# Nordic collaboration in mathematics education research

## The Nordic collaboration continues

The funding of the Nordic Graduate School in Mathematics Education (NoGSME) has come to an end but the Nordic collaboration continues. In 2003 NordForsk decided to support a Nordic graduate school in mathematics education with 5 Million NOK over a period of five years. The activities started in January 2004 and thanks to good economic planning the board managed to get the money to last for six years. Thus in April 2009 the final seminar for supervisors took place in Kristiansand and the final summer school for doctoral students took place in Denmark in September. A number of doctoral courses and conferences have been offered during 2009 to all students and supervisors by announcement via the email lists. The board had its final meeting in December in Lund and the chair of the Nordic Society for Research in Mathematics Education, NoRME, took part in that. One of the aims of NoRME is to ensure the survival of the NoGSME network and the kind of activities that has been built up during the NoGSME years of Nordic collaboration, and thus the meeting acted as a kind of hand over of the baton for leading the Nordic collaboration (see Grevholm, 2009a). NoRME will take care of keeping the email lists updated and make them available to members of the Nordic network. Thus announcements of courses and conferences can still be made to all members of the NoGSME network. NoRME will also initiate new applications for funding of Nordic summer courses and other doctoral courses and seminars. The database created by NoGSME will be run by NoRME. In addition to that NoRME will act as a network for the national societies for research in mathematics education in the Nordic and Baltic countries. See the NoRME-webpage at www.norme.me

NoRME is currently led by Frode Rønning, professor of mathematics working at NTNU in Trondheim. He has earlier worked at Sør-Trøndelag University College and initiated the master programme in didactics of mathematics there some years ago. He is active in and publishing research

**Barbro Grevholm**
*University of Agder*

Grevholm, B. (2009). Nordic collaboration in mathematics education research. *Nordic Studies in Mathematics Education*, 14 (4), 89–100.

in didactics of mathematics. Frode has promised that NoRME will continue to produce the traditional article about Nordic and Baltic collaboration in each issue of NOMAD, which started in 2004 via NoGSME. Thus this is my last message to you in NOMAD in the name of NoGSME and I am very pleased to hand over the responsibility to NoRME and Frode Rønning.

## Final report to NordForsk about NoGSME

The board has sent its final report to NordForsk about the activities in NoGSME. Here are some parts of this report summarising the activities during 2004–2009.

The NoGSME network consists of about 40 institutions in the Nordic and Baltic countries and over the years more than 130 doctoral students have been included and around 140 supervisors. Over the years 2004–2009 we estimate that more than 50 doctoral students successfully defended their dissertations and entered as prospective supervisors.

Many of the students have received travel support to courses and thus been enabled to take courses at several different universities. About ten students have applied for mobility stipends and were grateful to receive them and for the opportunity to study abroad for a month. A NoGSME webpage has functioned as center of information (see www.nogsme.no). Here is a selection of the doctoral courses that have been offered via NoGSME over the years:

- Theory of science from a mathematics education perspective.
- Meta-perspectives on mathematics and the learning of mathematics in a technological environment.
- History of mathematics with emphasis on modern mathematics.
- Theoretical aspects of mathematics education with emphasis on the *French school*.
- Problem solving in mathematics education research.
- Methodology in mathematics education research.
- Theories of learning and teaching mathematics.
- Research design and research methods in mathematics education.
- Perspectives on *Identity* in learning and education research.
- Views of knowing and learning: constructivism and socio-cultural theory.

– Gender and mathematics education.

– Justification in mathematics and science education research with special emphasis on the role of theory in justification.

– Research on assessment in mathematics education.

– Conceptions of mathematics.

– Mathematical literacies: construction of concepts and the implication for design of research and education.

– Learning, thinking and communicating: development and controversy in understandings of "coming to know" in mathematics education research.

Five workshops on different important themes have taken place and in one case led to the creation of a new network of researchers on mathematics textbooks. Themes of the workshops were:

– Classroom research in mathematics education (Goodchild, Jaworski, & Måsøval, 2007).

– Research on textbooks in mathematics.

– Research in use of ICT in mathematics education.

– Research on mathematics and language.

– The use of ICT in mathematics education – neither salvation nor catastrophe? What can we learn from research and what are our conclusions? (Grevholm, 2009b).

The first summer school in 2004 was organised in collaboration with YERME. Another five summer schools have taken place, alternating in all the five Nordic countries. The summer schools attracted between 25 and 35 participants. Eleven seminars for supervisors were separately organised in order to systematically offer competence development for active and prospective supervisors. Between 25 and 38 supervisors took part of the seminars. These were the places and titles of the seminars:

– September 2004 in Vasa – *Quality in research in mathematics education*.

– April 2005 in Korsør – *Quality of theses in mathematics education*.

– September 2005 in Trondheim – *Supervision of doctoral students*.

- November 2005 in Lund – *Reviewing of papers in mathematics education.*
- May 2006 in Vasa – *Research programmes in mathematics education.*
- October 2006 in Magleås – *Critical situations in supervision of doctoral students in mathematics education.*
- February 2007 in Trondheim – *The review process of papers for scientific journals.*
- October 2007 in Lund – *Quality issues in mathematics education research.*
- April 2008 in Tallin – *Publishing policies of scientific journals and criteria.*
- October 2008 in Gentofte – *Local, regional and international perspectives on mathematics education.*
- April 2009 in Kristiansand – *Critical reviewing of methodologies in mathematics education research.*

Additionally in Denmark one seminar for supervisors was co-organised with one of the doctoral courses with the theme *Justification in mathematics and science education research with special emphasis on the role of theory in justification*. Another seminar was primarily intended for the Baltic supervisors and took place in Riga in 2007 and dealt mainly with academic writing and publishing in scientific journals.

In both the summer schools and the seminars a number of important international scholars in mathematics education have participated and contributed at the most excellent level of performance. The willingness from the international colleagues to contribute has been crucial and is probably to be found in the fact that NoGSME from the very start established cooperation with five important centers of excellence in mathematics education around the world.

Both doctoral students and supervisors express how valuable the work of the Nordic Graduate School is to them. A growing network of students and supervisors has come to reality. The Nordic cooperation will continue through NoRME and the informal network and many in the network are working to make it viable for the future. We want to express to NordForsk the gratitude from all involved for the funding of NoGSME and all the opportunities it opended for development of the cooperation in Nordic research in mathematics education. The effects of this funding will probably be seen for many years to come.

## A Nordic summer course for doctoral students in May 2010

In May 2010 a Nordic summer course for doctoral students in mathematics education will take place in University of Agder, Norway. NordForsk has granted the course, which is given in collaboration between five Nordic universities. The second announcement was sent out in January 2010 and students are invited to register. There will be 30 participants and the programme will follow the traditions from earlier summer schools given by NoGSME. International scholars will contribute to the course, among them Jo Boaler from Sussex University. Students who take part of the examination of the course will be rewarded by 7,5 ECTS.

## New doctoral dissertations in the Nordic countries

Anna Klisinska presented her work, at Luleå University of Technology on May 29, with the title *The fundamental theorem of Calculus – a case study into the didactic transposition of proof.* A fundamental question in mathematics education is the relationship between academic mathematics at universities and classroom mathematics (the mathematical practices in classrooms at all levels including undergraduate university education). The study investigates the relationship seen from the perspective of mathematics education and by researching mathematicians. The fundamental theorem of Calculus (FTC) and its proof provide an illustrating example. The propositional content of the statements connected to FTC varies and the proofs differ. The formulations of different versions of FTC are seen from historical and institutional context. The study comprises a historical account of the invention of the FTC and its proof and appearance in calculus textbooks. Interviews with eleven researching mathematicians from different sub-fields (9 from a university in Canada and 2 from a university in Sweden) provide an image of what meaning and relevance they attribute to the FTC. The outcomes of the historical account and the data about the mathematicians' views are discussed from the perspective of the theory of didactic transposition. The study implies reflections about specialisation of knowledge and the distance between the producers and distributors of mathematical knowledge.

Elisabeth Persson defended her thesis in the Department of Didactical Science and Early Childhood Education at Stockholm University in October 16. The title is *It takes time: from teacher student to mathematics teacher* (in Swedish). The purpose is to investigate how future pre- and primary school mathematics teachers change their approaches to mathematics and mathematics education during their subject studies and how this view affects their teaching of mathematics after graduation. The method was qualitative interviews combined with

observations, notes, sound recordings and video recorded mathematics classes and materials produced by the teachers. Institutional theory was the theoretical framework and in part two of the study a design theoretical perspective was added. The first part of the investigation indicated that the language used by the students is changing during the studies. The observations later showed that for 80 % of the teachers there exists a clear relationship between the kind of teaching they claim to perform and the teaching they actually perform. One year after the graduation the teachers describe that they feel well prepared for teaching mathematics in preschool and primary school. Directly after graduation they were dissatisfied because of limited emphasis in concrete recommendations and "tips". They claim that in practice it turned out that their education provided more stable and secure foundation than they described shortly after graduation. They developed knowledge and skills that enabled them to be better prepared for future work roles than they believed themselves to become.

Lili-Ann Sackerud Kling defended her thesis, at Umeå University on November 16, with the title *Students' opportunities to assume responsibility for their own learning with regard to mathematics: a classroom-based study in a postmodern era* (in Swedish). The study investigates compulsory school students' opportunities to assume responsibility for their own learning in mathematics. How the school system in general and the teaching of mathematics in particular are organised and carried out was studied initially. There was also an intention to understand the social circumstances affecting students' opportunities to assume responsibility for own learning. The empirical material consists of interviews and observations and forms a case study with students from school years zero to nine in compulsory school. Additionally the way students, teachers and school leaders express and implement the idea of students assuming responsibility for their own learning in mathematics is studied. The theoretical starting point was the concept of phenomenological lifeworld. A design theory perspective was also important in carrying out and analysing the observations. Individual use of the mathematics textbook was the most common work form. The textbook influenced the course and nature of the teaching and the teacher's task was to assist and support the students to proceed through the book. A trend became visible from earlier group-based, collective activities toward more individual forms of work, labelled individual study. The author points to some challenges in school when it comes to the individual in relation to the group – How can the exchange between teachers be made possible across levels and how can mathematics education reduce its dependence on textbooks and strengthen the didactical tasks and duties of the teacher?

At the University of Helsinki on November 20 Liisa Näveri defended her thesis with the title *From arithmetic to algebra. Changes in the skills in comprehensive school over 20 years*. In recent decades the emphasis was on the understanding of calculation in mathematics teaching. Studies have found that better understanding supports applying skills in new conditions and that the ability to think on an abstract level increases the transfer to new contexts. In the thesis Näveri takes into consideration competence as a matrix where content is in a horizontal line and levels of thinking are in a vertical line. The know-how is intellectual, strategic flexibility and understanding. The resources and limitations of memory have their effects on learning in different ways in different phases. Therefore both flexible conceptual thinking and automatisation must be considered in learning. The research questions examined are what kind of changes have occurred regarding mathematical skills in comprehensive school over the last 20 years and what kind of conceptual thinking is demonstrated by students in this decade. The study consists of two parts. The first part is a statistical analysis of the mathematical skills and their changes over the last 20 years in comprehensive school. In the test the pupils did not use calculators. The second part is a qualitative analysis of the conceptual thinking of pupils in comprehensive school in this decade. The study shows significant performance differences in algebra and in some parts of arithmetic. The largest differences were detected in the calculation skills of fractions. In the 1980s two out of three pupils were able to complete tasks with fractions, but in the 2000s only one out of three pupils was able to do the same tasks. Also remarkable is that of the students who could complete the tasks with fractions, only one out of three pupils was on the conceptual level in his/her thinking. This means that about 10 % of pupils are able to understand the algebraic expression, which has the same isomorphic structure as the arithmetical expression. Näveri claims that this finding is important because the ability to think innovatively is created when learning the basic concepts.

Erika Stadler presented her doctoral work on December 18 at Växjö University and the title is *The transition secondary school to university: mathematics education as seen from a student perspective.* Stadler has followed five beginning student teachers during their introductory university courses in mathematics. Rather than using a theoretical framework the study takes its starting point in a situation close to reality: the students' learning of mathematics in a new connection. The empirical everyday reality leads to a theoretical description of this reality. The main results from the study indicate that from a student perspective the transition is about small qualitative and relational changes in the teaching situation. The changes are often tacit, implicit and partly hidden. For

the beginning students this means they are supposed to learn extensive mathematical content at the same time as they should crack the codes and find the ways to do it. There are individual differences and for some students it is easy to accommodate to the new learning situation but for other students this becomes a hard task. They experience clashes between the old way to participate in mathematics teaching inspired from upper secondary school and find it opposing to the new teaching situation and the demands presented there.

The Nordic theses in didactics of mathematics presented in NOMAD during 2009 make up 13 so far but there might be others, which are not known to me. As mentioned in the latest issue of NOMAD, Erkki Pehkonen has reported some Finnish studies in didactics of mathematics, which have not been exposed here. He lists 35 doctoral studies in mathematics education in Finland during 1984 to 2008. Seven of them are from 2006–2008 and three of them have been presented before in NOMAD. The other four will be shortly mentioned now.

Seija Hassinen presented her thesis in University of Helsinki in 2006 with the title *Idea-based school algebra* (in Finnish). The study presents a model for algebra teaching in 7$^{th}$ grade in order to make algebra more natural and useful to students. The basic ideas in the Idea-based Algebra (IDEAA) are to combine pupils' own informal mathematics with scientific mathematics, and to structure algebra content as a map of big ideas, not as a sequence of powers, polynomials, equations and word-problems. The design research project has three goals: research, design and pedagogical practice. The result of the study is threefold. The main result is the instruction model. The second result is the theory developed of the teaching, learning and algebra. The third result is knowledge about the design process. The instruction model has four main features of good algebra teaching: the situationality of learning, learning as knowledge-building, the emergence and diversity of algebra, and the development of high performance skills at any stage of instruction.

Sinikka Räty-Záborszky took her doctoral degree at University of Joensu in 2006 and the title of her work is *The conceptions of Finnish and Hungarian teachers and mathematics textbook writers about geometry and geometry teaching and learning in primary school (classes 1–6)* (in Finnish). The study aims to investigate the conception of Finnish and Hungarian teachers and textbook writers have about geometry and geometry teaching and learning in primary classes and to discover to what extent they are similar or different. Initially the different conditions in education and cultural differences are clarified. Mathematics as an information structure is discussed and the teaching and learning of geometry examined. The study is qualitative and data consists of semi-structured

interviews based on themes. Six Finnish teachers and six Hungarian teachers and two textbook writers in each country were interviewed. The teachers used the textbooks the writers had produced. A phenomenographic approach is taken. All teachers regarded geometry as thinking and spatial perception, but the Hungarian teachers understood it in a wider sense. The textbook writers had the same conceptions about geometry and saw it in the following categories: thinking, language, relation to everyday life, experiencing geometry and the amount of weight given to geometry in textbooks. Teachers saw teaching and learning geometry as one's own environment as a starting point, the importance of conceptual knowledge, the pupils' individual needs and the importance of language skills. Some categories that differ are also pointed out in the study. A new curriculum comes into force in 2006 with a strengthened position of geometry and the study can then be expected to be of use to support teachers in teacher education and in-service training.

Henry Leppäaho defended his thesis at University of Jyväskylä in 2007. The title is *Teaching mathematical problem solving skill in the Finnish comprehensive school. Designing and assessment of a problem solving course* (in Finnish). The thesis describes the teaching of a mathematical problem solving course in the Finnish comprehensive school. The method is design research. The intention of the course was to create a learning environment that develops pupils' mathematical problem solving skills. Data was collected by assessing the teaching intervention, interviewing pupils and through a quasi-experimental design. The experimental group consisted of 17 pupils and the control group of 35. The results are 1) the design procedure of a viable problem solving course itself for use in grade 6, 2) a concrete outcome of the design process was created and published as a textbook, and 3) the creation of a productive learning environment. The results were shown by using pre-, post- and delayed tests. The motivation of the experimental group improved during the course.

Pirjo Tikkanen presented her dissertation at University of Jyväskylä in 2008 with the title *"Easier and more fun than I thought" Mathematics experienced by fourth-graders in Finnish and Hungarian comprehensive schools* (in Finnish). The study compares the experienced mathematics curriculum of Finnish and Hungarian 4$^{th}$ grade comprehensive school pupils. The experienced mathematics curriculum comprises emotions, attitudes to and beliefs about mathematics, the learning and teaching of mathematics, and the mathematical self-concept. Three groups were compared: 20 Finnish pupils, 23 Hungarian pupils taught with the Varga-Neményi method and 21 Finnish pupils taught using the Finnish approach. Data consisted of pupils' writings and drawings and were analysed using a hermeneutic-phenomenological method. Three

experience types were identified: the argumentative problem solver in the Finnish Varga-Neményi-group, the contrasting equation solver in the Hungarian Varga-Neményi-group and the stating calculator in the Finnish group. Regardless of teaching method the Finnish and Hungarian pupils have a positive attitude towards mathematics. The pupils consider mathematics to be easy and important. Most of the pupils have a positive mathematical self-concept. The author claims that the experienced mathematics curriculum provides teachers with important feedback on how to develop mathematics teaching and learning.

These presentations might invite readers to study the theses more in detail. The hope is that the theses in Swedish and Finnish will be made available for a wider audience of non-Finnish/Swedish speaking readers by papers in NOMAD for example.

## The future of research in the Nordic and Baltic countries

The board of NoGSME had a discussion in its final meeting about what can be the expected future of the Nordic and Baltic research in didactics of mathematics. The development of Nordic mathematics education research has been strong and positive over the last 10 years. It is not obvious that this development will continue. From some of the countries reports were given about hard conditions to achieve funding for research and for doctoral students. Demands are raised that part of the funding must come from industry and it is not easy to motivate industry to support research in mathematics education. Mathematics education research must probably continue to defend its position and prove that there is a need for its existence. A sign of improvement was reported in the fact that there is a broader acceptance of didactics of mathematics among mathematicians in some areas. The great need for qualified mathematics teachers and teacher educators might be one fact that supports the development of research. In Sweden the situation seems to be changing according to the political conditions. The graduate school in mathematics education could not find further support but financing is given to the "Lärarlyftet", including parts of research education. In Norway mathematics education still seems to be an expanding field with a large number of professors and doctoral students. In Iceland there are many Ph.D. students in education, some of them academic teachers, who are under pressure to take a doctoral degree. Most of them have an outside supervisor from another country. In the Baltic countries there are two centres for mathematics education, the University of Tallin and the University of Tartu. Financing of doctoral students is a problem in the Baltic countries. The labour market for new doctors is limited.

We must ask if the golden days of Nordic expansion for mathematics education are over. There will be collaboration in journals, conferences and meetings but it is harder to find financing for a wider and more long-lasting cooperation. There might be a need to be creative and invent new forms for constructive cooperation? The hope is that the large number of new doctors in didactics of mathematics that have been produced in the new millennium will have the strength to keep up the spirit of Nordic and Baltic collaboration and ensure that new generations of doctoral students can enjoy the NoGSME-spirit.

With these hypotheses about future development I want to thank my colleagues in the board for fruitful collaboration during the six NoGSME years and for all the altruistic work they have done for mathematics education in the Nordic and Baltic countries. These are the members of the NoGSME board:

*The NoGSME board from left to right.*
*Trygve Breiteig, Ole Björkqvist, Barbro Grevholm, chair, Gudny Gunnarsdottir, Christer Bergsten, Mogens Niss, Madis Lepik.*

Barbro Grevholm
Director of NoGSME
University of Agder

## References

Goodchild, S., Jaworski, B., & Måsøval, H. S. (2007). Workshop on classroom research: analysis of transcript data. In C. Bergsten, B. Grevholm, H. Måsøval & F. Rønning (Eds.), *Relating practice and research in mathematics education. Proceedings of Norma05, Fourth Nordic Conference on Mathematics Education* (pp. 133–147). Trondheim: Tapir Akademisk Forlag.

Grevholm, B. (2009a). Creation of a Nordic society for research in mathematics education, NoRME. In C. Winsløw (Ed.), *Nordic research in mathematics education. Proceedings of Norma08, Copenhagen, 21–25 April, 2008* (pp. 387–389). Rotterdam: Sense Publishers.

Grevholm, B. (2009b). *The use of ICT in mathematics education – neither salvation nor catastrophe? What can we learn from research and what are our conclusions? A workshop organised by the Nordic Graduate School in Mathematics Education (NoGSME) during the Norma08-conference*. In C. Winsløw (Ed.), *Nordic research in mathematics education. Proceedings of Norma08, Copenhagen, 21–25 April, 2008* (pp. 273–286). Rotterdam: Sense Publishers.