, p. 127). Thus our main unit of analysis in these projects was on *inquiry as a developmental tool* leading to *inquiry as a way of being in practice*, with focus on the development of teaching for pupils' more effective learning of mathematics.

We agree strongly with Skovsmose and Säljö that "inquiry processes must be understood as interactional achievements and as parts of the joint construction of meaning. So, if one wants to document that an inquiry process has taken place, in-depth analyses of interactional processes are necessary" (2008, p. 39). The interacting participants in the KUL projects are didacticians (D) and teachers (T), as well as teachers and their students (S). So, to chart inquiry processes in the projects we analyse dialogue from interactions as follows: $D \leftrightarrow T$; $T \leftrightarrow T$; $D \leftrightarrow D$ (where \leftrightarrow means "interacting with") and of course $T \leftrightarrow S$. However, we see the latter interactions ($T \leftrightarrow S$) as a consequence of the other three, rather than as primary in their own right. This is relevant in addressing the first of the criticisms tackled below.

An inquiry culture versus an exercise culture

We agree, again strongly, with Skovsmose and Säljö in their reference to "an exercise paradigm" as dominant in the culture of mathematics classrooms widely. They write:

This [the *exercise paradigm*] implies that the activities engaged in the classroom to a large extent involve struggling with pre-formulated exercises that get their meaning through what the teacher has just lectured about. An exercise traditionally has one, and only one, correct answer, and finding this answer will steer the whole cycle of classroom activities and the obligations of the partners involved.
(Skovsmose & Säljö, 2008, p. 40)

They write, a little later (p. 40),

The ambition of promoting mathematical inquiry can be seen as a general expression of the idea that there are many educational possibilities to be explored beyond the exercise paradigm.

The promotion of inquiry in the KUL projects may be seen in these terms although never actually expressed in this way. An inquiry mode,

for both didacticians and teachers, involved seeking new visions for classroom mathematics which led to more open questions and tasks and less concentration on narrowly focused instruction.

The didacticians designing the projects wanted to promote a developmental research approach to mathematics teaching in which didacticians and teachers together would explore possibilities for classrooms. This had to start somewhere, and it began with inquiry-based mathematical tasks, created for workshops by didacticians (Jaworski, 2005). Interactions between didacticians and teachers led to teachers designing tasks for their classrooms, often as adaptations of workshop tasks (we provide examples below). A characteristic of such tasks was that they encouraged participants, whether didacticians, teachers or students, to engage with mathematics, ask their own questions, and decide their own directions for inquiry (Jaworski, Goodchild, Daland & Eriksen, in press). Thus, by their very nature such tasks avoided the "exercise paradigm". In using and developing such tasks, there was a clear challenge to the exercise paradigm in project classrooms. An aim of the projects was to develop a community of inquiry between didacticians and teachers to promote "inquiry as a way of being" – i.e. to promote an inquiry culture which would influence activity in classrooms. Such an inquiry culture would be a challenge to an exercise culture if such a thing could be seen to exist. The degree to which this challenge led to new practices varied according to school level, with the higher secondary schools being least willing to change the exercise culture to incorporate inquiry-based tasks.

The tasks that were designed and used came from a range of sources, either in the published literature, or in the experience of didacticians and teachers in the project. Many of these came from within mathematics itself, or used real world situations to create opportunities for engaging with mathematics. For example:

An example of a task within mathematics:

Given a number, such as 10, write the number in different ways as a sum (e.g. $10 = 1 + 2 + 3 + 4 = 2 + 2 + 2 + 2 + 2 = 5 + 5$).
For each sum, find the product of its elements and explore what is the largest product you can find.

An example of a task from a real-world situation:

A square picture is framed using framing material 1cm wide. What length of framing material is needed for any particular size of picture?

Further examples of KUL tasks can be found in Skovsmose and Säljö (2008, p. 43). The tasks used had to be seen to contribute to topics within the mathematics curriculum used by teachers at the different levels. The projects were set within the Norwegian educational system, in

Norwegian schools, and with responsibilities on teachers to work within the Norwegian national curriculum (Kirke-, utdannings- og forskningsdepartementet, 1999). It was not a purpose of the projects to adapt or change this curriculum, but to find ways of working within it – perhaps to provide “educational possibilities to be explored beyond the exercise paradigm” (Skovsmose & Säljö, 2008, p. 40).

The mathematics curriculum

Before going to our next section, it seems appropriate to say a few words about the mathematics curriculum as extant in Norway. In their arguments in the article, Skovsmose and Säljö (2008, p. 44), with reference to Dewey (1966), write: “The aim of an inquiry-based mathematics education becomes to bring students into mathematics and to make students appreciate mathematics”. We agree with this aim and believe it is highly relevant to activity in the KUL projects. Skovsmose and Säljö go on to speak of the “modern conception of mathematics” which “dominated the Modern Mathematics Movement, which was initiated during the late 1950s, and which concentrated the teaching and learning of mathematics within the structures of mathematics itself” (p. 44). Although they offer no references to support these statements, their use of terms here leads us to interpret their words as referring to a modern mathematics curriculum which we both experienced in the 1970s (in places as diverse as Norway (Fuglestad) and The Philippines (Jaworski) where we worked at this time). This was rooted in a set-theoretical approach to mathematics (Kirke- og undervisningsdepartementet, 1971, 1974).

The idea of modern mathematics was introduced in Norwegian schools in the 1970s, with a new curriculum plan. The intention was to replace the existing curriculum plan from 1939 with a new plan for 9 years compulsory school. In a new plan, introduced temporarily in 1971, two versions of mathematics were presented, the second being an optional alternative with elements and the form of presentation from modern mathematics visible (Gjone, 1985). This included set theory, logic, and extended use of mathematical symbols and formalism together with examples of Venn diagrams, truth tables and lists of symbols (Kirke- og undervisningsdepartementet, 1971). The local school board could decide which alternative to use.

In teacher education, new mathematics books with “modern mathematics” of this kind were introduced at different levels depending on the degree of specialism in mathematics, e.g. Stoll (1963) was used as part of a one year specialisation over three years, and Alvin & Anderberg (1968) was used for a shorter course in teacher education. In addition, courses

and a series of TV programmes taught modern mathematics, based on set theory and logic, to teachers and interested others (Gjone & Onstad, 2000). These events contributed to a conception of modern mathematics for Norwegian teachers that mathematics uses a lot of symbolism and difficult/formal language and created a lot of discussions concerning the mathematics in schools. The modern mathematics influence on the curriculum was considerably modified in the permanent curriculum from 1974, visible as "supportive concepts from set theory and logic"³ (Kirkeog undervisningsdepartementet, 1974, p. 143), and was replaced entirely in 1987, by a new curriculum with emphasis on problem solving and use of computers as a new topic (Utdannings- og forskningsdepartementet, 1990).

Our intention in the references above is to emphasise that curricula evolve over time and teachers, at any stage in history, have responsibility to interpret the curriculum as it stands. The so-called modern mathematics curriculum was not different in this respect. However, few remnants of this curriculum remain today. In all of these curricula the teaching of mathematics focused, perhaps unsurprisingly, on mathematics itself. It was up to teachers to decide how to approach this mathematics, particularly in terms of the sorts of examples they used and how they used them. Text books were influential in guiding teachers' choices of such examples (e.g. Melhus, Guldbrandsen & Løchsen, 2007; Oldervoll et al., 2009a, 2009b; Torkildsen & Maugesten, 2006).

Responding to criticisms

We see the article of Skovsmose and Säljö as offering two principal criticisms of the KUL projects. The first is, to quote:

[I]t is a surprise to us that there are only a few cases where substantial documentation of inquiry processes or patterns of classroom interaction between teachers and students, or among students, has been provided. (Skovsmose & Säljö, 2008, p. 38)

And the second follows later in the article, to quote:

In the KUL projects, we see a clear dominance of landscapes of investigation which refer to mathematics domains or to invented examples where real world events serve as background illustrations for mathematical exercises such as in the case of word problems. We see only few attempts to use real life environments as a basis for establishing inquiry processes. (Skovsmose & Säljö, 2008, p. 43)

We tackle each of these in turn below.

Patterns of interaction in inquiry processes in the KUL projects

Skovsmose and Säljö, in their criticism, refer to interactions between teachers and students and between students in mathematics classrooms (i.e., $T \leftrightarrow S$ and $S \leftrightarrow S$). They take no account of interactions involving $D \leftrightarrow T$; $T \leftrightarrow T$; or $D \leftrightarrow D$. However, within our prime unit of analysis, these latter interactions take precedence over the former. Such interactions have been extensively analysed and documented. We provide a few examples here.

Early interactions between didacticians ($D \leftrightarrow D$), in meetings in which project planning took place, were documented in Cestari, Daland, Eriksen and Jaworski (2006). The particular focus here was on the ways in which didacticians should interact with teachers, either in workshops or in schools. Many didacticians were, or had also been teacher educators, in which their role was to guide or advise teachers. Analysis of dialogue revealed that such a role was thought to be inappropriate in the KUL projects. In particular, a didactician's role in a small group in a workshop was discussed: it was felt that the didactician should be a participant but not a leader or coordinator. A role as "facilitator" could be acceptable.

In two articles (Goodchild & Jaworski, 2005; Jaworski & Goodchild, 2006), teachers' representations to didacticians, regarding respective roles in the projects, were analysed ($T \leftrightarrow T$ and $T \leftrightarrow D$). Some teachers had felt that the projects did not take sufficient account of their own interests and desired outcomes from the projects. These views were documented and analysed using Activity Theory perspectives (Engeström, 1999; Leont'ev, 1979).

An important event, early in the life of the projects, involving interactions between teachers and didacticians ($T \leftrightarrow T$ and $T \leftrightarrow D$) arose from a request by teachers in one higher secondary school for help from didacticians in planning inquiry-based tasks relating to linear functions. Meetings took place between teachers and didacticians to discuss the concept of linear functions and discuss what tasks might be possible. Subsequently, teachers designed tasks and used them with pupils in three classrooms, recorded on video by didacticians. Analysis of the video material, of a reflective meeting between teachers and didacticians, and of an oral report by teachers at a workshop led to three articles in which analyses were documented and reported (Fuglestad, Goodchild & Jaworski, 2007; Hundeland, Erfjord, Breiteig & Grevholm, 2007; Jaworski, 2007c).

The KUL Book, *Learning communities in mathematics* (Jaworski et al., 2007), contains chapters in both Norwegian and English relating to many aspects of the KUL projects. Many chapters within this book document interactions between teachers, between didacticians and teachers, and in some cases between teachers and students (for example, Bjuland, Cestari

& Borgersen, 2007; Daland, 2007; Erfjord, 2007; Goodchild, 2007). The focuses of these interactions are diverse.

Fuglestad (2007) documents how interactions between teachers and didacticians (T ↔ D), including didacticians' engagement with pupils' work as participating observers, promote development of computer based tasks to stimulate pupils' inquiry related to fractions, percentages and decimal numbers. The inquiry followed the steps of a developmental cycle with the elements plan, act, implement, observe, reflect and feedback, with a new cycle starting as ideas for improvement and further development were generated from the inquiry.

More recent publications in which dialogue between teachers and between teachers and didacticians (T ↔ T and T ↔ D) is analysed are Erfjord (2008) and Bjuland and Jaworski (2009). Erfjord, in his PhD thesis, offers detailed analyses of lower secondary teachers' dialogue in talking about their use of technology in their mathematics teaching. Principally, their focus is in their initial use of the software *Cabri géomètre* and issues arising from their planning and classrooms experiences. Erfjord analyses such dialogue using Activity theory and Instrumentation theory (Trouche, 2005a, 2005b). Bjuland and Jaworski analyse data from focus group interviews between didacticians and teachers after two years of the KUL projects. These analyses reveal teachers' perspectives on the projects and on their participation in the projects.

These various examples have all used data collected during the projects and analysed dialogue from interaction, mainly between didacticians and teachers. Since the central activity of these participants was inquiry into the teaching and learning of mathematics, they all document analyses of inquiry processes within the projects. It can therefore be seen that, in accord with the main unit of analysis in these projects, extensive analysis of interactions took place. That these were, in the main, not interactions between teachers and pupils in classrooms, is not a fair criticism. However, we acknowledge that extending data capture and analysis to interactions in classrooms more widely could have been extremely valuable. Of course any project has to focus, and no project can do everything that mathematics educators widely would like to see.

The dominance of landscapes of investigation

Skovsmose and Säljö (2008) introduce the term "landscapes of investigation" with which we are familiar from earlier work by Alrø and Skovsmose (2002), referred to extensively in Skovsmose and Säljö's article. They indicate that a landscape of investigation "refers to a learning milieu different from those structured through exercises" (p. 41) and they mention three

types of landscape: those located within mathematics, those that include references to non-mathematical domains and those that include real-life references. They provide an example in each of these domains and also refer to documented examples from the KUL projects which might be seen as providing landscapes of investigation. Their judgment on the KUL projects, quoted above, suggests that KUL landscapes fit within only the first, or possibly the second of the three domains. They conclude:

It appears that the KUL projects have operated within a rather narrow set of landscapes for mathematics learning. We find this to be a problematic limitation of the scope of the inquiries, [...].

(Skovsmose & Säljö, 2008, p. 43)

They go on to discuss "Alternative conceptions of mathematics and inquiry". In this they draw on Dewey's (1966) perspective on inquiry "as a principle of education that is grounded in people's experiences of living in a complex world" (Skovsmose & Säljö, 2008, p. 44). They contrast Dewey's perspective with "a modern conception of mathematics" which we have tried to locate historically (see our discussion above). As we understand their argument, a "modern" conception seeks to locate inquiry within mathematics itself, dependent fundamentally in a rationality that is rooted within mathematics and mathematical structures rather than in real world situations and problems. Such isolating of mathematical inquiry within mathematics allows mathematical rationality to develop independently of its relation to the big issues of the real world and to be used indiscriminately for good or evil. Thus, they extemporize to horrific uses of mathematics as pointed out by D'Ambrosio (1994), that "we experience the whole spectrum from wonders to horrors when mathematics is put to use" (Skovsmose & Säljö, 2008, p. 45).

We struggle somewhat to relate this big debate about mathematics to our KUL projects. On the one hand there are the so-called "landscapes of investigation" (not our term) within the KUL projects. On the other hand is the contrast between what is referred to as "modern conceptions of mathematics" and the Deweyan notion of inquiry "as a principle of education that is grounded in people's experiences of living in a complex world". We try to take these issues together.

One example from a teacher's work in the KUL projects, not quoted by Skovsmose and Säljö, is referred to in Harstad, Heggem and Sandberg (2007), a chapter written by teachers and published within the KUL book. Here, a teacher at grade 3 presents a task from her classroom focusing on geometry. She used a geometrical "blind man's buff" to introduce measurements involving short and long steps and changes of direction. Children were blindfolded and had to move according to instructions

from their peers. The activity went further to focus on planning and estimating how many steps to move in relation to a prescribed route. It took place in the school playground and continued later in the classroom moving on from practical activities into recording measures on paper. Further development led to inquiry into relations between long and short steps, tabulation of results and development of the activity to spot parts of a multiplication table. The activity as a whole showed interactions between the teacher, her pupils and a didactician who was visiting the school (Fuglestad, 2009).

We might characterize this activity in terms of a landscape of investigation involving a real world situation and its mathematical interpretation. We think that it probably fits into Skovsmose and Säljö's second category of landscapes that include references to non-mathematical domains. We would not place it in the third category because it does not involve the use of mathematics to address a real world problem. The "problem" is contrived to enable the addressing of the mathematical concepts of distance and angle. However, the children are able to enter the landscape and to formulate their own questions. A video, recorded by the didactician is available to provide evidence of children's involvement and a finer-grained analysis of their dialogue if desired. There is evidence of children's enjoyment of their activity and of understanding of the subsequent mathematics. As far as we are aware, a finer-grained analysis has not (yet) been done to substantiate the nature of learning and relations between the activity and the emerging mathematics.

We can make an argument here that this inquiry activity, directed as it was towards certain mathematics as required by the Norwegian curriculum, allowed the children to engage, to enjoy their engagement and to learn some mathematics. Other tasks, as referenced by Skovsmose and Säljö (2008, p.43), could be argued to have similar effects. The extent to which the (pseudo) real world aspects of the tasks were dismissed by students at their point of entering a mathematical world is something on which we have no evidence, but might be addressed as a research question in the future.

We return to the distinction between a "modern" and a "Dewyan" perspective. Of relevance seem to be the following considerations. Schools and teachers have responsibility to attend to the national curriculum in their teaching. This curriculum requires certain mathematical topics and concepts to be addressed. As has been pointed out there are numerous ways of addressing such concepts other than an "exercise" approach. Inquiry in classroom tasks seeks to engage participants, to encourage dialogue, and to develop understanding of mathematics. Inquiry in teaching seeks to design suitable tasks, to try them out in classrooms and observe

outcomes, and to analyse the process and its outcomes. Given that teaching is a real-world process, the participants in this activity, the teachers and didacticians, gain insights into educational issues and develop important awarenesses about the educational process. These awarenesses feed back into future action and allow more informed action (Mason, 2002). This real-world scenario might not parallel grave incidents in atomic power stations on a humanitarian scale, but they are nevertheless central to educational principles of which Dewey speaks.

Further considerations on inquiry

Given that "landscapes of investigation" were neither a term used nor a concept in the KUL projects, we now put them on one side and return to the projects themselves and their aims. The projects were designed to enable didacticians and teachers to learn more about approaches to teaching mathematics through inquiry. We can see this in two ways:

- a) to learn (through inquiry) more about approaches to teaching mathematics, and
- b) to learn more about approaches to (teaching mathematics through inquiry).

One of the outcomes from the KUL projects was a recognition of power differences between didacticians and teachers (Jaworski, 2005, 2008) and their influence on activity in the projects. Despite a rhetoric of collaboration and partnership, it had to be acknowledged, at least in the early phases of the projects, that the power in design and decision-making rested with the didacticians. However, it gradually became clear that teachers would and could do only what their established school communities would or could support. Thus considerable power rested with the teachers. Didacticians had to learn about issues and conditions in schools and how schools and teachers were able to respond to design and decision-making by didacticians. Such learning was highly significant to these projects. It formed an important basis for the conceptualisation of a new proposal to the funding council, the TBM project (Teaching better mathematics). This project was funded as a further development to LCM and ICTML, and is ongoing to the end of 2010. A major difference in the new project was that school leaders and teacher leaders within schools were part of the formulation of the project from the schools' point of view from the beginning and schools obtained their own funding for the project. So, the new project was more genuinely a partnership in terms of the initial design and its implementation.

Thus, the inquiry basis of the original projects working its way through collaborative activity between didacticicians and teachers led to important new learning at a range of levels. These levels pertain to *macro* and *micro* elements within the projects (Jaworski & Potari, 2009; Lerman 1998). At the macro levels, we see established communities of schools and university and their respective practitioners forging relationships to inquire together into mathematics teaching and learning in schools. At micro levels we see small groups and individuals working on aspects of mathematics learning and teaching, designing tasks, trying out tasks in workshops or classrooms, and analysing outcomes. Inquiry has permeated all these levels and contributed to learning by individuals, groups and the project community as a whole. This learning has gone considerably beyond the learning of mathematics, although mathematics learning has been at the centre of it all.

The criticisms of Skovsmose and Säljö (2008) lead to research questions which can valuably be taken up in future projects. The following can be seen as indicative:

- 1) What is the nature of interactions between teachers and students, and between students themselves, that result from the design and use of inquiry tasks in mathematics classrooms? In what ways can students' work on these tasks be seen to contribute to their developing understandings of mathematics?
- 2) How can inquiry in mathematics classrooms be directed more towards real world issues and related critical thinking on a humanitarian scale? What changes to the curriculum are required to make such inquiry a viable approach to the mathematical learning desired by society and the educational establishment?

Skovsmose and Säljö quote Säljö and Wyndham (1993) in saying:

And we must not forget that a dialogue is embedded in institutional traditions of what it means to communicate, learn and know in the classroom.
(Skovsmose & Säljö, 2008, p.38)

The KUL projects have demonstrated clearly the force of such a reminder. We believe that it has been important that these projects have not become bogged down in the minutiae of the micro. While micro considerations are clearly of importance, trying to maintain a macro scale is essential to the depth of understanding of the big issues in developing mathematics teaching and learning. Our exchange of views with Skovsmose and Säljö is one more contribution to these understandings.

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Notes

- 1 At the time of these projects Barbara Jaworski was also employed at the University of Agder, Norway.
- 2 The Research council of Norway (NFR): <http://www.forskningsradet.no/>
The KUL programme: <http://www.forskningsradet.no/servlet/Satellite?c=Informasjonstekst&pagename=utdanning%2FHovedsidemal&cid=1224697827586>
The Agder projects: <http://fag.hia.no/lcm/> and <http://fag.hia.no/ikttml/>
- 3 In Norwegian: *hjelpbegreper fra mengdelære og logikk*

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Barbara Jaworski is professor of mathematics education in the Mathematics education centre at Loughborough University, UK where she teaches in mathematics and mathematics education. Before this she worked at the University of Agder, Norway in doctoral education and at the University of Oxford in teacher education. She edited the international Journal of Mathematics Teacher Education for 6 years, was one of the editors of the first *International handbook of mathematics teacher education* (2008), and was President of ERME (European society for research in mathematics education) from 2005–2009. Her research principally is into the teaching of mathematics at all levels (currently at university level) and in the development of teaching in which research is a developmental tool. The use of "inquiry" in collaborative processes between teachers and researchers is central to her work in exploring approaches to teaching and their contribution to students' learning of mathematics.

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