

Developing mathematics teaching and learning through research

Following the idea that started in the previous volume, of having a number per year with a special topic, it is our pleasure to present this number on the relationship between research in mathematics education and the development and improvement of teaching and learning practices. When we decided about the relevance of this topic for NOMAD, we considered, on the one hand, the importance of promoting actively one of the aims of NOMAD, namely, developing mathematics teaching and teacher education in theory and practice at all levels of the educational system in the Nordic region. On the other hand, we took into account our knowledge about the state of affairs in the mathematics education community in the region and wanted to open a publication space for the very many projects existing at the moment, where teachers and researchers collaborate in order to provide well-reflected and solid educational alternatives for the improvement of mathematics teaching and learning.

The issue of the relation between the development of practice and research has been well discussed in the international community of mathematics education. A central discussion related to this has to do with different views about the main aim of mathematics education research. Some people (e.g., Hart, 1998) argue that mathematics education research emerged from the interest of mathematics educators to intervene in practice in order to better it. Therefore, an essential feature of research in the field is a close connection with the work of teachers for devising teaching methods leading to effective mathematics learning in students. This driving aim has been at the heart of, for example, design-research (e.g., the work of Paul Cobb and collaborators in the USA), the realistic mathematics approach in the Netherlands, and the French didactical engineering approach. As the discipline has advanced, other people have argued that research, in the consolidation of a field of study, does not only have the intention of improving practice, but rather of explaining, understanding and theorizing it (e.g., Ernest, 1998). The improvement of practice, then, is a secondary aim subordinated to the main goal of explaining, understanding and theorizing. Thus, the collaboration between researchers

and teachers is not a necessary condition for the realization of research. The formulation of theoretical propositions about mathematics teaching and learning is seen as belonging to the realm of the intangible world of research with little contact to the concrete world of practice. In the case of research *for* practice, the connection between research and improvement of practice is strong and evident. In the case of research *about* practice, the connection may not be immediate. Thus, the connection between the two is often formulated in terms of the divide between theory and practice, and many people have written about it. For a comprehensive and updated view on this discussion we recommend to see the results of the survey team "The relation between research and practice in mathematics education" at <http://www.icme10.dk/>.

We prefer to define mathematics education research as a field of study that has the double aim of explaining, understanding and theorizing mathematics teaching and learning, as well as improving it. From this perspective both aims are equally important. If this is the case, several forms of systematic inquiry are needed: the work of teachers-as-researchers (e.g., Zack, Mousley & Breen, 1997) doing inquiry in their own practice for changing it; and the collaboration between researchers and teachers in setting up environments for action-research (e.g., Atweh, 2004) and for collective inquiry (e.g., Jaworski, 2006). From projects of this nature important understandings about practice are generated, and innovative alternatives to mathematics education practices are effected.

The papers in this issue represent different approaches to and aspects of developing the practices of mathematics teaching through research and reflections. The paper by Ulla Runesson, *A collective enquiry into critical aspects of teaching the concept of angles*, reports a learning study involving three classes of fourth and fifth graders. A learning study is a method for systematic collaboration between a researcher or a group of researchers and a group of teachers with a common intention of studying the teaching and learning of a particular concept or piece of knowledge. The method is closely related to the Japanese lesson studies, which is an institutionalized format for in-service teacher training in Japan. The learning study, however, puts more emphasis on studying the learning effect of variations in the teaching. Through a cyclic process of developing, testing and analyzing variations in the teaching of the concept of angles, the study group (the researcher and three teachers) developed knowledge about the pupils' learning difficulties and about how to deal with them in the teaching situation.

In the second paper, *Design of a didactic situation – mathematical experiments in linear algebra*, by Thomas Vils Pedersen we are presented to the development of a course in linear algebra for life-science, university

students. The course includes an exam project which is designed explicitly on the basis of the French *Theory of didactical situations* (developed by Guy Brousseau) and *Didactical engineering* (developed by Michelle Artigue), with the purpose of creating situations enabling the students to experiment with Leslie matrix population models and eventually (re)discover that the limit of iterating multiplication with the matrix can be expressed by the dominant eigenvalue and the corresponding eigenvector. The paper is rounded off with a section of reflections on the usefulness of the two theories from the perspective of a course designer. It is argued that, in the case at hand, the theories were helpful both on an operational level in designing the course and as a basis for understanding and analyzing the students' learning difficulties.

In the third paper, *Some aspects of web-courses in mathematics based on PC screen recorded video lectures*, by Dag Lukkassen, Lars Erik Persson and Anna Sierpínska, an innovative and comprehensive design for web-based mathematics teaching is described and analyzed. Two master degree courses, one in complex analysis and one in partial differential equations, were designed and subsequently further developed since 2001. More than 200 students have taken the two web-based courses so far, and the format has now been institutionalized at Narvik University College as the only way of teaching these courses. Furthermore, the design has been used extensively in Ph.D. courses in applied mathematics. Students generally welcome the web-based format, which gives them a lot of flexibility in their study process. However, it is argued by the authors that the effects that the web-based format may have on the students' learning need to be researched more closely. They call for collaboration between mathematicians teaching and designing web-based university courses and researchers in mathematics education in order to develop the quality of mathematics teaching using modern information technology.

The three papers offer examples of various forms of relationship between teaching practice and research, in developing possibilities for improving teachers' practice. While the first paper addresses collaboration in the basic school, the other two papers show university mathematics teachers engage in design and reflection on their own design and implementation of innovative teaching strategies. Their contact with the tools of research has been a fundamental part in their advance.

New member of the editorial committee

During 2007 we have engaged in engaging new members in the editorial committee of NOMAD. The last new member is Guðný Helga Gunnarsdóttir, assistant professor at the Iceland University of Education. Guðný is

also a member of the board of the Nordic graduate school in mathematics education. With Guðný we complete the enlargement of the editorial committee, which gives support and advice to our work as editors.

(Multi)culturality and diversity in mathematics education

The next thematic issue, planned for December 2008, will be addressing the challenges of (multi)culturality and diversity in mathematics education. In this issue we will be dealing with the challenges that an increase in diversity of students from different cultures and backgrounds posed to the teaching and learning of mathematics in educational institutions in the Nordic region. While for some decades ago it was able to consider the population in most of the region as homogeneous and mono-national, the increase in migration of peoples in the world has changed the composition of the student body. Students from different nationalities, ethnicities, languages and religions meet in mathematics classrooms. Such diversity has implication for the work of teachers, in particular for how individual students are met by existing, dominant teaching and learning practices which have been based on an assumption of homogeneity.

We invite submissions of research papers addressing these challenges, providing understanding and illuminating practice. The deadline for submission is the 15th of August 2008.

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