Historical aspects on special education in mathematics

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Research on failure to master mathematics (often used term: disability) is a modest speciality, compared with related domains. Research on low attaining persons appears to be too much directed towards a small number of topics en vogue while other issues, often more impressive ones, are mainly left unnoticed. Not the least disquieting is the excessive concentration on computation with small natural numbers in a setting of formalism. This presentation has the aim to demonstrate that the number of parameters (factors, vectors, dimensions) is great in the field of research on education and learning mathematics and that research is a problematic matter, due to the complex relations between mathematics, individual and environment (MIE).

About my personal involvement

From my early thirteen years as a schoolteacher in various types of schools I found no student so hopeless in maths that some tutorial would not make him at least get a pass judgement. I came to know better when I began studying failure. It was my academic teacher Professor John Elmgren in Göteborg who suggested that difficulties to learn mathematics might be a suitable specimen of research after completing my doctoral thesis in 1952. He gave me to understand that neither in Sweden nor elsewhere was it known how many, nor why schoolchildren were low achievers in mathematics. I met authorities who ought to know, as Heinrich Bauersfeld, Germany; Bent Christiansen, Denmark; Zed Dienes, Canada; Hans Feudenthal, The Netherlands; Ernst von Glasersfeld, USA; Edyta

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Grucsczek-Kolczyńska, Poland; Ladislav Kočš, Slovakia; Zonya Krygowska, Poland; Jean Piaget, Switzerland; Fred Schonell, Australia; Tamas Varga, Hungary, but they often had the most differing explanations to boys' or girls' failures in mathematics. The observant reader may observe that many of these scholars have attachment to Europe, not as many would believe the United States of America. It has been so from the very start.

Since then I have studied failures from many aspects. In a Göteborg project I gathered information on about 6,000 students aged 7 to 15 years in the year of 1953 (Magne 1960). Another Göteborg study gave me information of a different sample of 600 students aged about 9. The next big project was carried out during the period from 1962 to 1980 on teaching some 5,000 low achievers in so-called Mathematics Clinics (Magne 1973). A study at Medelby (Middle Village) compared results of the same arithmetic test of about 600 8-year old youngsters in 1955 with the same number of the same age in 1984. They were taught according to very different curricula. A still not completed project concerns the development of 6,000 students' mathematical knowledge (aged 7 to 16). They belonged to the school system of the Swedish town Medelsta (Middletown), the project began in 1977 and has so far continued up to 2002 (Engström/Magne 1990, 2003, 2006). Due to the outcome of these two investigations it was hypothesised that efficacy of teaching tradition is stronger than curricular reforms.

Last but not least a bibliography was introduced, analysing the thematic field of the literature on Special Educational Needs in Mathematics (Magne 2003), although still under the process of being completed. I will present glimpses from these studies and compare them with observations from other sources.

Introduction

It seems that the first publication on failure to master mathematics appeared as a short notice by the Austrian physician Oppenheim in the year of 1885. He met a brain injured person who could not write and read short messages but was able to do simple arithmetic calculations. After him physicians began to take an interest in arithmetical errors among patients with brain injuries. During the First World War soldiers met with shots in the head and in 1918 the German neurologist Peritz described new types of miscalculations. The anatomical ground theory was founded, by the Swedish Professor of Medicine Salomon Eberhard Henschen, together with the medical terminology.

Some pioneers, for instance the Hungarian psychiatrist Paul Ranschburg began to speculate on children who are bad at arithmetic in school (about 1905) and psychologists started with arithmetic tests in the nineteen tenths. To begin with, backwardness in arithmetic was associated with general mental backwardness or even imbecility. So the theory was backed up by an opinion that failure in arithmetic was caused by a ”deficiency”, a syndrome of illness or defect.

Furthermore, this view was still more strengthened when in the year of 1921 the American psychologist Clara Schmitt found a youngster who was weak only in arithmetic but made ordinary performance in all other respects. By that was the so-called specific arithmetical deficiency brought to light. During the following century this phenomenon was predestined to get into the focus of the debate on failure in mathematics performance.

Still, it was physicians and, to some extent, psychologists who were interested in weak skills in ”number work”. Note that these investigators mainly studied the most elementary kind of arithmetic, whether they had to do with school children or brain injured patients. Also today the main interest is directed towards the most simple ”number skills” or ”number habits”. It seems that we still have too little knowledge of schoolchildren’s lacking talents in mathematics and too many questions why schoolchildren are low achievers in mathematics. Research has more or less let us down.

Terminology and definitions

In this presentation I mainly use *special educational needs in mathematics* (SEM) as a comprehensive technical term, or shorter, special mathematical needs. The term SEM was made public in the British Warnock Report (1998) and, after that, has been often used by the EU. Operational educational definition:

> *Special educational needs in mathematics* (SEM) means low achievement in mathematics. A SEM student is an individual at school who has got marks in mathematics below the pass standard according to the valid marking system.

The practical application of the definition is often based upon:

– either a statistical measurement, such as 1 to 1.5 standard deviation units below the mean of a normal population as regards a given mathematical type of task,

– or a fixed mark below the pass standard of an approved marking scale,

– or a certain fixed criterion, irrespective of population, for example that the student shall achieve 95 per cent of a certain measurable aspect of mathematics.

In connection with brain studies I use the term acalculia. Another word I will use is low attainment. The literature reveals a great variety of terms and definitions. See Lorentz and Radatz (1993); Lunde (1999); Magne (1998) and Sjöberg (2006).

In American literature the expression *mathematical disability* is often used because it covers the widest possible range of topics in school matematics. Terms like *arithmasthenia* (Ranschburg, 1905), *acalculia* (Henschen, 1920) and *dyscalculia* (Gerstmann, 1924) usually are restricted to low attainment in elementary arithmetic. Etymologically, the word dyscalculia is a linguistic monster, being a compound of the Greek element ”dys-” and the Latin element ”calculus”.

The inconsistency that exists in the terminology also applies to concepts as failure and low achievement in mathematics. My view is that no definition actually determines the precise significance of this condition. All definitions are descriptive of the consequences of this condition.

According to the author, low achievement is a social construct. The definition will vary with the point of view. I will suggest at least five standpoints, following the arguments of Tredgold (1952) and Wallin (1949). Each standpoint adds an important facetto the total picture. The following approaches seem to be valid (from Wallin):

A Anatomical (neurophysiologic) definition.
B Psychological definition.
C Socio-occupational definition.
D Educational definition.
E Eugenic definition.

The educational definition belongs to school systems and has to do with conditions in schools due to legislation. Special educational needs in mathematics (SEM) is not a fact but a human interpretation of relations between mathematics, the individual and his/her environment. Special educational needs in mathematics must be looked upon from a relativist view. Reusser (2000) concludes that special mathematics needs first of all must be compared with weaknesses (or not) in all other academic subjects. Landerl et al. (2004) show in their experimental study that mathematical achievement ought to be balanced against all other results in academic subjects. D’Angiuli and Siegel (2003) also conclude that low achievement in mathematics shall be defined on the basis of differences in school-related achievement. The assessment refers to a set of achievement elements and depends on the instructional criteria due to the prevailing educational conditions, traditions or legislation in a given school system or educational level. Operationally, the instructional criteria refer to the objectives in a curriculum. The students are expected to learn the specified objectives of the syllabus. Their achievement is assessed according to a marking scale. The marking system may contain provisions of marks over and below the pass standard. Thus, a student below the pass standard is supposed to fail in mathematics. Operationally such a student can be characterised as a low achiever, a student with special educational needs in mathematics etc.

Obviously, standards may vary from one school system to another or from one nation to another. The condition may refer to a given occasion or to a defined period of the individual's life. Low achievement seems to be assessed by different criteria in and outside the school systems. In a Swedish official report on conditionally able-bodied in or outside the labour market the relativity in the concept of working capacity is stressed (SOU 1977:89). As to the definition of labour handicap it was found necessary to adapt the specification of criteria for determining the degree of labour handicap to the competence demands and needs of the various places of work. For mathematics competence in the working life similar variable competence demands may be required, specific for each branch. I propose the expression Special occupational (personal) needs in mathematics (SOM) for conditions on the labour market or else in a person's private life. In this case we adopt the socio-occupational definition.

Prevalence of low achievement is low in preschools, at least as a registered entry in population statistics. The frequency increases during the compulsory school age and amounts to ten to twenty per cent but varies from country to country. For adult persons prevalence is unknown, probably very low.

The expression *special educational needs in mathematics* (SEM) is often used for the condition when a student fails in his/her efforts to master one or several main areas of mathematics according to set school standards. Applied to education, the low achievement is often considered to be *general*, related to the whole set of mathematical topics or even to all other school subjects as well. The low achievement sometimes is *partial*, meaning that a student manages a subset of the specifications in the mathematics curriculum and this refers to knowledge of some mathematical topics but not to others. In exceptional cases the special need is observed in mathematics but not other school subjects (often termed *specific*), a case which is often called underachievement or special educational needs only in mathematics (Lewis et al. 1994). In the two latter cases the Germans use the word *Teilleistungsschwäche*.

There are other classification systems than the ones dependent on school conditions. According to WHO's International Classification of Diseases, 10th revision (ICD-10, 74), and the American Psychiatric Association's Diagnostic and Statistical Manual of Mental Disorder (4th Edition DSM-IV,2) the main defining criterion of the intended condition (low achievement in mathematics) is a significant discrepancy between specific mathematical abilities and general intelligence. It is a case of anatomical or neurophysiologic definition.

ICD-10 recommends terms as specific arithmetic difficulties and DSM-IV mathematics disorders. An unofficial term for the condition is ”dyscalculia” and in some cases ”developmental dyscalculia”. Since the 1970's already this discrepancy method of definition has been heavily criticised by researchers, organisations and authorities in many countries. Some critics have waved aside the idea of discrepancy as the confused reasoning appears to be contaminative about the unexplained relations between general intelligence and mathematical ability. Above all, the classification system also seems pointless, from an educational point of view, because of the meaningless assumption to exclude what might be called ”general dyscalculia” (Bleidick & Heckel, 1970; Grobecker, 1996, 1998; Röhrig, 1996; Magne, 1998; Sjöberg, 2006; Timm, 1977; Wittoch, 1996). The implied name ”specific mathematical ability” is undefined and, thus, without content.

Strangely enough, in these non-educational classification systems the most natural discrepancy criterion is disregarded, namely between the individual´s mathematical competence and his/her general achievement.

A word should also be said about the term *developmental dyscalculia* which was introduced by Bawkin and Bawkin (1960), Cohn (1968), and Kočš (1974), probably influenced by Soviet defectology. It was hypothesised to be a congenital condition which is changing continuously, but not necessarily in a harmonious way. It may have existed since its conception and grown into its present shape. Obviously the meaning of the word indicates that it is a subset of something (possibly a certain form of general low achievement, however undefined), but certainly not ”normal”.

On the origin of the term developmental dyscalculia Professor Kočš told me an astounding story. He had written a report on the results of investigations with an arithmetic test he had constructed. Now, Czechoslovakia had adopted the Soviet defectology as base for their special education, and difficulties in mathematics did not exist in it. Thus, it was impossible to study a defect called difficulty in mathematics. Kočš had to find an alternative political basis. The solution would be to find an expression which was not forbidden. It was suggested to treat the low attainment in mathematics as something that had ”developed” to something related to mental handicap for instance imbecility – which was a lawful ”defect”. Developmental dyscalculia was such a neutral expression. So Kočš could publish his results.

Arithmetical aphasias

We assume that mathematics behaviour depends on brain functions. It is obvious, however, that not only do cerebral activities affect development of mathematical knowledge, but experience also affects the cerebral structure and functions. There has been rather little research on individual differences among average people (Dowker, 2005). Most research directly related to the brain has involved grown up persons, usually brain-injured hospitalised patients. Some studies show that mathematical failures due to brain damage also occur in children. Prevalence is unknown, possibly less than five per cent of the whole school population and less than twenty percent of the SEM school children (Magne, 1958).

As the pioneering work in this field I want to mention the presentation of low achievement syndromes in mathematics by S.E. Henschen in the fifth volume of his *Klinische und anatomische Beiträge zur Pathologie des Gehirns* (1920). Henschen's great scientific contribution was his remarkable finding that every point on the retina corresponded to a certain region of the brain in the back lobes. He found that the loss of mathematical skill following brain injury could be nearly total and therefore called it acalculia. Other syndromes he called numeral deafness, numeral blindness, numeral aphemia (inability to pronounce digits or numeral statements), Reihenzifferaphämie (inability to count 1, 2, 3 and so on), digit agraphy (inability to write the digits), paracalculia (difficulty to choose number operation), and amnestic acalculia (inability to recall the result of simple computations, for instance tables). Henschen studied about 1,700 cases from the medical literature and observed 260 patients closely. Henschen was a ”localist”, and later research conceded that the brain functions are not so mosaicly inserted in the cerebral cortex (McCarthy & Warrington, 1990; Kahn & Whitaker, 1991). Thus the Belgian neurologist R. Collignon (1977) has remarked that different lesions in the brain lead to the same kind of symptoms in a patient but the same lesion may result in different symptoms. The extensive material gave Henschen reason for the following conclusions:

- As a rule linguistic and arithmetical aphasias coexist.
- In many cases acalculia is independent of language aphasias.
- The size of an arithmetical aphasia is proportional to the damaged brain substance.
- In all probability numeral and calculation centres are wholly or partially separated from language centres.
- The main acalculia centre is placed at the left occipital cortex, near the visual centre.

It may be worth adding that one of the leading specialists on head wounds among soldiers, the German physician Peritz (1918) considered the Gyrus angularis near the left occipital cortex to be the main arithmetical centre. Henschen´s work was never followed up by other Swedish neurologists, although other problems in connection with acalculia have been treated from a methodological point of view, for instance Professor David Ingvar's work on neuroimaging technology together with Lassen and others (1977). From that time brilliant electronic machines were introduced for various measurements of brain activity. Magnetic resonance imaging (MRI) is a noninvasive procedure that produces a two-dimensional view of an internal organ or structure, especially the brain and spinal chord. Multiple MRI images can be combined to effectively provide three-dimensional representations. Functional studies on the living brain can use noninvasive technique with the help of electroencephalographic reproduction of neural activity (EEG) (used since the beginning of the 20th century), magnetoencephalography (MRG) and functional magnetic resonance imaging (FMRI). Our future understanding of brain functions can be still more enlarged with moderately invasive techniques of positron-emission tomography (PET) and related single photon emission tomography (SPECT). These techniques mostly work through detecting how blood flow patterns change in different areas during different mental activities.

In 1924 the German neurologist Josef Gerstmann happened to open a controversal field of research that still attracts much attention. Gerstmann got a 52 years old woman as a patient. She was unable of recognising her own fingers, name them and at request point to a finger. This was called finger agnosy. The patient showed four symptoms, in addition to finger agnosy also failing mathematical skills. The condition was named the Gerstmann syndrome. Many neurologists have experimented with the syndrome, among others the English neurologists Macdonald Critchley (1953), and Marcel Kinsbourne and Elizabeth Warrington (1962). They accepted it while others as Strauss and Werner (1938), and Benton et al. (1951) rejected it. Critchley speculates over the seemingly accidental symptom set. He supposes the hand to be a parietal organ, also used to determine the number of a set of things. Thus, representation of quantity may have one localisation in the parietal lobes.

Gerstmann may be didactically useful. The syndrome is believed to result from an injury to an area (areas) near the left side of the crown where sensory and motor impulses take place. This may indicate that common mathematical knowledge may have a sensory and motor origin and also that the teaching ought to engage the learner's manipulative functions.

In 1952 the English neurologist Grewel suggested that the available literature stressed the role of three main areas (figure 1):

1 An occipital group of dysfunctions (Henschen-Peritz acalculia) which he personally considered the result of grave injuries in or near the visual region.

2 A parietal type, among others the Gerstmann symptoms, and several similar symptoms where sensory or motor defects were included.

3 Frontal injuries, mainly affecting higher problem solving in mathematics.

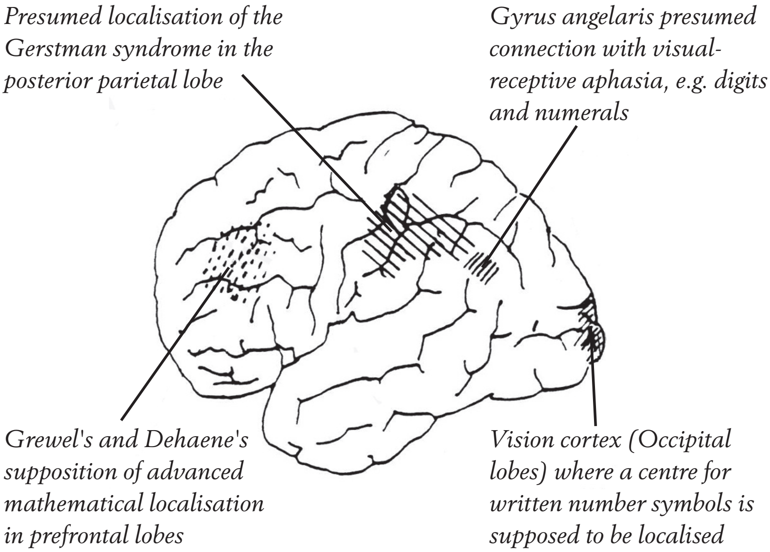


Figure 1. *Diagram showing the cerebral cortex from the left, illustrating possible localisations supposed to be related to arithmetical operations*

In a report from 2005 the French neuropsychologist Stanislas Dehaene et al. present a similar picture of localisations founded on brain-imaging observations according to his own model. Their report only applies to the domain of the most elementary arithmetic.

In his attempt to explain acalculias Dehaene (1997) suggests that there are at least three ”modules” constituting a ”triple code model” for number processing: (1) a module of number words (verbal word frame), (2) a module where numbers are transformed into approximate numbers, called ”analogue values” or analogue locus on an ”internal number line” (analogue magnitude representation), and (3) the Arabic number form where numbers are transformed into the ”Arabic code”. They suggest that ”approximate mathematics” has language independence recruiting bilateral areas of the parietal cortex involved in visuo-spatial processing, while ”exact mathematics” requires language-specific format. This view is apparently adopted by many neurologists. However, it has also been criticised, among others by Paul Spiers (1987) as being speculative and not very adapted to mathematics or mathematical didactics. Spiers says that Dehaene is unable to prove correspondence between the modules of his theory and cerebral location.

According to several neurologists the system of acalculias is independent from dyslexias. Studies in differential symptomatology supports this view (see fig. 2). Recent research indicates that subgroups of children displaying failure in mathematics versus reading/spelling, respectively, differ in significant respects (Hembree, 1992; Lewis et al. 1994; Ostad, 1995; Magne, 1998; Ansari & Karmiloff-Smith, 2002; Katzir & Paré-Blagoev, 2006). To exemplify these relations Landerl et al. (2004) compared dyslectic, dyscalculic and double deficit children and were able to establish the fact that there were distinctive differences. The differential symptomatology is illustrated by comparing figures 1 and 2. This has been demonstrated by the Canadian neuropsychologist Byron Rourke (1991). In his comparisons between visual-spatial and auditory-perceptual skills he has pointed to the existence of observed differences in achievement between low achievers in mathematics and low achievers in spelling and reading. Mainly, those who fail in mathematics seem to have visual difficulties while those with low skill to read and spell disclose auditory or phonemic difficulties.

The French neurologist Hécaen (1976) has in his shrewd neuropsychological analyses modernised the aphasia theory about the mathematical disorders. He assumed a comprehensive view on the neural functions and drew up the disintegration model, namely that ”acalculia” only seldom should be interpreted as a total dissolution of the functional consistency in arithmetical reasoning, but more often as a partial disintegration. This

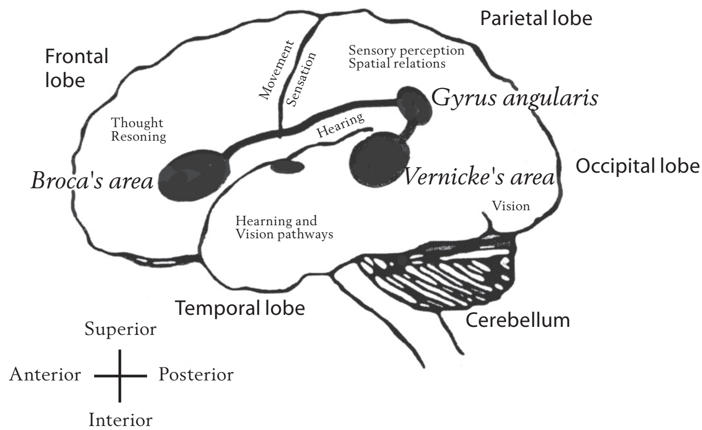


Figure 2. *Left hemisphere of the human brain from the left. Regions marked with black have been implicated in dyslexia functional studies (from Katzir and Paré-Blagoev, 2006, p. 58)*

view has been pursued by many modern researchers in the field, for instance R. Collignon (1977), Gérard Deloche and Xavier Seron (1987), Aleksandr Luria (1969), and Elisabeth Warrington (1982). Note that most neurologist restricted themselves to the absolutely most simple arithmetic.

We find similar cognitive models in the Russian neuropsychologist Luria (1969) and in the French cognitive psychology. To cite the great Frenchman Henri Wallon: ”The normal child is to be found in the deviant child” (Bärbel Inhelder, 1944, p. 33), by that intimating that there are more resemblance than divergence between normality and handicap.

Numeracy deficiency is often found in genetic disorders, although not very thoroughly investigated, for instance in Down’s syndrome, William’s syndrome, the Turner syndrome, the Fragile X syndrome, Spina bifida and idiopathic epilepsy. Very little is currently known about the genetic disorders of low mathematical achievement, although accessible twin and familial studies suggest a heritable risk for the development of low mathematical ability. Possibly, this risk is only expressed under unfavourable environmental conditions, but these are not yet understood (see for instance Buselmaier & Tariverdian, 1999).

The studies by Gérard Deloche and Xavier Seron (1987) are important. They studied aphasic persons’ linguistic logic in single-case studies on numerical transcoding. What struck Deloche and Seron was certain arithmetical integrated Gestalt patterns of cognitive nature in their patients, so-called *stack notions*. "Stack structure" is explained as an integrative relation between two independent variables (often called codes) in the mathematical transcoding process. They assumed that mathematical production is a coherent cognitive process which consists of reactions on several reasoning levels. Arithmetical disorders were looked upon as consequences of mental disintegration of the connected whole.

Let me clarify Deloche's and Seron's point of view with a summary of their own argumentation about the number system. For the sake of simplicity I chose English vocabulary (French is more complicated). The set of quantities 0 – 999 is represented by two codes of the number system, (1) the number names (words) and (2) the Arabic numerals. Both codes contain three lexical basic systems: (a) the units, (b) numbers within the second decade, and (c) the tens, including the hundreds etc. The units (the first decade) are the words zero (number 0) to nine (9). The second decade words are ten (10) to nineteen (19). The tens words are ten, twenty, ... , ninety. The authors classify the even hundreds, and also the following even thousands etc. to the tens words lexical system, because they are constructed according to the same logical and semantic rules as the smaller numbers. All the following four, five, six etc. digit numerals belong to the tens transcoding system. Example: to express a number word representing a four digit numeral as two thousand (2,000) it is enough to add "thousand" to a units word. In a similar way, the three digit number words can be built up with elements of the three basic systems. Example: 235 consists of a units words (five or 5) and elements from tens words (two hundred (200) and thirty (30)) together with grammatical rules for composition of stack notions.

Knowledge of the two coding systems is constructed by the learner. Number words as well as the numerals are based on stack notions. Stack notions are interplay patterns on the basis of the individual's abstract thinking. Numbers should be conceived as abstract matter, constructed by a person's logical thinking. The code of number names as well as the code of Arabic numerals are supposed to constitute a specific and coherent microlinguistic system of particular use for experimental investigations because it contains (1) a restricted lexicon, (2) a formalisable syntax and (3) semantic qualities free from ambiguities. This treatment of numbers according to mathematical language rules is applicable to specific brain processes operating on numbers and, in neuropsychology, leads to the empirical question of selective breakdown due to brain injuries.

According to Deloche and Seron, the transcoding process is the union of mental activities that transform a number presented in a certain code (the source code) into another code (the target code). Numbers can be represented in various modalities. They can be written by a double code (both digital and alphabetic) and by a phonemic code. Thus, six transcoding possibilities may occur.

The transcoding working processes may be impaired in one or several respects. Example: Deloche and Seron have found failing number sense in consequence of impairment to the lexicon (the number words or numerals), a formalised syntax (rules for logical reasoning or communication with the ten base system) and a logical semantic component (which has to do with the individual's capacity to understand number notions). Here are a few examples:

– To transcode a units name as ”six” (6) to a second decades name, as ”sixteen” (16) or to a tens name, as ”sixty” (60).

– To confuse tens, especially among Broca patients (with a motor aphasia), such as 16 and 60, 16 and 61, or sixteen and sixty.

– To rotate or reflect a mathematical sign, such as to write a ”3” upside down, back to front or turned ninety degrees.

– To keep the separate lexical forms in stead of composing them into an integrated whole, as ”one thousand nine hundred” (1 900) is transcoded into 10009100.

– To use hybrid forms, as ”two hundred four” (204) which is transcoded into 2 104.

The different brain shortcomings can affect mathematical knowledge in a seemingly inconsistent way (as hybrid forms). That a cortical function is intact, corresponds to an adequate solution of a problem, for instance to recall a correct multiplication fact. If the person is affected by a circumscribed brain injury disintegration will appear, for instance as inability to tell a correct number fact. In her single case studies Elizabeth Warrington (1982) has demonstrated what she has called *semantic inaccessibility*. Her patient DRC had a selective impairment of arithmetical skills which was not secondary to any other cognitive deficits. She suggests that in an acalculic patient letters, colours, words, objects etc. can be selectively impaired (broad semantic categories) and, consequently, that subcategories within a major system also can be made selectively inaccessible (1987, p. 253). She thinks that inaccessibility is at the core of DRC:s acalculia rather than damage to the semantic entries of arithmetic facts.

To sum up, these findings exemplify that number sense spans the total semantic field of the mathematical structure, not merely single automatic numerical computation habit or spelling habits.

When this is said, it is also important to deplore that research often is prejudiced and aimless, so to say laissez-faire, and has occupied itself with a number of casual and cursory titbits of experimentation. It seems possible that curiosity and craving for sensation is the driving force behind the interest for popular topics like number anxiety, errors and testing of deviant behaviours. On the other hand, crucial topics have been neglected, for instance inclusive teacher education, individualisation in mathematics teaching, prevention of difficulties.

School mathematics

The earliest investigations on children's failures in school mathematics started as late as around 1910. One pioneer was the Budapest physician Paul Ranschburg (1905) who mapped out the skills of intellectually handicapped children in special schools. School teachers in Austria, England, France, Germany and the United States began to construct arithmetical tests and conducted studies of children's computation errors.

His studies gave Ranschburg the impulse to draw up a theory that computation is an intellectual activity. Ranschburg distinguished low achievers (whom he considered mentally deficient) from the normal children as they might have a different background for their mistakes or misunderstandings. He thought that good teaching might put the things right. Ranschburg became convinced that not even the mentally handicapped were hopeless in maths. He felt that slow learners had capacity to understand to some extent how to reason about maths and to gradually develop their logical understanding.

This line of thinking was rejected by the contemporaries of Ranschburg. His opponents used drill and small step methods, for instance Americans as Thorndike, Watson, Skinner and Englishmen as Schonell, and many others. They used associationistic and behavioristic teaching methods in which conditioning was the model procedure. First of all the teacher must diagnose the child's calculation habits. Serious cases should undergo psychological and medical examinations. After being diagnosed, the children had to be given a new chance. Thorndike's laws of learning would be followed. The most important law was called the law of effect, in other words, a modifiable ”bond” is strengthened or weakened according to satisfaction or annoyance which attends its exercise. A second important law was said to be the law of exercise. According to this law repetition was a necessary condition for strengthening the ”bonds”. The youngsters get their second chance with series of tasks of a similar type in which they had made errors before. They should train and would gradually get firm automatic computation habits.

Various training programmes saw the light of day. In many cases experiments were conducted with experimental and control groups. During the classical special education era from about 1910 right up to our time, different research and development projects of this type prevailed. Common names were remedial arithmetic or remedial mathematics.

On the whole, the successes of remedial instruction were few and far between. In the sixty's the Danish psychologist Finn Rasborg (1961) studied the efficacy of about thirty remediation experiments for low achievers in mathematics, but claimed that only two or three of them showed significant improvement. Lorrin W. Anderson and Lennard O. Pellicer are hard in their criticism:

> Current research on compensatory and remedial education programs shows that their goal of bringing academically deficient student back into the academic mainstream is not being accomplished. In fact, even though these programs are far more costly than regular programs a whole lot of money is being spent on them, they remain unsuccessful for the long term and are only slightly effective for the marginal student. (Anderson & Pellicer, 1998, November, p. 6)

Many others have criticised the classical remedial mathematics.

In the Nordic countries a more student centered series of experiments were launched. Its origin was a project by Olof Magne at Göteborg on psychological and social factors in low mathematical achievement, This investigation was completed in 1958. It showed that it was great need of special education for those students in the folkskola (primary education) who failed in mathematics. For tackling the problems, plans were already prepared by Olof Magne, now county school inspector at Karlskrona, together with Leif Hellström, teacher and research student, and Sven Green, headmaster at Karlskrona. The two cities Arboga and Karlskrona determined to organise small-group education for students with specific mathematical need of education. Already on August 28, 1963, the Swedish government published directions on "education at mathematics clinics" for primary school students with "specific difficulties in mathematic". To be eligible for education at a mathematics clinic the selected students would have marks that indicated that they had failed in mathematics, but were successful in other academic subjects. The education got State subsidies. Magne was one of the project leaders. The project also extended to experiments in other Nordic countries. Several reports were published by Magne and other members of the team (Magne, 1973; Magne; Bengtsson & Carleke, 1972).

Education should take place according to directions in the curriculum. While the subject content was kept within the traditional frame of the syllabus, the teaching and learning methods should be modernised. The method was considered laboratory, rational and constructivistic. Laboratory meant to start from everyday problems and find mathematical solutions by reasoning with the help of the mathematical language. Rational implied logical reasoning. Constructivistic signified active thinking on the part of the student. Thus the students were supposed to be active, reason by themselves, cooperate in groups and aim at practical everyday competence. According to the several evaluations, many students were able to acquire up to 1.5 school year knowledge in one year's attendance at ”the clinic”. Those who succeeded seemed to be ambitious, well motivated and mentally healthy. But for some students the result actually was negative. Various kinds of maladjustment created difficulties to learn.

During the 1960's and 1970's the project was gradually expanding until the subsidy unexpectedly was stopped around 1980.

Programmes for low achieving students are rare nowadays. There are more efforts to construct intervention programmes targeting children who are perceived to belong to high-risk conditions, for instance those living in poverty. To this programme type the American Head Start and Follow-through belong (see Arnold et al., 2002). An example from Britain is the Peers Early Education Partnership (Roberts, 2002). In the United States three intervention programmes focus on mathematics aiming at the needs of inner-city preschool children. They are the Rightstart programme by Griffin et al. (1994); the Berkeley Maths Readiness Programme (Starkey and Klein, 2000) and Big Little Kids Programme (Ginsburg et al., (1999). A similar programme was designed in the Netherlands by Van Luit and Schopman (2000). In this project children with special educational needs in mathematics underwent early intervention. For low-attaining 6- and 7-year-olds the Australian mathematics teacher Robert John Wright et al. (2000, 2002) devised their Mathematics Recovery Programme. It has also been used in the United States. All these projects are confined to learning arithmetical topics. The authors maintain that their experiments mainly show positive results.

There are many German projects on learning mathematics with a broader syllabus than the American-English experiments (Braband and Kleber, 1983). One programme is a series of lessons on problem solving in everyday mathematics by Margarita Wittoch (1973, 1996, 1998). Another example is a project on Exploring Mathematics (German; Entdeckende Mathematik) and Productive Mathematics by Petra Scherer (1995, 2000, 2003). For the time being, German research on mathematics teaching and learning of students with special educational needs in mathematics seems to be very progressive.

Which is the state of research on special teaching today?

What do the mathematics teachers say? Quite recently I read the book by Rudolf Strässer on research in teaching and learning mathematics (Strässer, 2005) which contains an overview of dissertations related to the didactics of mathematics. A remarkable observation is that he mentions no dissertations which explicitly deal with the low achieving students at school. Another interesting question concerns the policy of various groups of research workers. One such group is the Psychology of Mathematics Education (PME). PME has worked out a list of 28 research fields on mathematics education (Cockburn and Nardi, 2002). In this list failure in mathematics is missing. Another observation: When the Swedish Matematikdelegationen (Mathematics delegation) summarised their suggestions on actions for improving mathematics education (SOU 2004:97) there were no comments on the critical situation for students with low achievement. Another issue: Is teachers' education up to date concerning low achievers' opportunities to learn mathematics? All this seems to intimate that low achievement does not really fit into the picture. Would this field of research be better suited to be classified among psychiatric problems? Linguistic intervention? Or in school psychology? Or what else? There is a much more enterprising power and promising developmental work in the research actions to promote progress in teaching and learning for dyslectic students.

Who is a low achiever in school mathematics?

The aim of the presentation in this section is to describe the low achieving student related to mathematics and his/her environment. The starting-point will be school mathematics. This standpoint makes it convenient to adopt an educational operational definition of low achievement (same as previosly discussed):

> *Special educational needs in mathematics* (SEM) means low achievement in mathematics. A SEM student has got marks in mathematics below the pass standard according to the valid marking system.

An explicit or implicit definition of this type often seems to be effected in school research on low achievement in mathematics.

Until the middle of the 20th century it was in vogue to study errors, mistakes or misunderstandings. Drill and repetition was used for the purpose of rehabilitation. It was called remedial training. Leading countries were Habsburg-Austria, England, Germany (before Hitler) and the United States. After the Second World War there was a distinct reorientation of educational values in many respects, such as change from traditional dual school systems to comprehensive organisation, from segregation to inclusion, from intellectualism to social awareness. Research on the behaviour of the SEM students also changed. Earlier the interest had mostly aimed at failures of achievement. Studies in behaviour problems increased. Now social and emotional behaviour problems came into focus. Educational prevention prevailed.

Magne's two Göteborg projects (1958, 1960) introduced objectives from this movement. They can be said to be two of the first modern studies on low achievement of children in ordinary after-war schools. Project 1 or the 1953 Göteborg Inquiry was an investigation in which three random school districts were selected for these studies. 6 268 students aged 8 to 15 years were of school age in these schools (3 205 boys and 3 063 girls). The number of children *below the pass standard* in mathematics was 362. There were 209 boys and 153 girls. An estimation of the total proportion of low attaining students in the whole city of Göteborg showed that 12 per cent were low attainers. The number of low achievers increased significantly from age 8 to age 15.

In addition to this overall estimation of special educational needs in mathematics it seemed significant to find the prevalence of ”specific” needs in mathematics. We required the teachers to give us information of the students' marks in all subjects to find the number of low attainers with marks below the standard in mathematics but over the standard in all other subjects. These were defined as *specific low attainers*.

It turned out that out of the total of the 6,268 students 15 students belonged to the subgroup of low attainers. As the marking system implied that the teachers had passed these boys and girls in all subjects except mathematics, it was assumed that they had no difficulties to learn spelling, writing, reading or other areas in the syllabus. This meant four per cent of the low-achieving group and three students per thousand of the total population. It is an exceptionally low prevalence of failure.

In Project 2 or the 1954-55 Göteborg Inquiry we studied all 600 students belonging to grade 3 in one school district and 78 were categorised as low attainers according to our educational definition (13 per cent). To compare, in Project 2 we estimated the number of students with difficulties in reading/spelling to be 21 per cent. Those assessed as specific low attainers were not more than 5 students (0.8 per cent). The interpretation of these observations is that low attainment was a very considerable problem in Göteborg in the 1950's, mostly in the upper grades. It also indicated that a phenomenon existed which might be defined specific low attainment, but this phenomenon was exceptional at that time. The later Mathematics Clinic Project resulted in similar frequencies from the period 1965 to 1970 (Magne, 1973). The still later

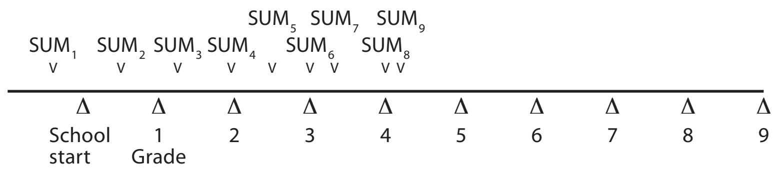
Medelsta Project (Middletown) confirmed the data from the earlier investigations in general.

In their reports on ”developmental dyscalculia” Shalev et al. (see Sjöberg's analysis) obtained sensationally high frequencies of about 6 per cent in a study designed from the discrepancy criterion in DSM-IV. Related findings are reported from other investigations. I refer to the criticism of Shalev et al. (1993, 1995) by Gunnar Sjöberg (2006). Von Aster et al. (2005) announced that only a small minority with ”tief greifende Rechenstörungen” had Lese-Rechtschreib-Probleme. Lewis et al. (1994 reported frequencies of 1.3 per cent for their group of 1,206 children, aged 9-years. Similarly, Ansari and Karmiloff-Smith (2002) interpret the literature as indicating low frequency of ”pure dyscalculia”. It seems plausible that the DSM-IV criterion leads to spurious high frequencies of the ”specificity”.

Students with special educational needs in mathematics have a prevalence not less than about 12 per cent. We hypothesise that specific low attainment has a prevalence of less than one per cent. There are no indications of increasing prevalence during the period 1950 to 2000.

The Medelsta Project showed that an average low attainer, defined as the lowest attaining fifteen per cent (called SUM students), at school leaving age at 16 reached the average level of grade 4. Figure 3 demonstrates the average level of the 15 per cent lowest attainers in every age group compared with the mean attainment of each total age group.

Let me go back to the Göteborg projects 1 and 2. Each teacher teaching a low achiever was asked to assess every student in various respects. We found that low achievers belonged to the weakest fifth in his/her class as regards: intelligence 74 % and marks 77 %. The family situation was estimated as lower for the low achieving children than for the average children, but the difference between the two groups was insignificant. About a fifth was reported to have a possible lesion in the nervous system, although the symptoms were doubtful in many cases. Otherwise the health situation was about the same for low achievers as for the average child. Nearly every low achiever showed more symptoms of maladjustment than average children. A remarkable difference was observed concerning some symptoms, and particularly low motivation. Many felt anxiety, hate and loathing for mathematics (one in four). We noted also a great difference concerning drive (for instance energy, will-power and stamina). About 75 per cent were afflicted with passivity (indolence, daydreaming and inhibited activity). These students were averse to trying hard, to exerting themselves to the utmost, to strain every nerve when necessary. We interpret this striking observation as a crucial symptom of low achievement, particularly of specific low achievement. We named it *reduced working capacity*.



Key to the signs: SUM$_1$, SUM$_2$ etc stands for the various performance means of the SUM students (lowest attaining 15 %) in relation to the means of the average students.
The symbol v marks the position for the mean of the performance of the SUM-students at the end of each grade.
The symbol Δ marks the end of each school year (and the beginning of the next).

SUM$_1$	Mean after the first school year is lower than the mean of the school beginners
SUM$_2$	The mean is equivalent to the knowledge level after the first half school year.
SUM$_3$	The mean has reached the level at the beginning of second school year.
SUM$_4$	The mean has almost arrived at the finishing tape of grade 2.
SUM$_5$	Manages the first half of grade 3.
SUM$_6$	Reaches the goal of Grade 3.
SUM$_7$	Has reached the beginning of Grade 4.
SUM$_8$	Near the target of Grade 4
SUM$_9$	Has just passed the transition from Grade 4 to Grade 5.

Figure 3. *Diagram illustrating the performance level of the SUM-students at Medelsta at the end of the spring term 2002 for the nine grades of the Grundskola in relation to average performance level in each grade.*
(Revised from Engström and Magne, 2006)

Math phobia, impact of dyslexia and sequels of brain injuries are three themes that attract public interest. Best known is math phobia. There are several types of anxiety related to the learning of mathematics. One is called ”specific mathematics agony” and is supposed to be congenital in some way. Another type is called test anxiety and may be conditioned by series of misfortunes. A third variety is associated with repeated experience of stress at mathematics problem solving and, thus, is linked up with logical reasoning and intellectual work. The incidence of math phobia is much lower than is usually believed, in the two Göteborg projects about 20 per cent. There is a preponderance of males in the lower grades, but of females in the upper grades. Stress and anxiety are not always detrimental to mathematics achievement, and more often signals, a challenge to hard work.

The two British authors Stephen Chinn and Richard Ashcroft (1993) have presented the hypothesis that failure in mathematics might be caused by previous failure in spelling/reading and related linguistic backwardness. The standpoint is supported by other authors, in Sweden Gudrun Malmer and Björn Adler (1996). From what has been said earlier in this article their opinion is not borne out by actual facts. Lyon and Shaywitz (2003) suggest a new definition of dyslexia, emphasising that ”observations of commorbidity do not detract from the specificity of the proposed definition of dyslexia since the cognitive characteristics of deficits in attention and mathematics are quite different from the cognitive characteristics associated with deficits in basic reading skills” (p. 3). The Swedish dyslexia experts Görel Sterner and Ingvar Lundberg (2001) are more cautious in their view and particularly point to the fact that there are no conclusive studies of correlations between language and mathematics. In his survey on the same issue Hembree (1992) suggests that thinking ability (intelligence) is the common general factor controlling linguistic as well as mathematical ability. In short, research is needed.

Regarding brain injuries, school children sometimes have lesions to the brain accompanied by loss of mathematical skill. About their prevalence at school age we know very little.

On symptoms and causes

It has been proposed that there are three categories of causes to low attainment in mathematics, (a) genetic factors, (b) apparent injuries or disorders in the nervous system and (c) social inhibitions (Magne, 1998). Research on these factor groups have advanced considerably during the last decades, above all in consequence of the growing body of knowledge concerning the human genome and the neuroimaging instruments. However we still know very little about causes to low attainment in mathematics. Progress in mathematics learning is regarded as a function of social and biological factors. The individual variations are enormous in mathematics. The success is due to the life itself. Low attainment is due to the student, mathematics and the society. Good brains, good care, good teaching, support by a nice family and just the right economy add to the success.

Student and environment

A fairly new concept is didactogenic factors. According to Dieter and Barbara Ellrott (1995, p. 3-8) it refers to the harmony between general teaching concept and individual learning predisposition. Referring to school mathematics, the concept implies that the educator simultaneously considers (a) the content of the *mathematics* syllabus, (b) the *individual's* possible capacity to learn the stuff and (c) the *environment*, including the frames of education, provided by law (MIE). Didactogenic factors are disharmonic if there is a disparity between the individual's personality and the demands or expectations of the school system. It seems likely that the educational and legislative interests and demands sometimes come into conflict with another (Reusser, 2000). The state stipulates certain skills, but the state cannot dictate what the students learn or are able to learn. Learning depends on the students' ability and motivation, the teacher's competence and the favourable dispositions of the administration. The broadening view of the mental health service may stress three major areas for reorganising the tuition:

– Changing attitudes and expectations, including politicians' demands.
– Changing the environment, including curricula and school systems.
– Changing the child and his/her home.

Maladjustment is sometimes established outside the school system, but in other cases by the school organisation. It may begin when mathematics is not too easy for the student. Self-confidence is lost. Eventually a crisis begins. The end product may be failure and delusional fantasies; a self-destructive status choice (Linnanmäki, 2002).

Such a student runs the risk of becoming a passive consumer. Being a consumer means to be filled with skills, perhaps to have 'competence' in facts and routines; to have been taught to know that there is always an answer, usually only one, and it is the authority which declares that answer; to have been taught to listen much and to question little; to have been taught that a success comes from passively following the authority's predetermined paths; to have been taught in formal contexts but not filter out life skills, to have been taught much but not learnt much that can lead to independence.

The exact opposite is a student who is critical, involved in decision-making and actively strives after meaning in what he/she wants to learn. As a leading principle it means that knowledge is acquired when the student tries and looks, reasons and tests, creates and invents (Brian Donovan, 1990; Petra Scherer, 1995; Margarita Wittoch, 1973, 1996).

When students fail in mathematics it is often part of a vicious circle of disappointments. A typical element of intervention programmes is to restore the student's self-esteem. An ingredient of this process is "le contrat didactique" between student and teacher (Guy Brousseau, 1998). It is suggested that the educational agreement shall build upon "a therapeutic alliance" (Veerman 1983; Magne, 2000). That is a process based upon the tutor's empathy and competence and the student's conscious and unconscious wish to succeed in his/her effort to learn and to co-operate with other individuals and to accept the tutor's aid in overcoming the mathematical difficulties.

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Olof Magne

In 1952 Olof Magne was appoint docent at the Göteborg College, from 1954 Göteborg University, after his doctoral thesis and was 1952 to 1960 responsible for the Department of Education at the College (from 1954 University). Ped. Dr h.c. (Åbo Akademi). County School Director (Karlskrona) 1961 to 1971. Assistant Professor and Professor at the Malmö School of Education 1971 to 1983. After his retirement Olof Magne has continuously acted as a consultant, researcher and author. Among his chief research interests the following ones could be mentioned: mathe-matics education, learning and memory, special education. Magne has been engaged with missions in various parts of the world and entrusted with national and international assignments. In Norway the Olof Magne Foundation has been created in order to support teachers' special education studies in mathematics.

Sammanfattning

Forskning om lågprestationer i matematik (ofta använd term matematiksvårigheter) är en blygsam specialitet i jämförelse med besläktade vetenskapsområden. Studier om låga prestationer i matematik – både i skola och utanför skola – är få, ofta godtyckligt utvalda och ibland subjektivt evaluerade. De kan stå för lättillgängliga smakriktningar snarare än vardagligt anspråkslösa teman med vetenskaplig tyngd och djup. Ett annat oroande drag är en överdriven koncentration på enkel aritmetik med små naturliga tal i en formalistisk tankekostym. Komplexiteten i matematikens struktur går förlorad. Vardagsmatematik försummas trots dess betydelse för den lågpresterande räknarens livskvalitet.

Denna framställning ägnas främst åt (1) att visa och klarlägga mäktigheten av parametrar (faktorer, vektorer, dimensioner) i vetenskapen om matematikens undervisning och inlärande samt (2) att exemplifiera hur dess utforskande har fyllts av motsägelsefulla objekt till följd av de komplexa relationerna mellan matematik, individ och omgivning (MIO) – engelska: *mathematics, individual and environment* (MIE). Forsknings- och utvecklingsarbetet är mera energiskt i projekt för att förbättra undervisning och inlärning för elever med läs- och skrivsvårigheter.