Posing problems using Cabri

Lil Engström and Thomas Lingefjärd

The purpose of Engström's research was to investigate in which ways and to what extent three different teachers, one in Switzerland and two in Sweden, used a specific dynamical geometry software, Cabri Géomètre, in upper secondary school.

The method consisted mainly of field notes and audio recording during observations in classrooms. The field research was then analysed and evaluated according to the following questions: a) How do teachers pose problems and b) how do teachers make students' experiences useful when Cabri Géomètre is accessible? The result showed that one important factor enabling to challenge the pupils in learning mathematics when using Cabri Géomètre is the way the teacher poses the problems or the questions. The word 'challenging' means among other things, that the pupils continue asking themselves questions so that there will be continuous learning, not limited to finding just one correct answer.

The communication in any school mathematics classroom normally takes place between teachers and students, between students and students or between groups of students and the teacher. A specific mathematical content is usually in focus from the beginning. When software is included in this communication, it makes the classroom situation more complicated and the content might change towards open-endings due to the students experimenting and the way the problems have been presented by the teachers. This interaction is illustrated in figure 1.

The figure can be viewed in *two* ways. One way is that the content is in the centre of the work of the student, teacher and software. The other

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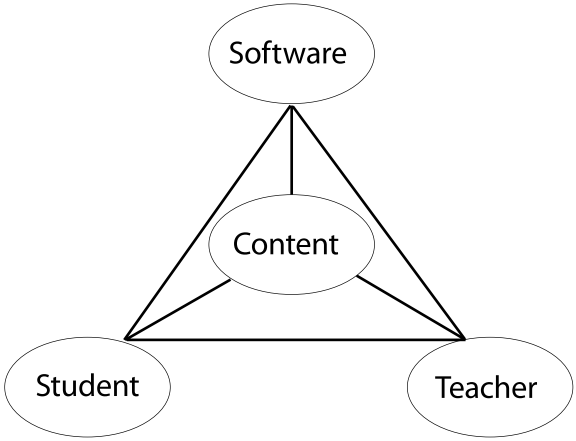

Figure 1. *Interaction in the classroom*

way is that all four corners in the tetrahedron are dependent on each other. The teacher is influenced by the software, by the student's knowledge and by the content which the teacher intends to teach, when she/he poses the problem. The content also directs in which way the software can be taken advantage of. The software can challenge both the teacher and the student to seek and find facts and relationships, depending on its way to present results.

The overall research question for this study was how the teachers actually pose problems when the students are expected to solve them by computer aid, how the teachers make use of the experiences of the students working with the computer, and to what extent the teachers use the software to best advantage. The particular software in this research is the dynamic geometry software (DGS) named Cabri Géomètre, which will be shortened to Cabri. The software can naturally be used in quite different ways, depending on the teacher's knowledge both of the mathematical content and the possibilities of the software. How a specific student handle the software might primarily depend on her/his acquaintance with computers and software in general and of dynamic geometry programs in particular. Furthermore, teachers' strategies might depend very much on the teacher's approach to learning and his/her view of the subject mathematics including teaching with computers and using specific software. Balacheff and Sutherland (1994) have described the aim of Cabri as follows:

> Cabri Géomètre had the aim, from the very beginning, of providing learners with an environment in which geometrical knowledge could emerge from their activity in a natural way.
>
> (Balacheff & Sutherland, 1994, p. 140)

As the expression 'traditional teaching' will appear, it includes among other things teaching and learning according to 'bottom up' basic first and designed to convey discrete and identifiable body of knowledge, information is identified and taught to learners, learning is externally driven by activities and practice (Hannafin, Land & Hill ,1997). A traditional lesson in mathematics can look like this: the teacher shows and explains a task on the blackboard, the pupils can ask questions and then the pupils solve similar problems from the textbook in the same way as was shown. Of course this is an extreme example and can vary a lot.

A consequence of the ideas expressed in the introduction above, was that the researcher decided to investigate what strategies teachers use when they are teaching mathematics with a DGS in upper secondary school. The teachers were chosen having different experiences of dynamic geometry software and the pupils were beginners with the actual software.

Background

Sweden is a fairly well computerized country, in school as well as at home. This does not necessarily mean that the teaching of mathematics has changed dramatically. Säljö (2000) concludes that the educational system does re-examine its way to work, but it generally is a very slow process. The text book has for a long time ruled the so called traditional teaching. In order to use a dynamical geometry software as a teaching tool, the teaching itself most likely must transform into a non traditional way to teach.

DGS were introduced in the beginning of the 1980s, mainly by the Geometer's Sketchpad from USA and Cabri Géomètre from France. Both these DGS can also handle elementary functions. There are today at least 70 different DGS available on the market (Laborde et al., 2006). The way dynamical geometry software is constructed calls for teaching aimed at something else than just facts and skills, for instance understanding and conjecturing through the construction phase. This might be illustrated by figure 2. New discoveries result in new knowledge that results in experiences. These experiences expand in new knowledge and act as a start for new discoveries. The circle is closed and knowledge can be constructed over and over again. The contrast to this is to solve a problem

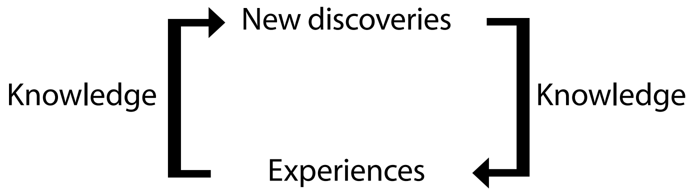

Figure 2. *The processes of investigations and conjecturing*

and getting an answer. If the answer is correct it is finished. There are no more chances to get new knowledge. The reason to show this figure is to illustrate: the possibility to get new knowledge over and over again.

The curriculum and dynamic geometry software

To learn geometry involves understanding artefacts and natural objects in our daily world. To solve problems includes interpreting and analyzing, developing and applying logical thinking, developing the ability to select different methods, identifying what kind of knowledge that is needed, to make conjectures, generalizing and formulating hypotheses and, lastly, testing them.

The Swedish curriculum contains general recommendations for using computers and calculators in the teaching and learning of mathematics. Implicitly there are incentives for using dynamic geometry software, although there are no explicit guidelines for using a specific kind of software. Instead, the curriculum contains statements of the following kind: The school should offer students the possibility to develop their knowledge, to deepen their thinking, to grow with respect to their qualifications and to reflect over their experiences (SKOLFS, 1994, p. 26).

These aims do correspond to the objectives that Cabri was designed for, but the ways in which the aims are actually realized depend on how the software is used by the teacher. The Swedish curriculum also stipulates that students should be given the possibility to see relationships. This is not far from giving them the possibility to discover and understand different relations, something that fits the objective for any DGS well. Finally, the curriculum stipulates that teaching mathematics should aim to give the students confidence in their own thinking; it should also develop their curiosity, analytical capacity, creativity and capacity to generalize.

The school curriculum in the canton of Valise in Switzerland makes no reference to the use of computers in the classroom. Nevertheless, special courses in using Cabri are available at all levels. The teaching of mathematics in the canton of Valise is described as permitting the student to use an "intellectual tool without which the student would not be able to develop the same level of scientific knowledge". Mathematics is described to be this tool. The curriculum also says that the teaching ought to show that mathematics not only is a scientific language, but also methods, reasoning and an exact and rigorous structure. Geometry is a much more important school subject in the canton of Valise, where it is taught throughout all five years, compared to Sweden where it is a minor part of school mathematics.

Methods

The purpose of the research was to investigate in which ways and to what extent three different teachers used a specific dynamic software, Cabri, in upper secondary school. The research was a qualitative case study based on a small non-randomized sample and the findings cannot be generalized to all teachers. The following criteria were used when selecting the teachers in the study: knowledge of and experience of mathematics, knowledge of and experience of teaching mathematics, familiarity with technology in general and a declared interest in using Cabri when teaching mathematics. The intention of the research was to describe in which way educated teachers with different experience of dynamic geometry used this tool in mathematics teaching. Unfortunately, it was difficult to find Swedish teachers who were well experienced in teaching mathematics with a DGS. One of the Swedish teachers had experienced Cabri in her teacher training program and she had also used Cabri to demonstrate mathematical relationships in her teaching. The other Swedish teacher in the study worked in the same school and was interested in learning more about the use of DGS in her teaching of mathematics. These two Swedish teachers belonged to a group of four teachers using Cabri at the same school. The lessons were given at the same time, though, so they could not all be visited. Due to the fact that the researcher also wanted to investigate the behaviour of a well experienced teacher in terms of using DGS, the Swiss teacher was chosen. The Swiss teacher had ten years of experience of DGS.

In table 1 below these differences between the three teachers are illustrated. The teachers are named André, Kristina and Erika. André and Kristina had studied mathematics at university level for five years. Erika had studied mathematics at the university for one and a half years after which she completed a training course at a Teachers' Training College. This is required for the Swedish upper secondary school and consequently Kristina had completed the teacher training course as well.

Table 1. *Description of the three teachers*

Name	Age	Teacher exam	Computer use when teaching mathematics	Teaching subjects
André	44	1982	about 10 years	mathematics
Erika	34	1994	about 10 lessons	mathematics, computer science
Kristina	50	1996	about 150 lessons	mathematics, computer science

The research was approved by the headmasters of the partner schools in question; in addition, the Swedish students signed an agreement allowing the researcher to make use of the results. In Switzerland it was not necessary to obtain the student's consent for studies of this kind. The Swedish lessons took place between 19th of August to 8th of October 2002 in the second year of upper secondary school where the students are about 18 years old. For practical reasons, the researcher attended two school classes in Switzerland for a period of only one week, 17th of November to 21st of November. The students were about 14 to 16 years of age and were similar to the Swedish students in terms of prior knowledge of mathematics and experience of Cabri. This was found by interviewing the teachers. The class room activities were tape-recorded and later transcribed. The researcher also took notes of what she considered significant for her study in the classroom; for example, what was the general atmosphere in the classroom, did the students listen to the teacher, did the teacher listen to the students, did the students come to class on time. A questionnaire regarding details in the teacher's background was sent out before the study (see appendix). This questionnaire was completed when meeting the teachers or by e-mail.

The analysis is based on the fact that Cabri was created and developed to give students the possibility to explore mathematical concepts, make generalizations and formulate hypotheses. It also allows the students to make their own constructions, which might very well influence their learning. If the teacher does not understand the potential and the limitations of Cabri, there is a great risk that they will use it traditionally rather than innovatively in their teaching. The focus of the analysis is on how and to what extent the teachers allowed and encouraged their students to explore and experiment using Cabri. The analysis of *How the teacher poses the problems* is divided into three parts: 1) the type of problems: pseudo questions, test questions or direct questions, and in which way the problems enable the student to inquire and explore; 2) how the posed problem takes advantage of the potential of the software; and 3) whether the teaching environment for the problem is traditional or open and if there are opportunities for discussion. The descriptions of the lessons illustrate the question: *How does the teacher put the students' experiences to use*? Also here the analysis is divided into three parts: 1) the potential of the software, 2) the teaching environment, and 3) the possibilities for discussion.

Previous research

There is a modest amount of research concerning the role of technology in teaching and learning mathematics in Sweden, with the exception of Hedrén (1990), Wikström (1997), Dahland (1998), Lingefjärd (2000) and Samuelsson (2003). Hedrén studied how children aged 9–12 learned to use LOGO. Wikström examined how model building and simulation facilitated the students' understanding of solving differential equations in upper secondary school. Dahland examined how the computer and calculator were used in secondary and upper secondary school. He found that the right software challenges the student's creativity and that problem-based environments have to be evaluated differently than the more traditional environments. Furthermore, in his view the teacher must have good knowledge of both the technology and subject matter of mathematics to be able to evaluate the students' knowledge satisfactorily. Lingefjärd found that mathematical modelling by aid of computer and calculator significantly affects the learning, teaching, evaluation and understanding of mathematical concepts at the university level. He also observed that the students' mathematical capability grew partly as a result of using the technology. Samuelsson, lastly, examined the possibility of using computers to change the teaching in upper secondary school. He found no significant changes in how the teachers taught mathematics.

How computers contribute to the learning of mathematics is of course influenced by the learning environment, which can vary considerably. Schofield (1995) found that some teachers have difficulty in changing their traditional way of teaching and using the computer. According to Säljö (2000), changing one's teaching methods easily causes problems. For example, instead of repeating information and presenting facts, as was done in the past, students are now expected to build up their own knowledge base. Hannafin, Land and Hill (1997) described these two learning environments as *Directed environments* and O*pen-ended learning environments* (OELE). Another factor that is important for learning mathematics is the mathematical language (Sierpinska, 1998). Therefore the teacher has to be well acquainted with the terminology and use it consistently.

There are also important differences between different software regarding the opportunities they offer the students. Schofield (1995) found that skill and practice software, which produce good skills in calculation and tasks done according to a prescribed pattern, often bring very little challenge to the teaching. Each task has just one correct answer. Schofield examined ten classes, eight of which used software called Geometry Proof Tutor. This software is similar in some respects to DGS. Schofield found that the software did not restrict the students in their work and those who needed more time with the teacher received the time they needed. The teacher was more of a coach and less of a person simply stating facts. Schofield also found a number of factors that increased the students' motivation when using the software, such as personal challenge and competition, any frustration the students experienced was directed towards the computer and not towards the teacher. Another advantage was that the students were not afraid of making mistakes. The importance of making room for the students to make mistakes has been argued for by Jones, (2002), and Goldstein, Povey and Winbourne (1996).

Apart from skill and practice software and DGS, there are also ready-made multimedia software which is often introduced as pedagogical software with beautiful pictures and seductive special effects. However, here the constructor has decided in advance what answers that could be obtained. Although it is true that there are often different ways to work through the software, the sum of facts is predetermined. Cabri, on the other hand, contains some 65 commands that can be combined in many different ways and which the students can use in an open-ended exploratory fashion. It is also possible to construct new commands. This makes the work with Cabri a highly dynamic process.

However, the openness of DGS should be not only a function of the software, but also mirror its use by the teacher. The important role of the teacher has been explored by Balacheff (1993), Laborde (1993), Kilpatrick and Davis (1993) and others. Their view is that knowledge about learning theories and familiarity with the software is essential for the outcome. Apart from openness, such factors as visualization and possibilities for interaction whereby the students can also learn something through their mistakes should not be neglected as they characterize dynamical geometry software.

> Tasks given to the students may require different strategies in a computer environment and thus call for different knowledge than in a

> paper and pencil environment. Changing tools leads to changing the way tasks are performed and the facilities offered by computers may offer a strong interaction between visualization and knowledge in geometry. (Laborde, 1993, p. 48–49)

Of vital importance is that the teacher poses the problem to be solved with DGS in such a way that the tasks will take advantage of the more open-ended processes it offers compared with a 'paper and pencil environment'. Watson and Mason (1998) claim that if teachers rely only on the type of questions given in the textbooks that are made for a paper and pencil environment, teachers will not develop their capacity to pose open-ended problems. The authors also claim that whether this development will occur or not depends on the teacher's attitude to mathematics and his/her awareness of learning theories. Furthermore, posing problems in more open-ended ways could very well develop deeper mathematical thinking.

Findings from the Swiss college

The Swiss school is a private monastery college. The students followed the general curriculum used in the canton. The students were 14–19 years old. The college prepares students for theoretical studies at university.

Andrés views on teaching mathematics

The following quotations are taken from the verbatim replies which the Swiss teacher André gave back, based on the questionnaire the teachers were asked to answer before the study (see appendix):

- Mathematics is the art of 'how to question'.
- Mathematics is necessary for the students to grow and become an adult.
- Mathematics is to go over the natural intellectual capacities and grow in competence.
- To show us how humanity has grown during all the centuries thanks to mathematics and how mathematics is connected with history, philosophy, religion, politics ...
- Mathematics is to give us good capacities in mental computation, algebra and all techniques in mathematics in computation, demonstration ... in order to prepare them for future studies.

- Mathematics is to show that everybody can do mathematics and also love it. It is not true that some are able and others are not.

When I asked André what he meant by saying mathematics is an art, he wrote the following in his reply:

> I do not want to give my students the answer with the questions ... I want to teach a language, some rules and how to reflect (with their own minds) in order to solve problems. Most important is that the students reflect on their own ... and the teacher is like a guide for them and not like somebody who gives them something to eat. I try to teach them how to fish, not give them the money to buy the fish ...!! So when teaching I speak a lot with the students and I try to teach using their reflections for explaining my courses.
>
> (E-mail, 2003–04–14)

According to André, knowledge of mathematics can be described in three steps: to learn, to understand and to create. He also said: ”You must not learn (only) by heart, you should use your head *and* heart”. The students always used an ink pen and sheets of A4 paper to write on. André's idea was that students should not have the possibility to erase their notes, mirroring their thoughts. He meant that thoughts are never wrong although they might have to be modified. André also told me, that he and his students could thereby follow the process involved in solving a problem. Furthermore, the students could then also learn something from their initial thinking about the problem. In a similar way, it is possible to replay every command used by a student or a teacher in Cabri, when investigating and solving a problem.

Regarding how he viewed using the computer in his teaching, André replied: ”The 'didactic' is different, obviously, but for me the principal problem when teaching mathematics is to give the mathematics 'life' and the life by spirit is movement ... this is the principle of the success of Cabri. Teaching is the possibility to create mental images – visuals, additives – in three dimensions; so the computer is very important.” André defended using the computer in order to ”be up to date”, to make the dynamics of mathematics and abstraction visible, to give the students the possibility to explore, hypothesize and make conjectures, and for students and teachers to work together in partnership, each with their own competencies and experiences.

The structure of André's lessons

The classroom lessons were closely connected to the work in the computer room. The classroom had one computer attached to a beamer. The teacher had prepared the schedule for all the lessons in advance, and published them on his Internet website.

The students worked in pairs. The instruction papers delivered to the students included the total work for the week. The purpose of the first lesson was to discover how the software worked by examining and describing four axioms. The instructions were very detailed, showing exactly which command to use for each axiom.

Axiom 1: The plane.
Axiom 2: The straight line. Construct several straight lines in the plane. Move them with the mouse and name them.
Axiom 3: The parallel lines. Construct through a point A, a line parallel to a given line d. Move its position.
Axiom 4: Construct a segment and a ray and move their positions.

The following two lessons in the classroom were devoted to the teacher's and students' discussion of their experiences of working with the software. On the last day of this first week, the students were asked to construct a triangle of three arbitrary side lengths (called axiom five) and to answer the following questions: Is this construction always possible? In which cases yes and in which cases no?

During the second week, as André informed me, the students were given a 'real-life problem' to solve: Where should an energy station (E) be placed in a river in order to deliver energy to villages A and B with a minimum loss of energy: on the same or the opposite side of the river? The students were then suggested to investigate where to place E if there were four villages but no river, and lastly, in three villages where there were special circumstances. No recommendations regarding how to solve the problem or how to use Cabri was given. The students could thereby investigate other possibilities. The problem was called Axiom 5: The economy of energy. Experiment, discover and make conclusions! The loss of energy distributed from a central to the village depends on the distance. The problem is to minimize the sum of the distances between the villages and the central.

The problem sheets were arranged progressively, both on the individual sheets and from one sheet to the next, starting with the investigation of lines and segments with all the commands described; proceeding to a more complex figure, a triangle to be constructed from three arbitrary segments, with only the new commands given; and ending with a practical problem with no commands given. In order to solve the problem the students had to make their own decisions and draw their own conclusions about how to proceed. They were then asked to describe in writing how they tackled the problem. The Swiss students were asked, in addition, to write down what they actually 'saw', not what they thought the teacher wanted them to see.

Here is another example of a problem for André's students in the second class. This class had to find a treasure by extracting facts from a text and translating them into a geometrical figure. To find the treasure, the individual in the problem had to walk a given path guided by special instructions. The problem could be solved, with some difficulty, without using dynamic geometry software. The students were astonished by the solution because even if the primary conditions were changed the treasure remained in the same place. In giving this problem, André's intention was to underline the importance of being curious about the answer and about how to prove that the answer one arrived at was the correct one. Judging from the students reactions, André achieved his aim.

Result

More or less all students remarked orally that it was frustrating to only *see* the mathematical relations on the computer screen, but also quite challenging to try to make formal proofs. The teacher's role, according to André, is to teach mathematical thinking by giving the students a mathematical language and a chance to develop their powers of logical thinking. He also stated that it was important to teach different techniques for exploring mathematics through concrete problems, sometimes without having to do calculations. The students continued to examine the above problems using Cabri, even after the teacher showed them one of the possible solutions. They kept working at the problem as long as they thought there might be more to discover. If they did not know how to proceed, they used trial and error before asking the teacher. It was evident that the students appreciated the opportunity to cooperate with each other. The teacher encouraged the class by telling them that they worked as true mathematicians and that what they had uncovered was of significant importance. This most likely gave them confidence and encouraged them to examine the problem even further. This was also noticed as all the pupils did not stop examining even if the lesson had finished

André constructed problems that either had to be solved, or were easier to solve, with Cabri than with just paper and pencil. Some of the exercises were an introduction to formally proving a theorem. André made clear to the students, that what they saw on the computer screen was not necessarily true.

André seemed to have taken advantage of the visualization and animation effects of the software in formulating the problems, which the students said they appreciated. This could very well help them grow – one of his intentions in the teaching.

Findings from the Swedish school

In Sweden, children enter compulsory school at the age of six or seven, and remain for nine years. Most of the students move on to the three-year Swedish gymnasium, i.e. upper secondary school. The two teachers were Kristina and Erika, both teaching in upper secondary school.

Kristina's and Erika's views on teaching mathematics

Kristina stated that one main reason for her to teach mathematics was that the students would be able to attain and pass the national exams in mathematics. She described knowledge and teaching in mathematics as involving doing calculations, using logical thinking, applying formulas and axioms in solving problems, describing reality by modelling, analyzing and making prognoses.

Kristina's intension was to use the computer to make her teaching more stimulating, more efficient and more comprehensible. She described Cabri as a facilitating tool in that it may help the students to understand concepts and relations and to develop their creativity.

Erika's intention with her teaching, on the other hand, was to help the students understand mathematics in general, to relax and have fun in class. In her view, mathematics entails thinking logically, doing calculations and confirming one's knowledge. She describes attaining knowledge in mathematics in terms of the students' developing greater self-confidence and their ability to think logically.

The structure of Kristina's lessons

The 18-year-old students worked in pairs. The lessons all followed the same pattern and were distinct from each other in terms of content. Each lesson started or ended with an example from real life. As an example of a real life problem, the lesson used a problem related to a 'Ferris wheel'. The main problem was to find the one correct answer to how many baskets there were in the wheel (se figure 4).

The purpose of the lesson was to find the relation between the angles subtended by an arc and their relation to the corresponding centre angle. The lesson began with the teacher reading aloud and explaining the instructions and then giving a short demonstration of Cabri. The teacher demonstrated how to draw the angles and the arc using Cabri. Then the students should do the same thing according to the instructions on their work sheets. The instructions were very detailed; for example the lesson with the Ferris wheel. The sheet was called Laboratory work with subtended angles and centre angles and the sheet had several tasks.

Task 1a) Draw a circle in Cabri, mark the points A and B. (a circle was on the sheet with M, A and B marked, M for the centre). The arc between two points on a circle can be chosen in two different ways. Examine and mark with the command *Arc*.

Task 1b) Get acquainted with the definitions. (The words subtended angle, centre angle and arc were illustrated by a picture)

Task 1c) Draw three subtended angles standing on the same arc. Mark and measure them. Write down what you discover. Save the file. Take a printout and give to the teacher.

Then there were Tasks 1d – f with similar instructions. The last task was 1g (see figure 3).

Task 1g) Discover and formulate the relationship between the centre angle v and the subtended angle z on the same arc BC.

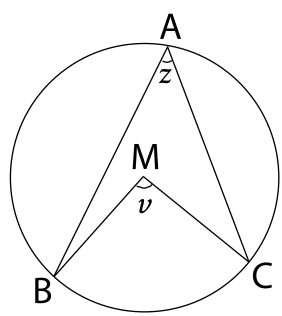

Figure 3. *Task 1g) on the student's sheet*

Students answered by filling in the missing words in the sentence:

_________is________as big as___________on the same_________.

The students seemed very anxious to fill in the correct words and they were afraid of not coming up with the correct answer.

Task 2, the *Ferris wheel problem* (figure 4). This problem was taken from another textbook and formulated like this. You sit in a ferry wheel and look at the baskets on the other side. When you see two baskets next to each other on the other side of the wheel the angle between them is

12 degrees. How many baskets are there on the wheel, if they are placed with constant distance between them?

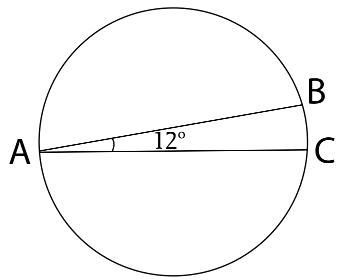

Figure 4. *Illustration to task 2 regarding the Ferris wheel*

Result

The students were given the same instructions at least twice. Kristina repeated in smaller groups what she had informed the whole class about at the start of the lesson and she also repeated what was written in the instructions. Nevertheless, several students anyhow asked her how to do the exercises. It is quite possible that the missing words in the sentences on the instruction sheets can restrict the students' creativity and the possibility of arriving at different answers. The mathematical concepts that are used in the exercise are supported by the figures on the instruction sheet, and a possible conclusion is that not much was left for the students to discover on their own or making mistakes and learn from them.

Kristina seemed to formulate the problems on the basis of the answers she wanted to obtain. This could make it difficult for students to discover unexpected answers. The students did not try any more when they thought they had answered the questions asked. Kristina had used Cabri in earlier classes for visualizing mathematical concepts. She rarely used the correct mathematical terminology and instead accepted the students' own terminology and means of expression.

Out of four teachers (using Cabri at the Swedish school), only one teacher, Kristina, actually gave an examination test to be solved with Cabri Géomètre The test was formulated more freely and consequently the students answered in a liberated and more creative manner, which also revealed their way of thinking. In this context, a creative manner means, among other things, that the students used many more words in their replies than before when they had to find the missing correct words. Now they had to formulate a whole sentence and a few pupils asked themselves new questions. When the students had finished the exercise on the computer, Kristina asked them to solve the problems in the textbook.

The structure of Erika's lessons

Erika's views on teaching mathematics have been mentioned above. In Erika's class a small group had been formed consisting of students who had experienced difficulties in understanding mathematics. The computer sessions typically meant repeating what had been prepared in the classroom with more or less the same kinds of problems. The students realized this and told Erika that they already knew the answers. The teacher ignored these comments and continued to teach in accordance with her aim of confirming the students' prior knowledge in mathematics. She generally used the same kind of problems as Kristina but Erika worked more traditionally with them. She did not use the problem about the Ferris wheel.

The instructions about subtended angles and centre angle started with a figure with a marked and measured subtended angle.

Task 1a. Open the file XX, mark the angle and measure it. Drag the point A along the circle. What do you discover?

Task 1b. Draw the centre angle BMC, mark it and measure it. Then the instruction was: Drag A along the circle and find a place where you easily can understand the relationship between the measurement of the subtended angle and the centre angle.

Task 1c. Drag B along the circle study both the subtended angle and the centre angle. Is the result the same as in the previous task?

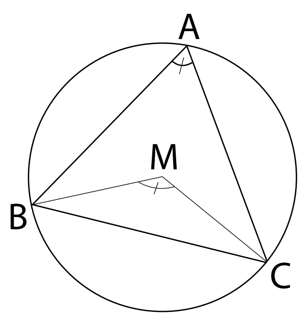

Figure 5. *Illustration from Erika's Task 1c*

Then the student should complete the sentence,

_______is_________as big as__________on the same___________.

This sentence is called______________________!

Erika preferred to illustrate mathematical relationships with Cabri Géomètre.

Task 2. Illustrate the following theorems. First construct a circle.

I. All subtended angles on the same arc are equal (they are half the corresponding centre angle).

II. The subtended angle on a half circle is 90 degrees (the corresponding centre angle is 180 degrees).

III. In a quadrilateral inscribed in a circle is the corresponding angles (Erika meant the opposite angles) supplement angles $x + y = 180$ degrees (the sum of the corresponding centre angles are 360 degrees).

Task 3. Solve the problems on p. 132–133 in your textbook.

On one occasion, for example, Erika asked the students to visualize written facts about the angles in a circle instead of letting them examine the relations in an exploratory fashion.

Result

The students told the teacher that they already knew the answers before working with Cabri, except for the exercise in which they examined a straight line. Here they were less restrained and could discover unexpected facts, just as Kristina's students did when tackling the same problem. The aim of the exercises in the computer room was the same for these two teachers; namely that the students should be able to solve the problems in the textbook according to the teachers agenda.

This part of Erika is quite short depending on the fact that her problems were more or less the same as Kristina's but fewer. Erika did add the visualization problem (Task 2) to her agenda though.

Results from the three teachers

How a problem is posed can affect the way the students use the DGS:

a it shows what the teacher expects the students to learn,

b it determines how the students are to work, c it is a potential source of challenges for the students, and

d it influences whether the students will make unexpected discoveries.

André's problems were in a way open-ended and the pupils could learn from their mistakes. For example constructing a triangle from three arbitrary segments might not succeed . Kristina's and Erika's posed problems where the pupils should fill in correct words in a sentence. Then the task was finished. It would be interesting to make a research if the pure mathematical knowledge has any influence on problem posing. Both André and Kristina have studied mathematics for the same time. Kristina was the teacher among the Swedish ones who took the initiative to start this project and also the only one who tried a different form of examination. André's problems differed from the Swedish ones.

How the problem is posed can be closely related to the teacher's attitude to mathematics, the teacher's knowledge of mathematics and last but not least, her/his knowledge of Cabri's potential. Samuelsson (2003) found no significant changes in how teachers taught mathematics when using the computer. In this research that is true for Erika and most likely for Kristina but André claimed he had changed his way of teaching. He said you can solve different sorts of problem and using Cabri needs a different sort of didactics. Both Kristina and André claimed that working with DGS helps students to take more responsibility for their own learning. Table 2 is a short summary and does *not* do full justice to the teachers.

Discussion

Different cultures

Swiss schools and Swedish schools belong to different cultures and should be considered differently, although there is much to learn from both. The two Swedish teachers told the researcher that their students did not regard mathematics as an important subject in school. The Swiss teacher had the opposite view of his students. All the three teachers had a mutual high regard for Cabri software as a support in teaching.

The importance of the role of the teacher cannot be emphasized enough, even when computers are used in the teaching. From what was observed, teachers need support if they are to take full advantage of the possibilities of the computer. To take full advantage of the computer means here to pose problems that take advantage of the dynamic properties of the software and that give students the opportunity to learn from their own experiences. For example, this is how Jones (2002) describes the role of the teacher.

Table 2 Summary of the teachers' opinions regarding some key concepts

RESULT	André	Kristina	Erika
Terminology	Very important: makes mathematics clearer and more distinct.	Considers it to be important but adapts it to the language of the students.	Is not aware of which terminology she used.
How was the problem posed?	Challengingly. Each task was part of a whole. Problems from daily life. The problem could be solved only with dynamic geometry software.	Separate tasks similar to those in the textbook. Problems from daily life. The test tasks were more challenging. The tasks about the 'Straight line' were more explorative.	Separate tasks similar to and often repetitious of those in the textbook. The tasks about the 'Straight line' were more explorative
How did the teacher use the students' own experiences?	Introduction to a formal proof. Awareness of the necessity of theory. The report should reflect what they had really seen, not what they thought was correct.	To solve the tasks in the textbook.	To solve the tasks in the textbook.
In which way was the potential of the software used to full advantage?	The tasks could be solved only by using the software. The instructions progressed from command to free exercise. No limitations. Instead student creativity was encouraged.	Drag mode and the visualization effects were used. The students seldom used their own initiative.	Drag mode and the visualization effects were used. The students seldom used their own initiative.
How does the teacher regard the computer compared with other tools?	One tool among others.	The computer software enhances understanding, although the textbook and doing calculations were central.	Most important were the textbook and doing calculations

> Indeed, classroom experiments have shown that the software itself does not grant the transition from empirical to generic objects, from the perceptive to theoretical level. The teacher plays a very important role in guiding students to theoretical thinking.
>
> (Jones, 2002, p. 20).

The *National Swedish agency for education* has defined the desire to learn mathematics as a combination of curiosity, imagination and an eagerness to discover (Skolverket, 2003). Dewey (1938/2004) points out that impeding students' desire to learn in school is to rob them of their willingness to learn in the future. This means that we are obliged to stimulate the students' desire to learn mathematics, which naturally depends very much on how we teach.

The Swiss teacher, André, who viewed mathematics as a humanistic subject that involves logical thinking, presented vivid and engaging problems and encouraged his students to find non-traditional ways to solve them. André often used figurative language in talking about his teaching. He saw the possibilities of the computer and used them to advantage. André also wanted to vary his teaching and to make the students work in the same manner as mathematicians; that is, to investigate, explore, conclude and test. André's way of giving his answers in a vivid way might be the reason why the description of André is longer than Kristina's and Erika's. They were notably shorter in their answers.

The Swedish teachers, Kristina and Erika, seemed to be quite bound to the curriculum and the textbook, although they too had expressed a desire to teach in a new way. It seemed to the researcher that they described their teaching in mathematics in the same way as they themselves had learnt the subject in school. Obtaining the correct answer was of prime importance; even if the computer was a useful tool for helping students to visualize mathematics and make the teaching more stimulating and comprehensible. The Swedish teachers used the computer as part of their traditional teaching. So-called traditional teaching is the dominant approach in school and has been described in detail by Schofield (1995), Hannafin, Land and Hill (1997), Lingefjärd (2000), and Samuelsson, (2003).

André adjusted the problems he gave the students to meet the capacity of the software. Several researchers have shown the importance of changing tasks when changing tools (see e.g. Keitel & Ruthven, 1993, Jones, 2002; Laborde, 1993), which is what André did. He supported and encouraged the students by telling them that the work they did and the results they obtained were very important. The students received a good deal of attention, had ample time to discover things on their own, and they used these possibilities to advantage. André was very anxious to maintain the language of mathematics. Furthermore, he made working on the computer an integral part of the two-week lesson plan.

Kristina used to advantage the dynamic effects of the software and put a good deal of effort, with some success, into getting the students interested in working in this way. Kristina seemed to want to change her teaching style. Although she was still clearly influenced by the traditional way of teaching. She used Cabri for laboratory work, which should have given the students the possibility to make their own discoveries and draw their own conclusions. However, the problems she presented were very similar to those in the textbook, since the students were expected to recognize them. Many of the students preferred the textbook and Kristina even said that they would not attend the lessons if they felt that the problems were too unfamiliar. Nevertheless, the test she gave the students contained problems that differed from those in the textbook. The majority of the students enjoyed working with the problems in the test, probably because they had become accustomed to the software by that time. Moreover, the test was obligatory and the students had to take it if they wanted to get good marks.

Erika, finally, taught in a traditional way. Generally, in her lessons the students were expected to take small steps along the path of mathematics. This is probably due to several factors: her earlier childhood schooling in mathematics, the fact that she had only one and a half years of mathematics at university level, and her lack of experience with Cabri.

Dynamic geometry software and the teaching environment

The purpose of the present research was to study the three teachers concerned from a variety of viewpoints. We have concentrated on how they posed the problems in the exercises, their teaching methods, how they met their students, how they used Cabri and their attitude to mathematics. How the teachers interpreted the curriculum also proved to be of vital importance. It goes without saying that teachers are dependent on the teaching environment which includes, among other things, the school system and the extent of the students' and teachers' competence in using the software. Similar to what Samuelsson (2003) found, the Swedish teachers did not change their way of problem posing. The problems they gave the students reminded of those in the textbooks, with just one correct answer. Kristina gave the reason (above) why the problems looked like the way they did. On the contrary the Swiss teacher posed problems that took extended advantage of the software. Even if his problems had just one answer, like the treasure problem, it made the pupils curious to think further. They even became frustrated, as I have mentioned earlier. In other words, André said he had changed his way of teaching when using the DGS software.

Consequences for teacher training

Stimulation of an interest in mathematics as something more than an instrument for doing calculations might very well be the result of changing one's way of working with the subject. Under the right conditions, laboratory work with Cabri, or any other dynamic software, is a valuable instrument in upper secondary school. But learning theories combined with the use of dynamic software definitively calls for a reformed teacher program for prospective teachers. Instead of using technical aids in traditional teaching, the teaching environment has to be changed in order to regenerate the students' desire to learn mathematics. With this open approach, students hopefully will discover that mathematics is much more than simply finding the only correct answer.

The findings of this study indicate that Cabri can be used in the traditional way of teaching as well as in more open-ended teaching. *These teachers, through their way of posing the problems, determine what possibilities the students have to learn mathematics*. To take full advantage of Cabri Géomètre, the teacher ought to pose problems in a way that does not limit the students' learning.

In closing, a reference to Sierpinska (1993) might be relevant in order to summarize and validate the research:

> Thus, the relevance of a research study may consist not only in a direct improvement of the practice of teaching, not in a growth of our understanding or knowledge, but in giving an impulse for further research, in pointing towards new questions and new avenues to explore them. (ibid., p. 46)

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Appendix

Questions to the teacher:

1. Name:
2. Age:
3. Examination year for the teacher exam:
4. Teaching subject(s):
5. The year of the pupils you teach:
6. For how long have you been using Cabri when teaching mathematics?
7. Describe your objectives when you are teaching mathematics.
8. How would you describe knowledge in mathematics?
9. Is your teaching strategy the same with or without the computer when teaching mathematics? Please explore your answer.
10. Why are you using the computer when teaching mathematics?
11. What does your curriculum say about the use of computers when teaching mathematics?

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Sammanfattning

Artikeln presenterar en studie av hur tre gymnasielärare, en i Schweiz och två i Sverige, använder dynamisk programvara, Cabri Géomètre, i sin matematikundervisning. Empirin består av ljudupptagningar och fältanteckningar från lektionsobservationer. Anlysen genomfördes med avseende på lärarens problemformuleringar samt hur läraren tar vara på elevernas erfarenheter när Cabri Géomètre används.

Resultatet visar på betydelsen av hur läraren formulerar problemen eller uppgifterna för att utmana eleverna, samt för att utnyttja programvarans särskilda förutsättningar. I en laborativ undervisningsmiljö i matematik kan ett dynamiskt datorprogram skapa möjligheter för elever att lära sig längs ej förväntade vägar.