Word problems in upper secondary algebra in Sweden over the years 1960–2000

Teresia Jakobsson-Åhl

This paper reports results from a study on school algebra at the upper secondary level. The study considered changes in the algebraic content as it was presented in mathematics textbooks in the second half of the twentieth century in Sweden. The paper describes the changes of word problems in terms of the way they are used for developing algebraic skills. It is shown in what ways the place of the problems, the context and the use of mathematical models changed in the period in question. The use of word problems is related to the curricular documents of the time and to different views on school algebra.

The importance of considering the history of mathematics education is emphasized by many authors. Howson, Keitel & Kilpatrick (1981), as an example, consider the history of school mathematics as necessary in order not to repeat the same mistakes all over again. The period of the 1960s and 70s was characterized by intense change in school mathematics and was called the 'new math'. These reforms were widely criticized and then quickly revised (Fey & Graeber, 2003); thus, a lot of variation can be observed over a relatively short historical period.

Research needs to consider the kinds of algebra that emerges in educational systems (Chevallard, 1985; Bolea, Bosch & Gascón, 2003). Chevallard (1985) studied the history of conceptions of algebra, represented in French textbooks. He provided a thorough description of what happened to traditional school algebra as the new math was implemented in the 1960s and 70s. Also, Jakobsson-Åhl (2006) undertakes a historical-epistemological perspective on the development of the algebraic content

Teresia Jakobsson-Åhl
Luleå University of Technology

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but focuses on the upper-secondary level in Sweden. This paper is based on that study. The main purpose of the paper is to describe the use of word problems in textbook approaches to algebra in the second half of the twentieth century. In particular, the place, or particular localization of the problems in the structure of textbook chapters, and the characteristics of the problems, i.e., the context and the role of the mathematical models in the problems are highlighted.

Different views on school algebra

In order to outline the historical development of school algebra, this section briefly brings up some views on school algebra identified in the literature. The views emphazise different aspects of school algebra, in various ways, and thus are complementary (c.f., Wheeler, 1996). The section centres on the role of word problems in each view. Accordingly, aspects such as the characteristic features of the problems are treated. Nevertheless, the placement of word problems in textbooks may not always be addressed explicitly. In fact, as a textbook chapter may involve a combination of varied views, or approaches to algebra, the localization of problems is not directly linked to a specific view.

Algebra as a separate school subject

Algebra as a separate subject comprises the idea of algebra as an extension of arithmetics and so algebra as generalized arithmetic. This view could be referred to as *traditional school algebra* and was common in textbooks from the first half of the twentieth century. Here, emphasis was placed on manipulative skills in algebraic expressions, equations, formulas, polynomials, rational expressions, powers and roots (Clements, 2003).

Algebra was, as stated by Chevallard (1985), used for solving problems previously solved with arithmetic. The solutions involve an analytic process (as opposed to a synthetic process, Charbonneau, 1996); the unknown was to be treated as if it was known, represented by a literal symbol and then operated upon. The relations among the quantities in the problems could be expressed in the form of equations that are supposed to be solved formally. Thus, it is possible to classify the problems into certain types. This kind of abstraction enables generalization; word problems of the same type could be solved by the same method.

Word problems in algebra textbooks were placed after a brief explanation of a particular topic. As an example, a section on formulas in a textbook of the time (see Donoghue, 2003), first states a formula and describes how to substitute values into this formula to find the unknown. Then students' exercises comprise word problems such as:

> P = .433*h* gives the water pressure in ponds per square inch for the height of water in feet. What is the water pressure per square inch at the bottom of a standpipe when the water column is 92 feet high? (Welchons & Krickenberger, 1949, p. 45, as cited in Donoghue, 2003, p. 355)

Word problems were used in applying algebraic concepts in various situations (c.f., Schoen, 1988).

Algebra in a unified view of school mathematics

In the mid-twentieth century the traditional mathematics curricula were questioned and led to the new math (Clements, 2003). The new math emphazised a theoretically unified view on mathematics; unifying concepts such as sets, structures and functions became central ingredients (Fey & Graeber, 2003).

In this view, two different ways of treating algebra are recognized. Firstly, in the new math, school mathematics was organized around algebraic structures. This meant that the contents of arithmetic and elementary algebra were organized differently than before. Algebra was found already in the study of fractions, previously a standard topic of arithmetic. The study of fractions was, for example, preceded by the study of the algebraic properties of positive and negative integers (Chevallard, 1985). Secondly, functions were viewed as a unifying concept for school mathematics and related mathematics to other school subjects (Clements, 2003). Functional approaches continue being of interest in projects on the teaching and learning of algebra (see Kieran & Yerushalmy, 2004).

A consequence of the stress on a unifying theme in the new math era was the fact that word problems played a minor role in the mathematics textbooks (c.f., Usiskin, 1997; Schoen, 1988). Also, at the time, school mathematics had no, or little, connection to the world outside of mathematics (Usiskin, 1997). Rather, the new math textbooks focused on the study of formal systems which meant that word problems, in line with Kline (1973), had an "artificial" character as in the following problem.

> According to the Law of Reflection: $i = r$. Given that $i = (2n + 30)^\circ$ and that $r = (4n + 10)^\circ$, find n. (Kline, 1973, p. 77)

This type of problems is characterized by intra-mathematical links and reasoning; in this example, the solvers, as highlighted by Kline, do not need to relate to the law of reflection as such at all.

Algebra as a problem solving tool

The teaching of abstract algebraic notions at the school level was strongly criticized and led to an emphasis on basic skills in which the idea of applying mathematics was involved. Subsequent curricular revisions stressed problem solving (Usiskin, 1997). Certainly, even though a course is not entirely organized around problem solving, word problems are nowadays important in school algebra.

In the view of algebra as a problem solving tool among others, the characteristics of problems vary; some curricula have traditional word problems, similar to those in a traditional algebra course, whereas others address real-life problems (e.g., Sutherland, 2002; Kendal & Stacey, 2004). Thus, algebra as a problem-solving tool, also allows students to study real-life situations and to express these by multiple representations such as equations, graphs, tables etc. (see e.g., Huntley et al., 2000).

Moreover, the view covers a modelling perspective on algebra. Modelling is, according to Burkhardt (1989), the part of applications which deals with non-routine tasks where the solvers formulate the mathematical models themselves. The author claims that it is important for students to know whether the aim is to show their mathematical abilities or to argue for the design of a particular model. Other authors focus on modelling as a process which includes interpreting a realistic situation, translating it into a mathematical form and being able to interpret the yielded results in terms of the original situation (e.g. Janvier, 1996).

In this view, with its weight on uses of mathematics, applied problems appear throughout the whole textbooks (Usiskin, 1997).

Algebra as a competence

More recently, algebra has been described in terms of competences and so is concerned with abilities of dealing with generality, functions and equations. To be more precise, Kissane's (2001) description involves three central features to be learned as 'algebra' at the secondary level: "representing situations and objects using algebraic symbols, dealing with functions and their graphs, and solving equations and inequalities" (Kissane, 2001, p. 127).

Generalization often appears as a competence in the context of algebra learning and comprises ideas such as generalizations about number properties (e.g., Bell, 1995) or patterns (e.g., Stacey & MacGregor, 2001).

The following problem illustrates the characteristics of word problems, used to establish generalization on numbers.

Example 1.

(a) Show that the sum of a number of four digits and the number formed by reversing the digits is always divisible by 11.

(b) The greatest and least of four consecutive numbers are multiplied together; so also are the middle pair. Show that the difference of the two products is always 2. (Bell, 1995, p. 41)

The distinguishing features of these problems cover the fact that the solver is expected to convert the given information into a representation by algebraic expressions, manipulate on the expressions and then interpret the result (Bell, 1995).

Another type of generalization problems is characterized by finding general rules as exemplified in the next problem.

Pond borders

Joe works in a garden centre that sells square ponds and paving slabs to surround them. The paving slabs used are all 1 foot squares.

The customers tell Joe the dimensions of the pond, and Joe has to work out how many paving slabs they need.

- How many slabs are needed in order to surround a pond 115 feet by 115 feet?
- Find a rule that Joe can use to work out the correct number of slabs for any square pond. (Swan, 1985, p. 72)

In this case, the solver is asked to first express the number of paving slabs in words and then in letter variables. This can be done by looking either at the pattern of numbers or at the pattern in the shapes (see further, Swan, 1985).

Theoretical considerations

The study was inspired by phenomenographical ideas and described the diachronic variation of the intended experience of word problems in upper secondary algebra as it is represented in Swedish mathematics textbooks. In terms of the phenomenograhical perspective, the intention is to study and describe potential variation in people's ways of experience 'word problems in algebra' (Marton and Booth, 1997).

The term 'experience' is used to explore different ways of experiencing phenomena in the learning process. Marton & Booth write:

> We prefer to describe learning in terms of the experience of learning, or learning as coming to experience the world in one way or another. Such learning inevitably and inextricably involves a way of going about learning (learning's *how* aspect) and an object of learning (learning's *what* aspect). (Marton & Booth, 1997, p. 33)

As the content of learning algebra is at the heart of this paper, possible outcomes, with respect to the content of this learning, are investigated. The paper subsequently addresses the 'what' aspect, i.e., an individual's way of experience a phenomenon and qualitative differences in experiencing the particular phenomenon (Marton & Booth, 1997). However, the paper is interested in neither teachers' nor students' experience as such but in qualitative differences in what is proposed for teachers and students as word problems in algebra, i.e., of what students should learn in relation to problem solving in school algebra.

The focus is on texts to be considered in education – mathematical textbooks – and qualitative differences that occur in these. The books reflect what their authors conceive of as 'algebra'. As such, the study follows the phenomenographical adage that "the way mathematics is written, textbooks are written, teaching is organized, evaluated, responded to, may tell us a great deal about how particular societies or those in power as well as students conceive of [algebra]" (a paraphrase of Marton & Booth, 1997, p. 116).

This implies that the experience-of-the-learner as such is not being considered but instead the experience-of-the-learner-as-intended-by-the-author-of-textbooks. If students are expected to become aware of something, certain aspects of this 'something' are pinpointed in the textbooks through the ways the terminology is introduced and used, the characteristics of the tasks, etc. The treatment of different kinds of word problems in the learning environment could emphazise a variety of knowledge in mathematics (algebra) such as varied techniques of solving problems. Also, problems could include certain types of equations or the use of different representation forms. Still, the notion of equation may not be an explicit object of teaching.

Methods

The data for the authors' licentiate study (Jakobsson-Åhl, 2006) was drawn from a selection of mathematics textbooks and were analyzed to provide a picture of the nature of school algebra at the upper secondary level. From this broad set of data, this paper concentrates on word problems and the way they are used for developing algebraic knowledge and skills.

Data sources

The selection of mathematics textbooks was guided by the aim of covering the changes that influence the nature of school algebra in upper secondary mathematics in Sweden in the period 1960–2000.

Two sets of mathematics textbooks were included in the study; these were written for Natural Science students in grade 1[1] of the upper secondary school, before 1994, or the courses labelled *Mathematics A* and *Mathematics B*[2] in 1994 and onwards. The textbooks were published in the years 1960–2000. Two criteria were used for selecting these textbooks: (1) the textbooks were written by the same author group with approximately the same authors over the years and (2) the sets of textbooks were commonly used. This was important because changes due to the fundamental views of different authors were not necessary to take into consideration. Further, changes across time in the respective set of textbooks were followed.

A list of the analyzed mathematics textbooks can be found in the appendix. The textbooks constitute a data pool where a variation of the algebraic content can be discerned.

Analysis of algebra in textbooks

The analysis process covered two steps. The first step was the identification of what counted as algebra in terms of concepts or skills in textbooks. The second step was descriptions of this algebra in terms of certain special categories of description. A more detailed account follows.

Descriptions of the algebraic content

The algebraic features were considered, in particular the authors' explicit definition of algebra (if included) and the algebraic content as it was presented in the books i.e., definitions, descriptions, worked examples and exercises for students. The algebraic content consisted of two themes: *literal calculi* and *algebraic theory*. Literal calculi are described in terms of the domain of reference of letters and the permissible operations on these letters. Thus, it is not a matter of real numbers only but the domains of sets and vectors as well. Furthermore, algebraic theory includes the notions of equations and algebraic structures. Textbooks differ in the relative importance of these themes and in what they include and stress in them.

Formulation of the categories of description

The outcome of a phenomenographical study is a set of related categories of description based on empirical data of a limited number of ways in which a phenomenon has been experienced (Marton & Booth, 1997). The variation of school algebra in textbooks is most often implicit in, for example, the changes of the organization of the mathematical (algebraic) content; in the ways concepts are defined; or in the role and importance awarded the algebraic operational symbolism. The result of the analysis of these changes is a classification of the 'textbook approaches' in terms of a number of categories of description. Although, no pre-determined categories were used, the terminology used for describing the categories is inspired by the literature. This means that the presentation of different views on algebra in educational settings, in a previous section, acts as a backdrop for a terminology for describing the role of word problems within the identified textbook approaches to algebra.

The empirical study was performed in three phases, each representing important elements in analyzing the data.

Phase 1. A tentative analysis of algebra in textbooks

To become familiar with the data, a tentative analysis of textbooks from different epochs was conducted. Algebra was considered both as an element of curriculum and ways of thinking; the objects of study were *algebraic expressions*, *equations*, *functions* and *number structures*. The identified categories constituted a first tentative approach to building up the variation of school algebra represented in textbooks.

Phase 2. Revision of categories

The results of the tentative analysis were reanalyzed to determine whether the categorization was incorrect in terms of qualitatively different ways in the intended experience of school algebra. The study was restricted to analyze algebra as an element of curriculum only. The objects of study were: *algebraic expressions*, *equations* and *algebraic structures*. The function concept as such was excluded because functions are treated within the theory of functions (pre-calculus) in Swedish curricular documents. New categories emerged.

Phase 3. Identifying relations between the categories
The relations between the categories were identified and described and led to main categories and their sub-categories. Thus, the categories were established and formulated. Each main category is related to a certain idea concerning the contents of teaching and learning algebra.

The main categories

The results of the study provide an image of qualitatively different ways in discerning school algebra, formulated in terms of a number of categories of description. The main categories point to various aspects of the nature of algebra in school mathematics and are concerned with the following ideas:

A Objects of study:

Which objects of study are primary in the textbook approaches? Emphasis could be placed on algebraic expressions, algebraic structures, on equations and algebraic expressions as equally important and on equations.

B Introduction of the objects of study

How are the objects of study introduced? The algebraic content could be introduced either by relating the new concept with previously acquired mathematics or by linking the object of study to a mathematical problem and then expressing it in terms of the situation connected to the problem in question.

C The tasks for practice of algebraic skills

What types of tasks are given for practicing algebraic skills? Attention could be given to either manipulation of algebraic expressions, finding the result of operations or using different forms of representation.

D The characteristics of word problems

What kinds of tasks are given for application of the learned concepts and skills? Word problems could deal with (1) applications within different areas, both mathematical situations and other school subjects or (2) with situations that are conceived as part of students' everyday experience. Additionally, the mathematical model could be given, possibly suggested, or formulated by the students themselves.

E Literal symbols versus numerical examples

What language is used in the presentation of the material? This category is concerned with the ways in which the properties of operations and rules for manipulating algebraic expressions are presented. It could be an emphasis on a use of literal symbols, ways of combining the use of literal symbols and numbers or a focus on numerical examples.

Thus, the topic of each main category is related to a certain question about the contents of teaching and learning. Textbooks respond in different ways to these questions; the different ways constitute sub-categories and consequently distinguish each sub-category from the others within the same main category. For a full depiction of the categories see Jakobsson-Åhl (2006).

The data are classified in three different trends of changes in the years 1960–2000: *Pre-new math* era (before 1966), *new math era* (1966–1978) and *post-new math* era (from 1978 and onwards). The shift to a new trend is guided by the year of publication of new textbooks.

As mentioned previously, this paper deals with the place and characteristics of word problems in upper secondary algebra. For this purpose the presentation of word problems in different trends builds mainly on the analysis of the main category called *The characteristics of word problems*, although aspects from the other categories are incorporated in the description as well.

Word problems in textbook approaches to algebra

The changed role of word problems is reported in this section. Each subsection is first devoted to the main ideas of school algebra in the particular trend, and then to the use of word problems. Word problems are described by considering the *localization* of the problems in the textbook and the *characteristics* of the problems, i.e., the context of the problem and the mathematical model. The presentation is illustrated by worked examples and students' exercises.

Pre-new math era (before 1966): Traditional school algebra

The main theme in the textbook approach to algebra is an emphasis on algebraic expressions. The use of literal symbols and manipulative skills consequently are important; these skills are trained lengthily.

Word problems appear in special sections after an extensive treatment of algebraic manipulation. This prepares the students for solving word problems in different situations such as relations among pure numbers.

> Determine four consecutive odd numbers, such that the difference between the average of the cube of the numbers and the cube of the average of the numbers is 900.
>
> (Nyman, 1961, p. 69, author's translation)

To solve this problem, students need to formulate a model and then use the manipulative skills they practiced previously.

Also, applications in chemistry, physics and finances are included. As an illustration – a velocity problem.

> A motorboat, which, in stagnant water moves with a maximum velocity of 9 knots, can go a certain distance up the stream in 11 minutes and the same stretch down the stream in 9 minutes. Calculate the stream velocity of the river, expressed in m/s. (1 knot = 1.852 m/h) (Nyman, 1961, p. 70, author's translation)

To calculate the stream velocity students need to interpret the information given in the text, express the situation by setting up an equation, and then solve the equation in question.

In the section on student exercises, which covers word problems, formulas are given or suggested in initial problems, and then students are supposed to formulate equations representing the situation from the information given in the text; the latter is prominent.

New math era (1966–1978): Algebra as structures

The subject matter is structured around number structures $\boldsymbol{N}, \boldsymbol{Z}, \boldsymbol{Q}, \boldsymbol{R}$ and the two-dimensional vector space $\boldsymbol{R}_2$. A consequence of this emphasis on algebraic structures is a decreased interest in algebraic manipulation. The introduction of algebra is characterized by a mixture of literal symbols and numerical examples as well as the use of graphs and tables in introductions of new topics. Two textbook approaches co-existed; one less and one more radically 'structural'.

One textbook approach first extends number structures to real numbers, then treats axioms that govern numbers and applies them to justify rules of operations on algebraic expressions. Next, calculations in real numbers follow. As in traditional school algebra, word problems constitute applications in mathematics and other school subjects and are placed in special sections. The models are suggested before students need to construct the models themselves. In comparison, there are fewer problems than in the previous trend.

In the other approach, on the other hand, algebraic expressions and equations serve as tools for the study of number structures; equations are used to justify the extension of a number structure to a larger one. The number structures $\boldsymbol{N}$, $\boldsymbol{Z}$, Q and $\boldsymbol{R}$ are treated separately one by one where the properties of operations on numbers are introduced for each number structure to emphazise important features. Word problems succeed the presentation of number structures in a special section. It is a matter of applied problems in mathematics and other subjects where the models are given or suggested. A velocity problem looks like this:

> The time of the car race winner was 1 hour and 12 minutes. His average velocity was 120 km/h. The average velocity of the second and third best contestants was 108 km/h and 96 km/h respectively. Calculate their times.
>
> (Brolin et al., 1966, p. 223, author's translation)

Before this exercise, the relationship between velocity (v), distance (d) and time (t) are discussed in a worked example. Therefore, it is clear for the students which mathematical model to use.

Post-new math era (1978 and onwards): A focus on equations

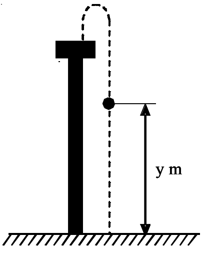
Equations take up a more dominant role than before. In one textbook approach, equations are viewed as equally important as algebraic expressions while, in the other, equations are the primary objects of study. Numerical examples and calculations are common. Basic laws and rules, for instance, are explained by means of numerical examples; general statements where literal symbols are involved are sporadically used. It is noteworthy that one of the textbook approaches not only is characterized by numerical examples but also by the link to geometry; the algebraic rules are related to visual (geometrical) representations.

The use of tabular and graphical representations continues being important but still representation by algebraic expressions is privileged. In 1999, the remaining textbook approach[3] emphasizes multiple representations. Representation by algebraic expressions no longer is privileged. Instead, students are expected to apply the most appropriate representation for the particular situation, regardless of representation form chosen (table, graph, narrative statements, algebraic expressions etc.).

In this trend, the use of problem solving pervades the whole course. This means that students meet problems early in the course and train algebraic skills in solving word problems. Thus, word problems appear throughout the textbook in the entire post-new math epoch. The mathematical models constitute equations and functions.

One textbook approach introduces algebraic concepts via extra-mathematical problems which lead to an intra-mathematical discussion of the new concept. The notion of polynomials, for instance, is introduced by first stating the following problem.

> From the top of a 325 m high tower a body is tossed upward with the velocity of 25 m/s. After x s its height above the ground is y m, and is calculated by the formula $y = 325 + 25x - 4.9x^2$



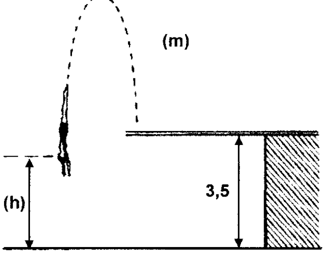
(Björk et al., 1978, p. 238, author's translation)

Then, the expression $y = 325 + 25x - 4.9x^2$ is the starting point for describing the notion of polynomials.

The characteristics of word problems, in the two textbook approaches, are alike well into the 1990s – applications in geometry and other school subjects where the model is given or suggested.

In 1994, applications in other school subjects no longer are in focus. In contrast, problems now are situated in everyday activities where the attention is given to situations in society.

> Eva jumps from a springboard, 3.5 m over the water surface. She jumps up with a velocity of 7.5 m/s. It is possible to show that after t s her height over the water surface is h(t) m, where
>
> $h(t) = 3.5 + 7.5t - 4.9t^2$
>
> How far from the water surface is she after 1 s?



(Björk, Borg & Brolin, 1994, p. 164; author's translation)

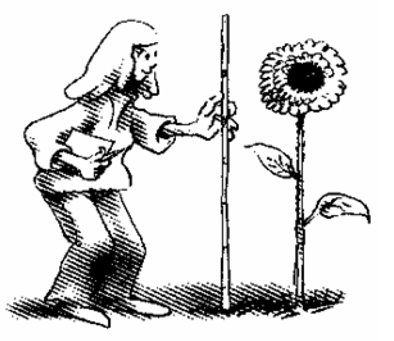
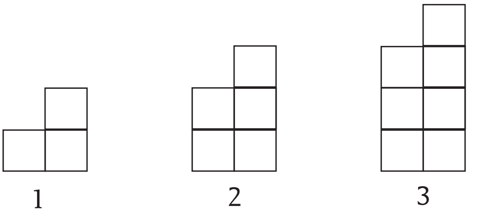


Figure 1. *Eva measuring a flower (Björk & Brolin, 1999, p. 200)*

In 1999, the corresponding example is about Eva who is measuring the length of a sunflower during the summer; the problem is located in a real-life context, illustrated by a picture (see figure 1).

Thus, in 1999, the trend is characterized by the fact that the situations of the problems are closer to the life of young people than before.

A new feature of school algebra, in 1999, is the formulation of relationships by looking at patterns. Different patterns are exemplified by matches, squares and circles. Patterns and formulas are used to train relational thinking such as the following example.



a) How many squares *K* are there in figure number *n*?

b) Calculate the squares in figure number 100?

c) What is the number of the figure with 49 squares?

(Björk & Brolin, 1999, p. 65, author's translation)

Students are expected to consider the pattern in the shapes and then express the relation between the number of squares and the corresponding figure.

Another new feature, in comparison to the early 1990s, is the idea of students setting up expressions and equations from information given in word problems. In general, the mathematical model is given or suggested. Nevertheless, in a small number of isolated problems students need to formulate the models themselves.

Word problems related to curricular documents

The changed role of word problems in textbooks approaches can be explained from changes in the corresponding curricular documents in the period of time studied.

In the *pre-new math* era, word problems succeed the extensive part on algebraic manipulation. The context of problems follows the curricular document of the time which points to the weight of applications in mathematics and other school subjects (Kungliga Skolöverstyrelsen, 1960).

Less attention is given to word problems in the *new math* textbook approaches. This could be a consequence of the emphasis on ideas such as algebraic structures, algebra of sets and vector algebra in the curricular documents (e.g., Skolöverstyrelsen, 1965). Besides, the mathematics content in the curriculum of 1965 was far too extensive in relation to the allotted time (Håstad, 1978). The contexts of the word problems correspond to the curricular documents which states that mathematics is supposed to "satisfy the demands of mathematical knowledge in other subjects." (Skolöverstyrelsen, 1965, p. 274, author's translation).

The textbook approaches are characterized by problem solving throughout the course in the *post-new math* era. Word problems continue being applications in school subjects. Accordingly, the curricular document of 1981 stresses the solving of applied problems in other subjects (Skolöverstyrelsen, 1981). Here, it is worth noting that the 1978 books were adapted to the new ideas in mathematics education and so belong to this trend.

In the 1990s, the primary focus moved from school subjects to other contexts: in 1994, word problems are situated in everyday situations while, in 1999, problems emerge as a natural part in the life of young people (real-life situations). These ideas are reflected in the curricular documents as students are supposed to: "Be able to formulate, interpret and use simple algebraic expressions and formulas and be able to apply these in practical problem solving" (Skolverket, 1994, p. 48, author's translation). Moreover, the new idea of isolated tasks of a modelling character, in 1999, is supported by the curricular documents that stresses students' ability to create models themselves (e.g. Skolverket, 1994).

Discussion

The aim of this paper is to describe the use of word problems in algebra in the upper secondary school in Sweden and was restricted to textbook presentations of algebra. The findings presented here manifested a changed role of word problems in the last fifty years.

In a nutshell: in the *new math*, word problems assumed a subordinate role in the textbook approaches to algebra. In contrast, the other two trends conceive word problems as significant, either as the goal of training algebraic skills or as something to be used throughout the course to highlight algebraic notions. The analysis reveals that word problems are grounded in different contexts such as mathematics, other school subjects and everyday and real-life situations. The context of the problems pay attention to mathematics and other subjects until the mid 1990s as another focus takes over, i.e., everyday situations. Therefore, the tendency in these changes of applications is towards more activities in everyday life. The results also indicate a changed role with respect to the localization of the problems and the use of mathematical models: In the *pre-new math* epoch, word problems were placed at the end of sections but in contrast, today, appear throughout the textbooks. The mathematical models were equations with ready-made formulas in the 1960s. Nowadays, though, the problems are more varied and involve patterns using equations and different representations of functional relationships.

The reported changes follow the historical development of school algebra. In the *pre-new math* era, word problems were placed at the end of sections in the same way as in Donoghue's (2003) description of school algebra in the first half of the twentieth century. In solving these problems, students were expected to construct equations themselves; the goal was for students to show their mathematical ability by setting up equations from given data, as described by Burkhardt (1989). The word problems on numbers, in the early 1960s, resemble the generalizing problems which are concerned with number properties in Bell (1995).

Overall, the identified *new math* textbook approaches are in agreement with the international movement (c.f., Chevallard, 1985; Usiskin, 1997), but is 'milder', i.e. not as radical in using abstract algebraic notions in school mathematics. As the notion of abstract algebra is significant, little attention is given to word problems. The contexts of the problems are the same as previously but here the model, to a greater extent, is given, or suggested. Nevertheless, the few included problems are not 'artificial' in the sense of Kline (1973). In the *post-new math* era, word problems are integrated throughout the whole course where algebra acts as a tool to solve problems. The contexts of word problems vary (c.f., Kendal & Stacey, 2004).

A new feature is recognized in 1999; i.e., a focus on relations or patterns. This suggests that algebra is conceived as a competence (see, e.g., Kissane, 2001) in school algebra. In addition, the new type of problems, i.e., the pattern-based problems, is of the same type as those discussed in the literature (Swan, 1985; Stacey & MacGregor, 2001). The paper demonstrates that students encounter multiple representations and are expected to select the appropriate form for the mathematical problem under discussion (c.f., Huntley et al., 2000).

In 1999, students are supposed to formulate models from given data. Accordingly, isolated tasks of a modelling character emerge again in the late 1990s and so, to a certain degree, re-enter in upper secondary mathematics. In this context, students set up equations or formulate functions from given data in solving problems. In other words, they are expected to show their ability in constructing mathematical models (Burkhardt, 1989).

This paper shows that, in the early 1960s, students are expected to show their ability in manipulative skills and to use formal methods in solving problems. However, this is not the case in the late 1990s where other skills are stressed. Take the generalization problems as an example. The problems on pure numbers in the 1960s require algebraic manipulation whereas, in the 1990s, using operational symbolism is secondary in solving the problems on patterns. The latter are similar to pattern-based problems with their weight on general rules, in the sense of Swan (1985).

The varied contexts of word problems and less attention to operational symbolism are supported by the results of Sutherland's (2002) comparison of algebra curricula in different countries, or regions. She writes: ”In general, where there is more emphasis on solving 'realistic problems' there tends to be less emphasis on symbolic manipulation” (p. 3).

The changes in ways of using word problems over time are, in the paper, related both to statements in the corresponding curricular documents and to the historical views on school algebra. The present focus on word problems as applications in everyday life seems to be a result of a long-term development in mathematics education.

To conclude, a historical-epistemological insight into the intended experience of word problems in school algebra, as described in the paper, is fruitful for the teaching practice. Mathematics teachers and teacher educators need not only to take issues such as cognition and social interaction into account but also the algebraic (mathematical) content as such. Teachers need to reflect over the kind of knowledge which is promoted through the use of different kinds of word problems. The distinguishing features of varied problems point to differences in school algebra and can be a source of inspiration for choosing relevant aspects for algebra (mathematics) education. Awareness of how the role of word problems may affect the learning situation will contribute to teacher's lesson planning and subsequently to students' acquisition of concepts and to students' mathematical performance as a whole.

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Notes

1 Grade 1 in the Swedish upper-secondary school corresponds to the tenth grade in organizing the school into grade 1–12.

2 In the early 1990s reforms in Swedish schools, school subjects were organized into successive courses at the upper-secondary level. This meant that natural science students, from 1994 and onwards, in general, take Mathematics A and Mathematics B in the first grade of the upper-secondary school.

3 One of the textbook approaches vanishes in the mid-1990s.

Appendix: Analyzed textbooks in a chronological order

Nyman, B. (1961). *Algebra för gymnasiet, del 1.* Stockholm: Svenska Bokförlaget.

Nyman, B. (1966). *Naturvetenskapliga och tekniska lärokursen, del 1.* Lund: Svenska Bokförlaget.

Brolin, H., Sjöstedt, C. E., Thörnqvist, S. & Vejde, O. (1966). *NT1 Matematik för gymnasiet, NT-kursen, Åk 1.* Stockholm: Natur och kultur.

Nyman, B. & Emanuelsson, G. (1968). *NT 1 Matematik för gymnasiet: Naturvetenskaplig och teknisk linje, årskurs 1.* Lund: Svenska Bokförlaget.

Nyman, B., Emanuelsson, G., Bergman, M. & Bergström, L. (1978). *Matematik för gymnasieskolan: NT1.* Solna: Esselte studium.

Björk, L-E., Brolin, H., Eliasson, L. & Ljungström, L-F. (1978a). *Matematik: Gymnasieskolans treåriga linjer. NT, 1 A.* Stockholm: Natur och kultur.

Björk, L-E., Brolin, H., Eliasson, L. & Ljungström, L-F. (1978b). *Matematik: gymnasieskolans treåriga linjer. NT, 1 B.* Stockholm: Natur och kultur.

Nyman, B., Emanuelsson, G., Bergman, M. & Bergström, L. (1982). *Studium matematik: NT1.* Solna: Esselte studium.

Björk, L-E., Brolin, H. & Ljungström, L-F. (1982). *Matematik: Gymnasieskolan NT 1.* Stockholm: Natur och Kultur.

Björk, L-E., Borg, K. & Brolin, H. (1994). *Matematik 2000: Naturvetenskapsprogrammet, kurs AB lärobok*. Stockholm: Natur och Kultur.

Björk, L-E. & Brolin, H. (1999). *Matematik 3000: Naturvetenskapsprogrammet, AB*. Stockholm: Natur och Kultur.

Teresia Jakobsson-Åhl

Teresia Jakobsson-Åhl is a doctoral student in mathematics education at the department of mathematics, at Luleå University of Technology, Sweden. Her special research interest is the nature of algebra in upper secondary mathematics.

Teresia Jakobsson-Åhl
Department of Mathematics
Luleå University of Technology
SE-971 87 Luleå
Sweden
teresia.jakobsson-ahl@ltu.se

Sammanfattning

Artikeln redogör för resultat från en studie där det algebraiska innehållet i svenska gymnasiala läroböcker, under åren 1960–2000, analyserades. Artikeln belyser de matematiska problem som utvecklar algebraiska färdigheter. Det framgår hur problemens placering, kontext och användningen av matematiska modeller förändrats under perioden. Dessa förändringar relateras till aktuella kursplaner och till olika synsätt på skolalgebra.