From problem solving to modeling

The emergence of models and modeling perspectives

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More than 25 years ago, a research project in the U.S investigated the question: "What is needed by students, beyond having a mathematical idea, that enables students to use the mathematical idea in everyday problem solving situations? (Lesh, Landau & Hamilton, 1983). The answer to this question has begun to emerge after 25 years of systemic work in the domain of modeling. In this paper, we chronicle the emergence of models and modeling perspectives (MMP) from the genre of problem solving research via a synthesis of the major strands in the extant literature.

An increasing number of mathematics education researchers have begun focusing their research efforts on mathematical modeling at the school level. This is evident in numerous research publications from groups of researchers in Australia (English, Galbraith and colleagues), Belgium (Verschaffel and colleagues), Denmark (Niss, Blomhøj and colleagues), Germany (Blum, Kaiser and colleagues), Netherlands (de Lange and colleagues) and the U.S (Lesh, Schoenfeld and colleagues). Among the questions that have been raised, is how well prepared are learners today to solve the problems that they will encounter beyond school, in order to fulfil their goals in work, as citizens and in further learning? (Christou, et al., 2005; OECD, 2004; Doerr & English, 2003; Schoenfeld, 1992). How can students work with problems that are less obviously linked to school mathematics and require students to deal with unfamiliar situations by thinking flexibly and creatively (Lesh & Doerr, 2003a, 2003b)? It is necessary to distinguish *problem-solving* activity as described by

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Schoenfeld (1992) and other researchers (for example, see English, 2003) from the activity of solving traditional word problems. Traditional word problems usually present simplified forms of decontextualized world based situations. The purpose of such word problems is only the exercise for specific type of mathematical learning, such as addition or subtraction (Wyndhamm & Saljö, 1997). The use of word problem solving in school mathematics hardly matches this idea of mathematical modeling and *mathematization*, which is the structuring of reality by mathematical means (Freudenthal, 1991). Student work with that type of solving problems is absent of heuristics and mathematical strategies and the result is a prevalence of mechanical and mindless solutions (Greer, 1997). A second, probably worse, consequence of student work in solving traditional word problems is the absence of high level cognitive and metacognitive processes involved. This absence is forwarding students to look for key words and employ direct translation strategies to solve a problem (Schoenfeld, 1992). Burkhardt & Pollak (2006) take the modeling perspective of applied mathematicians in contrasting the pure versus applied nature of mathematics relevant for today's world.

> While the 'pure' and 'applied' viewpoints have many things in common, including delight in the elegance of mathematics, they differ in others that are important for the design of school curricula – notably the central importance of teaching modeling. (p. 178)

This call is consistent with recent initiatives to emphasize the applied nature of mathematics used in emergent professions. Steen (2001) argued that in spite of the rich and antiquated roots of mathematics, mathematicians among others should acknowledge the contributions of researchers in external disciplines like biology, engineering, finance, information sciences, economics, education, medicine etc who successfully adapt mathematics to create models and tool kits with far reaching and profound applications in today's world. These interdisciplinary and emergent applications have resulted in the field of mathematics thriving at the dawn of the 21st century (Lesh & Sriraman, 2005a, 2005b).

Another important perspective to mathematical modeling is that it should also foster critical mathematics education (Skovsmose, 1994, 2000). Although the *National Council of Teachers of Mathematics* (NCTM, 2000) calls for purposeful activities along with skillful questioning to promote the understanding of relationships among mathematical ideas, this recommendation can be pushed further and modeling activities can be used as a way to cultivate critical thinking and critical literacy (D'Ambrosio, 1998; Gutstein, 2006; Michelsen, 2006, Skovsmose, 2000; Sriraman & Lesh, 2006). In other words, modeling problems situated in the real world can also be used to foster critical mathematical literacy. Critical mathematical literacy[1] holds that the aim of mathematics learning and teaching is not simply to impart procedural skills and functional literacy but create situations in which students are able to identify, interpret, evaluate and critique the mathematics embedded in social and political systems and claims. The literature also reports success of the critical approach using data from newspapers, maps, advertisements, and ecological data etc as a means of getting students to think about the basis for decision-making and the inequitable societal distribution of resources (Gutstein, 2006; Sriraman & Lesh, 2006). Lesh & Sriraman (2005a) contend the following in mathematics and science:

A Modeling is primarily about purposeful description, explanation, or conceptualization (quantification, dimensionalization, coordinatization, or in general mathematization) – even though computation and deduction processes also are involved.

B Models for designing or making sense of complex systems are, in themselves, important 'pieces of knowledge' that should be emphasized in teaching and learning – especially for students preparing for success in future-oriented fields that are heavy users of mathematics, science, and technology. Therefore it is important to initiate and study modeling, particularly those of complex systems that occur in real life situations from the very early grades.

This perspective traces its lineage to the modern descendents of Piaget and Vygotsky, but also to American Pragmatists like John Dewey, George Herbert Mead and Charles Sanders Pierce. The philosophy of this perspective (Lesh & Sriraman, 2005a, 2005b) is based on the premise that:

- Conceptual systems are human construct, and that they also are fundamentally social in nature.

- The meanings of these constructs tend to be distributed across a variety of representational media (ranging from spoken language, written language, to diagrams and graphs, to concrete models, to experience-based metaphors.

- Knowledge is organised around experience at least as much as around abstractions – and that the ways of thinking which are needed to make sense of realistically complex decision making situations nearly always must integrate ideas from more than a single discipline, or textbook topic area, or grand theory.

- The 'worlds of experience' that humans need to understand and explain are not static. They are, in large part, products of human creativity. So, they are continually changing – and so are the knowledge needs of the humans who created them.

The models and modeling perspective adapts Zoltan Deines' instructional principles to design model eliciting activities. An example will help better illustrate the notion of a model-eliciting activity. Suppose students are asked to 'rate' the quality of all the potential players on a volleyball team and then select the team based on a consensus on the ratings. This task requires students to gather/procure 'objective' data related to players speed, endurance, past performance, special abilities and reach 'subjective' consensus on the criteria most valued for the selection of the team. This is a model eliciting activity because it invokes the six instructional principles of Lesh et al. (2003) namely, (1) the *reality principle* (i.e., does the situation warrant sense making and extension of prior knowledge/experiences?), (2) the *model construction principle* (i.e., does the situation create the need to develop (or refine, modify, or extend) a mathematically significant construct?, (3) the *self-evaluation principle*, (does the situation require self-assessment?), (4) the *construct documentation principle* (i.e., does the situation require students to reveal their thinking about the situation? (5) the *construct generalization principle* (i.e., is the elicited model generalizable to other similar situations?) and finally (6) the *simplicity principle* (is the problem solving situation simple?). The construct of 'model-eliciting' circumscribes a problem solving situation, its mathematical structure, the mathematical models generated as well as the problem solving processes that are invoked by the given situation. Authentic examples of model eliciting activities based on the design principles of Dienes are found in Lesh & Doerr (2003a). A recent adaptation of the volleyball problem for the Danish context is the handball problem designed by Iversen and Larson (2006). The model eliciting perspective is based on the premise that modeling research should take into account findings from the realm of psychological concept development to develop activities which motivate and naturally allow students to develop the mathematics needed to make sense of such situations. More generally, *models and modeling perspectives* emphasize promising aspects associated with both sociocultural theories and theories of situated cognition. Neither of these latter perspectives necessarily forces researchers to re-examine their own basic assumptions about the changing nature of elementary mathematics – nor the changing nature of 'real life' problem solving situations in which some type of 'mathematical thinking' is useful. In fact, just as in traditional research on problem solving, both of these perspectives tend to begin with the assumption that the researcher already possesses clear and accurate conceptions about the nature of elementary mathematics – and about what it means to 'understand' relevant concepts and processes (Lesh, personal communication, November 23, 2005).

A modeling perspective to problem solving leads to the design of an instructional sequence of activities that begins by engaging students with non-routine problem situations that elicit the development of significant mathematical constructs and then extending, exploring and refining those constructs in other problem situations, leading to a generalizable system (or model) that can be used in a range of contexts (Lesh & Doerr, 2003a, 2003b; English & Doerr, 2006). For instance, in problem solving activities, referred to as model eliciting activities (Lesh & Sriraman, 2005a, 2005b), the products that students produce go beyond short answers; they include sharable, manipulatable, modifiable, and reusable conceptual tools (e.g. models) for constructing, explaining, predicting and controlling mathematically significant systems (Lesh & Doerr, 2003a, 2003b).

Students' descriptions, explanations, and justifications form an integral component of the models the students produce. In contrast to many of the problem situations students meet in school, modeling activities are inherently social experiences, where students work in small teams to develop a product that is explicitly sharable. Numerous questions, issues, conflicts, resolutions, and revisions arise as students develop, assess, and prepare to communicate their products (English & Doerr, 2006; Lesh & Sriraman, 2005a, 2005b).

In an attempt to review the related literature and provide a coherent state of affairs, the literature review in the present paper is organized into three major discussion strands. The first major strand situates mathematical modeling as being a problem solving activity and issues related to the teaching, learning and assessment of modeling are discussed. The second major strand presents some basic principles for designing modeling activities and finally, the third strand discusses the benefits for students and teachers in working with thought revealing modeling activities.

Mathematical modeling as a problem solving activity

Inadequacy of traditional teaching approaches

A number of researchers raise the question of the appropriateness of current teaching approaches in teaching mathematics and in mathematical problem solving in particular (Doerr & English, 2003; Sriraman & Lesh, 2006). The inadequacy of traditional approaches is even worse in the case of students' work with problems that are less obviously linked to school mathematics and require students to deal with unfamiliar situations by thinking flexibly and creatively (Lesh & Doerr, 2003a, 2003b). A necessary distinction should be made between 'problem-solving' activity, as proposed by Polya (1973) and Schoenfeld (1991) and relates to Dewey's *reflective thinking* and the activity of 'solving problems' and the traditional use of word problems in a school environment. Polya (1962) stressed that "in solving a word problem by setting up equations, the student *translates* a real situation into mathematical terms: he has an opportunity to experience that mathematical concepts may be related to realities, but such relations must be carefully worked out" (p. 59). National Research Council documented that mathematical problem solving rather deals with data and observations from science. Mathematical modeling is far more than just calculation or deduction; it involves observations of patterns, testing of conjectures, and estimation of results (Schoenfeld, 1992). Word problems appear in most textbooks usually represent recontextualized forms of decontextualized descriptions of everyday life situations. In most of the cases, such problems are just exercises for specific types of mathematical learning, such as addition or subtraction (Wyndhamm & Saljö, 1997). Hiebert et al., (1996) pointed out that solving such word problems cannot prepare students for everyday life, since students are not able to transfer the specific domain-related knowledge (mathematics) and also more general problem- and solution-related skills. Their recommendation was to base instructional design on problem solving in out-of-school situations, rather than on problem-solving models of limited applicability (Hiebert et al., 1996).

A significant number of recent research studies pointed out that the practice of word problem solving in school mathematics hardly matches this idea of mathematical modeling and mathematization (Reusser & Stebler, 1997; Kaiser, Blomhøj & Sriraman, 2006; Kaiser & Sriraman, 2006; Mousoulides et al., 2006). As a result, students readily 'solve' unsolvable, even absurd, problems if presented in ordinary classroom contexts. Students do not have opportunities to do mathematics; students look for key words, employ direct translation strategies when solving stereotyped word problems and their ability in solving these problems is influenced by contextual information (Greer, 1997; Verschaffel, De Corte & Lasure, 1994; Yoshida, Verschaffel & De Corte, 1997).

The role of context in mathematical modeling

Although modeling problems are contextualized by nature, and authentic problems always manifest themselves in a context, it is nevertheless important to explicitly address the role and the nature of contexts used. The role of context is very important in mathematical modeling, since modeling requires a context in which to 'frame' the problem and 'develop' the mathematics. This meaning of contextual teaching and learning is close to the notion of *situated learning* (de Lange, 1992). Researchers have documented a number of different functions a 'realistic context' can play in problem solving. De Lange (1987) listed the following functions: enabling concept formation, facilitating model formation, providing a wider range of utility and more interesting practice problems. Similarly, DaPueto and Parenti (1999) reported the following factors for using a realistic context in mathematical problem solving:

A facilitating collaboration, interaction and contribution of students who have different *styles* of exploration/understanding/use of concepts, different *levels* of formalized knowledge, etc,

B helping more balanced development of *reflective learning* and *experiential learning*, and

C facilitating the design and restructuring of the *schemata* through which knowledge is organized (p. 7).

A number of researchers commented on the importance of contextual based problem solving. Bohl (1998) identified the lack of relevance as a critical factor in engaging students in problem solving. Bohl (1998) documented that the use of contextual settings has the potential to assist students in actively be engaged in problem solving. Pace (2000) further pointed out that students need to experience real-world situations in the chosen context before they can create models for solving world based problems related to the same context. Contextual based teaching encourages the development of an active learning environment which in turn generates better mathematical understandings by the students (Pace, 2000). In line with previous findings, Gravemeijer and Doorman (1999) stressed that research on the design of Realistic Mathematics Education (RME) based activities has shown that the use of personalized contexts improved word problem solving by increasing the meaningfulness of contexts and enhancing student motivation. Gravemeijer and Doorman (1999) also claimed that "well-chosen context problems offer opportunities for the students to develop informal, highly context-specific solution strategies" (p. 117).

Research has also indicated a number of possible limitations related to the use of contexts in problem solving. The choice of context may be a turn-off to particular students or be so familiar as to inhibit some approaches judged as unreasonable (Choi & Hannafin, 1997). McNair (2000) pointed out that if the situation and experience used are not familiar to the students, then instruction may degenerate into the presentation of abstract ideas related to the context. Finally, Resnick (1988) addressed that there is a need for finding ways to create in the classroom situations of sufficient complexity and engagement that they become mathematically engaging contexts in their own right.

Affective factors and mathematical modeling

The importance of the affective domain and students' communication in classroom work is stressed in a number of research studies (Lesh & Doerr, 2003a, 2003b; Gravemeijer, 1997; Verschaffel, De Corte & Borghart, 1997). Yoshida, Verschaffel & De Corte (1997) reported that mathematical problem solving is now also focussed to student attitudes and beliefs and their capacity to apply their mathematical knowledge in non-routine problems. Verschaffel and his colleagues (1997) documented that student efficacy beliefs could act as a predictor of student achievement in solving authentic problems. The importance of considering student conceptions is also pointed out by Confrey & Doerr (1994), who argued that the use of learner-centered modeling tools and approaches can create positive beliefs in the mathematics classroom. The use of realistic mathematical modeling problems can enhance student sense-making, while bringing real-world situations into school mathematics is a necessary condition to foster a positive attitude towards mathematics. In line with previous findings, Bonotto and Basso (2001) interestingly reported that bringing real-world situations into school mathematics is a necessary condition to foster a positive attitude towards mathematics for many students.

While the use of modeling as a problem solving activity can positively change student beliefs towards mathematics, a significant hazard in teaching mathematical modeling is teachers' pre-conceived beliefs, and the projection of those beliefs onto their students. Verschaffel and his colleagues (1997) reported a strong and resistant tendency among teachers to exclude real-world knowledge when teaching arithmetic word problems. Teachers' pedagogical beliefs stressed that non routine problems are less important and the goal of teaching word problem solving is to find the correct numerical answer. Verschaffel and his colleagues (1997) concluded that these teacher attitudes and beliefs about the significance of real world knowledge in problem solving have a negative impact of their teaching practices and consequently on their students' learning processes and outcomes.

A quite interesting aspect of the relation between teachers' beliefs and implementation of authentic problems in the classroom is raised in the work of Doerr and English (2006). They documented that selecting and teaching appropriate activities whose features can promote powerful student learning can be a key component in teachers' negotiation of both mathematical knowledge and pedagogical content knowledge. The new roles in the practice of problem solving that teachers adopt and the negotiation of their knowledge can be the first steps in changing teachers' attitudes and beliefs towards mathematical modeling in a positive way (Doerr & English, 2006).

Assessment in mathematical modeling

Niss (1987) pointed that assessment of modeling could be problematic, since modeling is difficult to assess, let alone test, by traditional evaluation tools. Niss (1993) further clarified that assessment takes time and cannot be standardised. It does not imply that assessment cannot be exercised on a sound foundation of reflection and reasoning and articulate criteria and be subject to clear communication (Niss, 1993).

A number of different types of assessment being used to evaluate students' modeling abilities and understanding of models are found in a review of the literature. Crouch and Haines (2004) used a multiple choice format, in developing several test questions related to modeling, Kitchen (1993) proposed questions requiring students to set up a model, interpret a solution, or criticize a model based on the data, while Bell et al. (1992) and Hjalmarson (2005) suggested the use of an analytic scoring scale, by assigning point values to various dimensions of the modeling work.

Model eliciting activities: design and research

Modeling processes in problem solving

The modeling approach to problem solving suggests that there is not a single powerful procedure between givens and goals and a set of 'strategies' for overcoming any difficulties in this procedure. Indeed, the modeling approach indicates a number of trial procedures between givens and goals in order to succeed a solution. This includes a number of iterative cycles, in which students move back and forth from givens to goals, go back and again moving towards goals to test their hypotheses, refine their results and to improve their solutions (Lesh & Doerr, 2003a, 2003b).

A number of relevant works (Lesh et al., 2003; Blum & Niss, 1991) have documented the different processes involved in mathematical modeling as problem solving activity. In particular, students engage in the following processes: (a) Understand and simplify the problem. This included understanding text, diagrams, formulas or tabular information and drawing inferences from them; demonstrating understanding of relevant concepts and using information from students' background knowledge to understand the information given. (b) Manipulate the problem and develop a mathematical model. These processes included identifying the variables and their relationships in the problem; making decisions about variable relevancy; constructing hypotheses; and retrieving, organising, considering and critically evaluating contextual information; use strategies and heuristics to mathematically elaborate on the developed model. (c) Interpreting the problem solution. This included making decisions, analysing a system or designing a system to meet certain goals, and diagnosing a malfunction and proposing a solution. (d) Verify, validate and reflect the problem solution: This included constructing and applying different modes of representations to the solution of the problem; generalizing and communicating solutions; evaluating solutions from different perspectives in an attempt to restructure the solutions and making them more socially or technically acceptable; critically checking and reflecting on solutions and generally question the model (Blum & Kaiser, 1997; Lesh & Doerr, 2003a, 2003b).

Student and teacher models

A model is an internal conceptual system plus the external representations of that system used to interpret other complex systems (Lesh & Doerr, 2003a, 2003b; Lesh, Doerr, Carmona & Hjalmarson, 2003). Typically, this definition of model has only been used in reference to student or teacher thinking and learning (e.g., Doerr & Lesh, 2003). To provide a parallel construct at the researcher level, a design experiment carried out from a models and modeling perspective (a modeling design experiment) should be consistent with this definition. The design tested by the experiment encompasses two parts (similar to a model). Namely, the design includes theoretical assumptions (i.e., researcher-level conceptual systems about mathematical knowledge, models, teacher development, etc.) and external artifacts [2] (Lesh & Doerr, 2003a, 2003b; Lesh & Sriraman, 2005a, 2005b). Models consist of an internal conceptual system and external artifacts or representations (Lesh & Doerr, 2003a, 2003b; Lesh et al., 2003). In addition, models incorporate a number of external representations (e.g., a graph, a table). In constructing models, students identify, select and collect relevant data, express limitations and conditions of a model, interpret the solution in context, communicate effectively and describe situations using a variety of representation forms.

Characteristics of modeling activities

The different tools being designed and created to facilitate students' and teachers' externalization of their thinking and understandings of problem situations aim to elicit their thinking and thus researchers are referring to these tools as model eliciting activities (Lesh et al., 2003; Lesh & Sriraman, 2005a, 2005b). As stated earlier, in model eliciting activities students need to: (a) develop a model(s) that describes a real-life situation, (b) use their models to describe, revise, and refine their ideas; and (c) use a number of representational media to explain (and document) their conceptual systems. Model-eliciting activities can be designed to lead to significant forms of learning because they involve mathematizing – by quantifying, dimensioning, coordinating, categorizing, algebraizing, and systematizing relevant objects, relationships, actions, patterns, and regularities. An example of a model eliciting activity for students is intended to reveal the way students are thinking about a real life situation that can be modelled through mathematics. The solution calls for a mathematical model to be used by an identified client who needs to implement the model adequately. As a result, students must clearly describe their thinking processes and justify not a single solution, but rather all (or most of) the optimal and appropriate solutions (English, 2003). Students' engagement with such mathematical tasks results in developing math concepts through the need to develop powerful math ideas in order to solve a problem. Thus, they are given a purpose (and End-in-View) (English & Lesh, 2003) to develop a mathematical model that best describes, explains, predicts, or manipulates the type of real-life situation that is presented to them. In this way, model-eliciting activities allow students to document their own thinking and learning development. The aim of a modeling activity engages problem specification and validation, engage in critical usage of modeling, participation and communication skills; foster creative and problem solving attitudes, activities, competencies; provide the opportunity for students to practice applying mathematics that they would need as individuals in society; to contribute to a balanced picture of mathematics; to assist in acquiring and understanding mathematical concepts (Battye & Challis, 1997).

Types of the modeling activities product

Model eliciting activities include three types of products; tools, constructions and problems.

1 *Product as a tool*: Tools fulfil a functional or operational role and they include:

A *Models*. Models are used for ranking items, people and places; determining loan payments and may form the base of complex systems such as company's financial operations (ranging from productive systems to administrative systems and societal systems such as income tax and insurance models.

B *Descriptions and explanations*. Descriptions and Explanations illustrate and verify the results of an experiment or investigation or may describe why something that appears superficially correct is mathematically incorrect.

C *Designs and plans*. Used in all walks of life, designs and plans must meet detailed and complex criteria and must incorporate appropriate mathematical and representational systems.

D *Assessment instruments*. They are used in a wide range of contexts such as assessing learner's progress, and selecting staff. They normally undergo rigorous development that incorporates cycles of testing, refining and applying. (Lesh & Doerr, 2003a, 2003b)

2 *Product as a construction*. A construction normally requires students to use given criteria to develop a mathematical item. They do not define the nature of the product rather they set parameters for the design of the product. A construction can be in the form of:

A *Spatial constructions.*

B *Complex artefacts*. Inventions are a good example of complex artefacts. The criteria for their design frequently focus on deficits in existing artefacts or on perceived societal needs.

C *Cases*. Cases make use of persuasive discourse to adopt a stance on an issue, to recommend one course of action over another, or to highlight an issue in need of attention. Cases are especially effective when they draw upon mathematical data to support their claims.

D *Assessments*. They are the products of applying an assessment tool. Such products can serve a number of purposes and usually suggest or imply courses of action. (Lesh & Doerr, 2003a, 2003b)

3 *Problem as a product.* The ability to pose problems is becoming increasingly important in academic and vocational contexts. During modeling cycles involved in model eliciting activities students are engaged in problem posing, that is, they are repeatedly revising or refining their conception of the given problem. During the model eliciting activities, students find ways to judge strengths and weaknesses of alternative ways of thinking and whether a given response is appropriate and good enough (Lesh & Doerr, 2003a, 2003b; English & Lesh, 2003).

Principles for modeling activities

One defining characteristic of a design experiment is that the researchers create, test, and modify a design within the context of use (Design-Based Research Collective, 2003). For example, the researchers may be testing a new curriculum or teaching method in a classroom (e.g., Erickson & Lehrer, 1998; Verschaffel, De Corte & Borghart, 1997). This characteristic is consistent with model-eliciting activities that ask students to develop mathematical models to explain real-life situations. The development of a design or model is also often cyclic (Lesh & Lehrer, 2003). In a typical series of cycles, the student expresses thinking in some artifact or product, tests the artifact, and then revises the artifact. For example, a student creating a consumer guide for buying a car developed a spreadsheet for scoring characteristics of cars, asked other members of their group or class to test the appropriateness of their scoring guides (to test the product), and then revised the product based on testing results to improve their solution (improve the product) (Hjalmarson, 2005). The students' revisions are guided by a purpose (end-in-view) that describes the functions the final product should be able to perform (English & Lesh, 2003). Similarly, for modeling design experiments, researchers should have some end-in-view for the product under development. The end-in-view should guide researcher decision-making about revisions that are made to the product from research cycle to research cycle.

An important caveat is that for design experiments using a models and modeling perspective, the assumptions and understandings of the teachers (and researchers) may change throughout the study. It is imperative to document those changes as they are made (Lesh & Sriraman, 2005b). Often, researchers are interested in the students' development of responses or in how student models change within a session or between modeling sessions. So, rather than studying fixed constructs or examining snapshots of constructs in isolation, researchers may be studying changes in constructs over time and across problems and individuals. Capturing change and the effects of change can be a goal of design experiments with a models and modeling perspective. So, both components of the design (theoretical assumptions and artifacts) will change just as for students' models both the internal conceptual system and the external representations change. This characteristic is another example of how the researcher-level design experiment should be consistent with the student-level.

For model-eliciting activities, a crucial component is the local context that situates the task. The context guides the students' development of solutions, aids in their decision-making about whether a way of thinking is 'bad' or 'good', and helps them place the end-in-view in a context that is real to the students (English & Lesh, 2003). The context situates the usefulness of the design and aids development since the final product should be useful in that context (Design-Based Research Collective, 2003). However, this does not suggest that the products are not generalizable to other situations (or contexts). As with model-eliciting activities where students develop a product for a particular client that is generalizable to other (similarly structured) situations, designs should also be generalizable to other educational situations. This proviso means that the researcher needs to outline precisely the conditions under which the design was used and possible modifications that may need to be made for the design to be appropriate for different situations (Design-Based Research Collective, 2003).

Collaboration is also a component of design experiments following the modeling perspective that parallels assumptions about student learning. Collaborators may include researchers, teachers and students proceeding along multiple levels of development similar to multi-tiered teaching experiments (Kelly & Lesh, 2000; Lesh & Kelly, 2000; Schorr & Lesh, 2003). Researchers need teachers to help design, test and implement products. Products should be developed with teachers' questions about their own practice in mind (e.g. personal meaningfulness), and researchers can provide resources to aid teacher development (Design-Based Research Collective, 2003). There may also be multiple teachers or researchers involved in the development of any product. This characteristic can aid the triangulation of interpretations about results and the generalizability of results if products have been tested in multiple contexts. Collaboration also aids the documentation of results by requiring that strategies or tools need to be communicated to other people for comment (e.g., individual teachers develop a ways of thinking sheet or concept map to share with the group) (e.g., Koellner Clark & Lesh, 2003).

Appropriateness, usefulness and benefits of modeling activities

It is imperative that mathematics educators take students beyond the traditional classroom experiences, where problem solving rarely extends their thinking or mathematical abilities or where modeling competencies are developed. There is a strong need to implement worthwhile modeling experiences in the elementary and middle school years if teachers are to make mathematical modeling a successful way of problem solving for students.

Modeling activities have been found appropriate to enhance students' and teachers' capacities to engage in problem solving, thereby laying the foundation for exploring complex systems (Lesh et al., 2003). These activities are highly innovative learning experiences (English, 2003). A number of related features have emerged, indicating a number of benefits of modeling activities, both for students and teachers. Modeling activities provide a pathway in understanding how students approach a mathematical task and how their ideas develop; these activities appear to provide a strong basis for teachers to interact with students in ways that would promote their learning (Doerr, 2006). In the following part of the literature review we summarize the benefits for students and teachers while working with thought revealing modeling activities, including benefits in student mathematical literacy and conceptual understanding, in student social development, in student metacognition, and in teacher pedagogical approaches and teaching practices.

Mathematical literacy and student conceptual understanding

Related research in mathematical modeling indicated that student work with modeling activities assisted students to build on their existing understandings and to be successfully engaged in thought-provoking, multifaceted complex problems (Lesh & Doerr, 2003a, 2003b; English, 2003). Modeling activities set within authentic contexts, allow for student multiple interpretations and approaches, promoting intrinsic motivation and self regulation. A number of related research studies showed that the use of modeling activities encouraged students to develop important mathematical ideas and processes that students normally would not meet in the traditional school curriculum (English & Watters, 2004; Zawojewski, Lesh & English, 2003). The mathematical ideas are embedded within meaningful real-world contexts and are elicited by the students as they work the problem. Furthermore, students can access these mathematical ideas at varying levels of sophistication. Student work in modeling activities facilitates student development of generalizable conceptual systems. Students move beyond just thinking *about* their models to thinking *with* them for solving an important world based problem. English (2003) reported that there was considerable evidence that students' mathematical ideas had improved after they worked in a sequence of modeling activities. Mathematical language improved but also considerable fluency with the use of tables and data were acknowledged (English, 2003). However, there was an acceptance that students needed to know basic operations to be effective in these activities. Gravemeijer and his colleagues (2000) related their work in connection with Freudenthal's (1971) comprehension of mathematics as an activity that involves solving problems, looking for problems, and organizing subject matter resulting from prior mathematizations or from reality.

Lesh and Doerr (2000) have pointed out that modeling activities can promote students' conceptual understanding. They clarified that in modeling activities students are not simply working with ready made models. Since models are interacting systems based in more complex conceptual systems, Lesh and Doerr (2003a) claimed that models must be constructed in a meaningful way. This construction leads to conceptual understanding and mathematization (Lesh & Doerr, 2003a, 2003b; Sriraman & Lesh, 2006).

Harel and Lesh (2003) further stressed the importance of modeling activities by highlighting that student work on modeling activities can enhance student conceptual systems. They documented that conceptual systems are developed first as situated models that apply to particular problem solving situations. Then, these models are gradually extended to larger classes of problems as they become more sharable, more transportable, and more reusable. The aforementioned features of modeling activities helped students be successful beyond problem situations for which models were created (Harel & Lesh, 2003).

Lesh and his colleagues (2003) and English (2003) investigated the role of modeling activities with regard to student algebraic reasoning. Lesh et al., (2003) reported that modeling activities provide opportunities for students to explore quantitative relationships, analyze change, and identify, describe, and compare varying rates of change, as recommended in the Grades 3–5 algebra strand of the Principles and Standards for School Mathematics (NCTM, 2000). In addition, English (2003) pointed that elementary probability ideas emerging when young students linked the conditions and constrains of problems (e.g., drug pain relief activity). The above research studies have also highlighted the contributions of these modeling activities to young students' development of mathematical description, explanation, justification, and argumentation. Modeling activities are inherently social activities, and as so, students engage in numerous questions, conjectures, arguments, conflicts, and resolutions as they work towards their final products. Furthermore, when they present their reports to the class they need to respond to questions and critical feedback from their peers (English & Watters, 2004; Zawojewski, Lesh & English, 2003; Lesh & Doerr, 2003a, 2003b; Sriraman & Lesh, 2006).

An important parameter in students' work in modeling activities is students' use of their informal knowledge. Researchers have observed the interplay between students' use of informal, personal knowledge and their knowledge of the key information in the problem (Zawojewski, Lesh & English, 2003; Mousoulides et al., 2006). In a number of modeling activities, students' informal knowledge helped them relate to and identify the important problem information (e.g., understanding and interpreting the conditions for the solution of a problem). Doerr (2006) and Doerr & English (2003) also documented that students embellished their written reports with their informal knowledge and most importantly, many students recognized when their informal knowledge was not leading them anywhere and thus students reverted their attention to the specific task information (Doerr, 2006; Zawojewski, Lesh & English, 2003; Lesh & Doerr, 2003a, 2003b).

Concluding points

As the literature review has pointed out, it is vital that the mathematics education research community continually revisit the fundamental question: What does it mean for a younger student to understand models and modeling? Another important point we stress is that the models and modeling perspectives in North America and other parts of the world (like Cyprus and Australia) have evolved out of the limitations of problem solving research and the need to take into account the claim (which we hear from many) that the nature of problem solving (and 'mathematical thinking') has changed dramatically in the past 20 years (see Lester & Kehle, 2003; Lesh, Hamilton & Kaput, in press).We also think there is a real need for research about: (i) the nature of new 'real life' situations where some type of mathematical thinking in needed for success, (ii) what it means to understand relevant knowledge and abilities, (iii) how these ideas and abilities develop, and (iv)how development can be documented and assessed. There is also a need to focus on a call for research which takes into consideration what we already know about concept development in children. Sriraman & Lesh (2006) argue that today, when some kind of mathematical thinking is needed to solve real problems, the products that need to be produced often involve much more than short answers to pre-mathematized questions. For example, they often involve developing conceptual tools (or other types of complex artifacts) which are designed for some specific decision maker and for some specific decision-making purpose – but which seldom are worthwhile to develop unless they go beyond being powerful for a specific purpose to being sharable with others and re-useable beyond the immediate situations in which they were first needed. Consequently, solution processes often involve sequences of iterative development → testing → revising cycles in which a variety of different ways of thinking about givens, goals, and possible solution steps are iteratively expressed, tested, and revised (e.g., integrated, differentiated, or reorganized) or rejected. That is, the development cycles often involve a great deal more than simply progressing from pre-mathematized givens to goals when the path is not obvious. Instead, the heart of the problem often consists of conceptualizing givens and goals in productive ways.

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Notes

1 It is important to distinguish between mathematical literacy for critical citizenship (D'Ambrosio) and the formatting power of mathematical models in society (Skovsmose). This indicates a need for critical competencies towards modeling and the use of models.

2 By this we mean representations of the researcher-level conceptual system in the form of interventions, curriculum, etc.

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Sammandrag

Ett forskningsprojekt i USA undersökte för mer än 25 år sedan följande fråga: "What is needed by students, beyond having a mathematical idea that enables students to use the mathematical idea in everyday problem solving situations? (Lesh, Landau & Hamilton, 1983). Efter 25 års systematiskt arbete inom området "modellering" har svaret på frågan börjat framträda. I artikeln beskriver vi hur utvecklingen av "models and modeling perspectives" (MMP) har skett utifrån forskning om problemlösning. Beskrivningen sker med hjälp av en syntes av de viktigaste linjerna i befintlig forskningslitteratur.