Student reasoning constrained by the didactical contract

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This paper presents an analysis of an observation of student teachers' small-group work on a generalization problem in algebra. I begin my analysis by looking at the student teachers' attention to the teacher educator's thinking, at the cost of their own interpretation of the problem. Further analysis deals with the difficulties in changing representation from natural language to mathematical symbols. The analysis is based on Brousseau's theory of didactical situations in mathematics, and a semiotic approach to the problem of algebraic reference, informed by Radford.

Processes of generalizing and justifying in mathematics are often perceived as problematic to students (e.g. Chazan, 1993; Almeida, 2001). The research reported in this paper aims at examining a generalization process carried out by three student teachers, who are collaborating on a task designed by a teacher educator in mathematics. When I observed the small-group lesson presented in the paper, I perceived the interaction over lengthy periods as not being productive. Through the close examination of the interaction of the student teachers and the teacher, I got insights into the nature and complexity of the interaction. The objective of the paper is to show how the goal of the mathematical activity for the student teachers becomes the fulfilment of the didactical contract, and how this focus constrains the student teachers' sense making from a mathematical point of view. A better understanding of the phenomena related to the didactical contract is important knowledge for student teachers and teacher educators, as well as for pupils and teachers in school.

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Theoretical framework

In the episode to be discussed the students [1] are supposed to go from the particular to the general and then to justify a formula for a given pattern using processes of specializing, generalizing, and justifying as elaborated by Mason, Burton, and Stacey (1982). It is relevant that Mason (1996) has pointed, further, at teachers' and students' different comprehension of examples which are intended to illustrate a generalizing process. While a teacher might understand specific numbers and items in an example as placeholders, generic examples, the students interpret them as complete in themselves.

In students' investigation of the general term of a sequence, two main strategies can be identified (Mason, 1996). The first one focuses on the relationship between some terms of the sequence, usually a relation between consecutive terms. In this strategy perception and natural language play an important role. The relation between two consecutive terms can be seen and expressed in natural language, even if not in a stringent way concerning the naming of the terms. The general term is then represented by an implicit or iterative relation. The second strategy aims at an explicit representation of the general term. Here, perception is much less helpful. The production of a symbolic expression for the general term requires that a point of reference is chosen. This point of reference is related to the position of the term in the sequence, which is unperceivable. Radford (2000) refers to this as the positional problem. The analysis of the episode in this paper indicates that the students focus on an implicit relation between terms, while the teacher focuses on an explicit representation of the general term of a sequence.

The students in the actual episode are not driven by the need of justifying a conjecture. The teacher has revealed the connection between the sum of odd numbers and the square numbers, and the task involves representing this relation in terms of mathematical symbols. The students are concerned with answering the questions, and ensuring the use of the teacher's stated connection. Their motive for doing the task is interpreted in terms of fulfilling the didactical contract (Brousseau, 1997).

> In a teaching situation, prepared and delivered by a teacher, the student generally has the task of solving the (mathematical) problem she is given, but access to this task is made through interpretation of the questions asked, the information provided and the constraints that have been imposed, which are all constants in the teacher's method of instruction. These (specific) habits of the teacher are expected by the students and the behaviour of the student is expected by the teacher; this is the *didactical contract*. (Brousseau 1980, as cited in Brousseau 1997, p. 225)

According to Freudenthal (1973) the goal for mathematics education should be to support a process of guided reinvention in which the students can participate in negotiation processes that, to some extent, parallels the deliberations in the development of mathematics itself. Brousseau (1997) explains what such a process involves and requires when he writes that

> [a] faithful reproduction of a scientific activity by the student would require that she produce, formulate, prove, and construct models, languages, concepts, and theories; that she exchange them with other people [...] The teacher must therefore simulate in her class a scientific microsociety if she wants the use of knowledge to be an economical way of asking good questions and settling disputes [...] (ibid., pp. 22-23)

Brousseau defines an *adidactical situation* to be a situation in which the student is enabled to use some knowledge to solve a problem "without appealing to didactical reasoning [and] in the absence of any intentional direction [from the teacher]" (p. 30). The teacher's enterprise is to organize the *devolution*[2] of an adidactical situation to the learner. The negotiation of a didactical contract is a tool for this purpose. When the devolution is such that the learners no longer take into account any feature related to the didactical contract but just act with reference to the characteristics of the adidactical situation, the ideal state is accomplished.

A classroom can be said to have an institutionalized power imbalance between the teacher and the students. The analysis of the episode indicates how the students' enterprise is funnelled by the teacher's utterances. Cobb, Boufi, McClain and Whitenack (1997) claim that the teacher's authority can be expressed by initiating reflective shifts in the discourse, such that what is said and done in action can become an explicit topic of discussion. In order to make this possible, the teacher has to have a deep understanding of what is going on in action.

When learners mathematize empirical phenomena differently than expected by the teacher, the didactical contract is threatened. Such a situation may cause a conflict, which can not be solved by pure inferences. Voigt (1994) claims that "[t]his is one reason why mathematical meanings in school are necessarily a matter under negotiation" (p. 176). It is necessary to ensure that mathematics learners do not restrict their thinking to empirical evidence which is obvious to them. They should develop familiarity with mathematical rationality.

> Through processes of negotiation of what counts as a reason, the teacher can stimulate the students to develop a sense of theoretical reasoning even if empirical reasons are convincing and seem to be sufficient. (Voigt, 1994, p. 176)

Considering the enterprise of the students from the perspective of the didactical contract, their task is to give a solution to the problem given to them by the teacher, a solution which is acceptable in the classroom context. In this situation the learner acts as a practical person, for whom the priority is to be efficient, not to be rigorous. The aim is possibly to produce a solution, not to produce knowledge. Balacheff (1991) argues that beyond the social characteristics of the teaching situation, we must analyze the nature of the target it aims at.

> If students see the target as 'doing', more than 'knowing', then their debate will focus more on efficiency and reliability, than on rigor and certainty. (Balacheff, 1991, p. 188)

Methodology

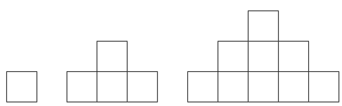
The participants in the research reported are three female students in their first year of a programme of teacher education for primary and lower secondary school, and a male teacher in mathematics. The students are medium-achieving in mathematics. The three students constitute a practice group, which is a composition of three or four students being grouped together to have school-based learning in a particular class in primary or lower secondary school. At the time the data was collected, they had been collaborating on several tasks in different topics during the five months they have been on the programme. Along with his colleagues, the teacher (who teaches mathematics to the group of students) is concerned about development of relational understanding (Skemp, 1976) for students in mathematics.

The episode described is a video recorded small-group work session at the university college, in which the students are supposed to collaborate on a generalizing problem in algebra. The teacher has designed the task aiming at developing competence in conjecturing, generalizing, and justifying. The data is collected in order to answer the research questions of my PhD project, which is about how mathematical knowledge is authored by the learner, and how mathematical meaning is negotiated through collaboration. I will analyze the episode from the perspective of the didactical contract and of a semiotic approach to the problem of algebraic reference.

Description and analysis of the episode

Three students, Alise, Ida, and Sofie (pseudonyms), are sitting in a group room adjacent to a big classroom, in which the rest of the students are working in groups on the same task for 75 minutes. There are two teachers present in the big room, observing the work of the students, helping them, and participating in dialogues with them. Only one of the teachers, the one who has designed the task, is in contact with the students during the episode described. The teachers are colleagues of mine, and I have also been involved in mathematics teaching in the class. I observe and video record with a handheld camera the work of Alise, Ida, and Sofie. My role is to be an observer and neither to interfere with their work nor to help them. This role is justified and explained to the students as necessary because the data collection should be in as naturalistic a setting as possible. Although my presence in the room, and the video recording, is indeed a disturbance, I try to minimize this as explained. The first part of the task handed out is shown in figure 1.

Here the first three figures in a pattern are illustrated.
You may use centicubes to represent the figures.



How many cubes are there in the fourth figure? In the fifth?
How many do you think there will be in figure number 10? And in figure number n?
What kinds of numbers are present in these figures? In each stripe, and totally in the figure?
Can you express, as a mathematical statement, what the figures seem to show? – With words? – With symbols?

Figure 1. *The first part of the task.*

The mathematical problem as fulfilment of the didactical contract

The students have been collaborating for 6 minutes. They have found out that the stripes in the pattern consist of odd numbers, and that each whole figure consists of a square number. They have agreed on $F(n) = n^2$ as a representation of the general term of the sequence of staircase towers, but have not revealed any connection between odd numbers and square numbers. There is uncertainty connected to the concept of 'a mathematical statement'. Sofie has focused on an implicit relation between consecutive terms in the sequence, asking Alise and Ida if they were supposed to show the increase (from one figure to the next). When the teacher enters the room, they ask him for help.

Excerpt I from transcript[3] (see Appendix 1 for transcription codes).

100 Alise: What is this ... what is that which you are thinking about ... (to the teacher)

101 Ida: A mathematical statement. We have made a formula for ... but how do you make a mathematical statement?

102 T: Yes, but a formula is a mathematical statement if it ...

103 Ida: Yes, because we have made it with symbols actually.

104 Alise: Yes, that is what we made here (points at her notepad), but with words – shall we tell what it is then?

105 Sofie: Is it for the increase, or is it for one here (about one figure) for instance?

106 Ida: It has to be for the increase.

Twelve turns between the teacher and the students.

119 T: So that ... mmm ... odd numbers which we build up – and what are we doing, and what is the result? There is such a connection here now ... (pause 5 seconds) Results in *n* squared, as you have said, it results in square numbers, but what do we do in order to make these square numbers appear?

120 Sofie: What do we do? We just square the figure? Or the number of the figure.

121 T: Yes, yes, you do that, when you ... but that may not be the most obvious, visual (character) of these towers.

122 Sofie: That you add a line.

123 T: Yes.

124 Alise: That you increase at the ends with one at each side.

125 T: Indeed.

126 Alise: This in order to have that staircase pattern.

127 T: And then we build it line by line ... So we are concerned with adding some numbers ... in order to get the total number (of cubes) ... (hesitantly). Figure number two is one plus three ... and the next figure is one plus three plus five ...

128 Alise: So you ... just increase all the time, so if – there is one (T: mm) – one plus three (T: mmm) – one plus three plus five

		(T: mmm) – one plus three plus five plus seven (T: mmm) – one plus three plus five plus seven plus nine.
129	T:	Yes, exactly.
130	Alise:	And like this the whole way upwards.
131	T:	And instead of saying this, what could you say that you are doing, in this adding process? ... Now you have said it with examples, one plus three plus five plus seven, but what is it you are adding here now? (Ida looks at Alise, then at Sofie's notepad, then she looks at her sweater, before she gasps discretely)
132	Alise:	The odd numbers in this series (she has a cheerless facial expression).
133	T:	Yes, it is so. Adding odd numbers. And what numbers do you get as an answer? (teacher in an excited voice, Alise strokes her eyes). What kind of numbers do you get as an answer?
134	Alise:	Square ... What kind of numbers I get as an answer ...?
135	T:	When you are adding the odd numbers in this way?
136	Alise:	Square num ... (hesitantly) (Sofie and Ida look down in their notepads)
137	T:	Then you get a square number, yes. This is almost a little discovery ... (pause 5 seconds) Which is, at least, such that ... if I have asked: What happens if I add – what kind of numbers do you get if you add the ten first odd numbers? (Alise: mmm) If I had asked you this question this morning, then you sure could not have answered: Then I get the tenth square number (Alise shakes her head and says: no). So this is nothing which is quite obvious, which you know without any more fuss. This, you can say, is the idea of a mathematical statement; that nobody knows it without any further thinking – there has to be done a piece of work. And that is the process which has been going on here now, which – which is resulting in (the formulation): It actually is like this, that if I add the three first odd numbers, I will get the third square number. (Alise and Sofie nod and say: mmm. Ida leans her head in her arm, looking down at the table). Yes. If I add the four first odd numbers I get sixteen, which is the fourth square number,

yes oh – connection in the world of numbers in a way. (Ida looks down at the table, nods) It looks like this – as if it is going to be like this.

138 Alise: []

139 Sofie: [] is it just this we are supposed to write, in a way?

Sofie follows up her concern with an iterative formula. In turn 105 she asks if it (the mathematical statement) should be about the increase, and is supported by Ida in turn 106. There are multiple interpretations of what a mathematical statement in this context might look like, and an iterative formula would be appropriate. A mathematical statement could then be formulated for instance as ”Square number (n+1) equals square number n, plus odd number (n+1)”.

The teacher's intention with the task is the formulation of the fact that the sum of the n first odd numbers equals n^2. The evidence which indicates this is the "funnel pattern of interaction" (Bauersfeld, 1988) in turns 121-137. The (from the teacher) expected notion 'square number' from Alise in turn 136 brings the teacher to the presentation of the solution of his interpretation of the task in turn 137, in which the teacher accomplishes a monologue lasting for one and a half minute. Here he reveals that the sum of the ten first odd numbers equals the tenth square number, representing in natural language an explicit relation between the position and the representation of the general term of the staircase tower sequence. When the teacher in turn 137 characterizes the outcome of the dialogue as ”almost a little discovery”, it is an example of the Jourdain effect, which is a form of what Brousseau (1997, p. 25) calls Topaze effect. The Jourdain effect is characterized by the teacher's disposition to ”recognize the indication of an item of scientific knowledge in the student's behaviour or answer, even though these are in fact motivated by ordinary causes and meanings” (ibid., p. 26).

Now the task implicitly is reformulated or narrowed, so that the problem is interpreted to be what the teacher originally had in mind when setting up the task: The students are supposed to represent with symbols what the teacher has stated generally; that the sum of the n first odd numbers equals the n-th square number. This process of step by step reduction of the teacher's presumption of the students' abilities and self-government is ”quite opposite to his intentions and in contradiction even to his subjective perception of his own action (he sees himself 'providing for individual guidance')” (Bauersfeld, 1988, p. 36).

After the last utterance of excerpt I from the transcript, there are several turns between the teacher and the students, during which the teacher tries to help the students expressing a general odd number in terms of mathematical symbols. After three minutes the teacher leaves the group room, and the students continue working on the problem. Seven minutes later the following conversation takes place:

Excerpt II from transcript

The students are collaborating. The teacher is not present in the group room.

191 Sofie: I'm lost (smiles). It should be *n* squared and we have to find out what we must do to *n* in order to achieve this, which fits a number. It is just taking a number then and write it as *n* squared. And then add and subtract till we are there ... (all three laugh).

192 Ida: What is *n* and what is *n* squared?

Then there are five turns between the students. Alise writes in her notepad:

$n + (n - 1) = n^2$
$3 + (3 - 1) = 3^2$

Alise finds out that the sum on the left side of the 'equalities' is the increase from the previous figure to the figure represented by the square number on the right hand side of the 'equality'.

198 Alise: This is a "two-in-one formula" (they all laugh loudly). Because by the formula you find both the increase and how many (cubes) there are totally. Actually it does not make any sense that 5 = 9 (all three laugh very load). But he (the teacher) *did say* that it was one of the sides! He said that *n* squared is the right hand side. But this formula of ours does not say anything about adding the odd numbers.

Turn 198 indicates that Alise understands well what is being asked for but not how to get to it. Table 1 gives an overview of the numbers and variables included in the actual pattern, and might have been helpful for the students when dealing with the problem. Alise considers the terms in the second and fourth column, and until turn 198 she denotes the terms of the same row to be identical.

The teacher has designed an adidactical situation which presupposes the students' mastering of a technique of expressing a general odd number. Because the students do not master this technique, there is a didactical problem. The teacher has not "contrived one [adidactical situation] which the students can handle" (Brousseau, 1997, p. 30);

Table 1. *Overview of numbers and variables in the problem*

n	$2n$–1	1 + 3 + 5 + ... + $2n$–1	n^2
1	1	1	1
2	3	1 + 3	4
3	5	1 + 3 + 5	9
4	7	1 + 3 + 5 + 7	16
5	9	1 + 3 + 5 + 7 + 9	25

the devolution of the problem specific to the construction of the target knowledge has not worked well.

In turns 119-137 the teacher's interventions can be seen to follow a kind of Socratic teaching method under the constraints of a didactical contract which faces the teacher with a paradoxical injunction: The more precisely the teacher tells the students what they have to do, the more he risks provoking the disappearance of the expected learning. A paradoxical injunction is also faced by the student: If she accepts that the teacher teaches her the result (according to the didactical contract), she does not establish it herself and therefore does not learn mathematics. If on the other hand she refuses all information from the teacher, then the didactical relationship is broken (Brousseau, pp. 41-42).

When Alise in turn 100 asks for the teacher's thinking, she takes into account features related to the didactical contract. This, together with the funnel pattern of interaction mentioned above, and the fact that the teacher does not succeed in hiding his will and intervention as a determinant of the students' focus and action, causes the collapse of the adidactical situation. Sofie illustrates in turn 191 an important aspect of the didactical contract: It is even more important to come up with the solution expected by the teacher, than making sense of the mathematics involved in solving the problem. The laughter and their facial gestures indicate that they are aware of the detrimental effect of such an attitude. Even if Sofie's utterance expresses despair, it can at the same time be interpreted to be ironic, which can be seen as an effect of the paradoxical injunction faced by the students.

Symbolic narratives and the problem of a point of reference

The challenge of the 'reformulated'[4] task is to transform the mathematical statement represented by natural language into a representation by mathematical symbols. An aspect of the complexity of this transformation is made evident through the problem of representing generality, of which excerpt III below offers an example.

Excerpt III from transcript

156 T: This is square number two (points at $1 + 3 = 4 = 2^2$, which he has just written in Alise's notepad). (Alise: mmm)
If I take the three first ones (writes $1 + 3 + 5 = 9 = 3^2$),
I will come out with square number three, and so on.
Then I take the *n* first ones, but how can I mange to tell about this? Then I continue here, and I shall up to ... odd number *n*. How shall I express odd number *n*? (He has written $1 + 3 + 5 + ... + = n^2$ in Alise's notepad) ...(pause 7 seconds, the students look at Alise's notepad)

157 Sofie: Can't we just take *n* (for the *n*-th odd number)?

158 T: Well, and then I ask: What is the fourth odd number?

159 Sofie: Eeemm

160 Alise: Seven.

161 T: One three five seven. Seven, yes. Odd number four is seven. Mmm. The fifth odd number is ...?

162 Alise: Nine.

163 T: Nine. So odd number *n* is ...?

164 Ida: $n + 2$?

165 T: Ok? But there is exactly such a structure you must try to search for now. Odd number *n* can't be *n*, because then odd number four would have been four. If you say *n* twice in a sentence, it has the same meaning ... (pause 5 seconds)

166 Ida: But it has to be something with plus two (T: ok?) because it increases with two each time.

In turn 157 Sofie suggests that they represent odd number *n* by *n*. The teacher offers a contradiction, but in turn 164 Ida follows up Sofie's suggestion by proposing $(n + 2)$ as a representation of the *n*-th odd number. For Sofie and Ida, the symbols *n* and $(n + 2)$ respectively, function as nouns in a referencing act, not as variables in the pattern. The symbols *n* and $(n + 2)$ appear as narratives, what Radford (2002b) calls symbolic narratives. Ida takes Sofie's narrative as a starting point, and develops it in accordance to the fact that we have to add 2 when we go from one odd number to the next. Ida's point of reference is here seen to be what I call *local*. Her narrative ($n + 2$) is related to the sequence of consecutive odd numbers, which are the focus of attention at the moment. She explains the choice of the narrative in turn 166 when she says: "But it has to be something with 'plus two', because it increases with 'two' each time." The indefinite pronoun 'it' appears twice in the quote, and refers to an arbitrary odd number, the general term of the sequence of odd numbers, which is at stake of turns 158-163. The incompatibility is caused by the choice of n as the number of the n-th figure in the sequence of square numbers built up from sums of odd numbers. This point of reference is what I call *global* and is chosen by the students as they have let $F(n) = n^2$ refer to the n-th figurate number in the sequence, a choice which is followed up by the teacher. The students fail to take this point of reference into account when symbolizing the odd numbers. Therefore their suggestions, n and $n + 2$, remain without link to the general term of the sequence of square numbers.

The above interpretation of the symbols n and ($n + 2$) as symbolic narratives, and a chosen point of reference being local or global, informs the interpretation of Sofie's and Alise's responses in turns 122 and 124 in excerpt I from transcript. Turns 121 and 127 indicate that the teacher is concerned with the global act of (constantly) adding odd numbers, building a developing sequence of staircase towers. In turn 127 he is aiming at an explicit formula for the general term of the staircase towers sequence. This focus is not in line with Sofie's local act of expressing the relation from one staircase tower to the next. Sofie's attention is on an iterative formula for the staircase tower sequence, considering one term of the sequence known and then getting the next term by adding a line (odd number). Alise's point of reference is local in a different meaning than Sofie's point of reference. Alise's attention is on an iterative formula for the sequence of odd numbers, considering one term of the sequence known and then getting the next term by adding "at the ends by one at each side" (turn 124).

The different points of reference which the teacher and the students have, are important in understanding the lack of success in the interaction between the interlocutors. The teacher puts a lot of effort in revealing the functional features of the objects in action, and he offers concrete examples aiming at the students' own conjecturing. But the students seem not to be sensible to the teacher's contributions due to the different focus in the generalizing act. Their focus is on the iterative relationship between the terms, manifested through the attention to the concept of 'increase', an attention which seems to be ignored by the teacher.

The interpretation of utterances in which generality is represented by natural language, has been informed by insights offered by the analysis of utterances in which generality is represented by mathematical symbols. This indicates the different effects of the two semiotic registers (Duval, 2002), natural language and mathematical symbols. When algebraic reference is manifested in the use of natural language it is more difficult to express and perceive nuances and exactness. The discrepancy between the teacher's and the students' point of reference is easier to perceive when generality is represented by mathematical symbols, as for instance in excerpt III.

Theoretical versus empirical reasoning

The interpretation of the situation from a reasoning point of view is that the students consider the statement that the sum of the *n* first odd numbers equals the *n*-th square number to be truth, and not to be a conjecture. The starting point of the task is the empirical phenomenon of the staircase towers in the task. The statement about the connection between the sum of odd numbers, and the square numbers appears for the students to be an empirical statement, not a theoretical statement. These two types of statements have different rational bases, and this may explain why the students do not feel the need to justify the statement, which for them is based in the empirical phenomena. The students mathematize the empirical phenomena differently than expected by the teacher; hence the didactical contract is threatened. Because the students are convinced by the empirical reason offered in the form of illustrations and hands-on material, they do not feel the need to justify the statement in a theoretical sense. This neutralizes the need of generalizing in terms of mathematical symbols, because the motivation of a general statement in terms of mathematical symbols is likely to be driven by the need of justification of a conjecture (e.g. proof by induction). The teacher wants the students to experience and deal with mathematical rationality, and he suggests theoretical reasoning when he says:

> If I add the four first odd numbers I get sixteen, which is the fourth square number, yes oh – connection in the world of numbers in a way. It looks like this – *as if it's going to be like this*. (Excerpt I of transcript, turn 137, emphasis added)

This utterance points at the uncertainty of the empirical reasoning. If the students were challenged to negotiate about what would ”count as a reason” (Voigt, 1994, p. 176) in this situation, they were likely to experience familiarity with mathematical rationality.

Conclusion

The situation referred to in this paper points at the necessity for the teacher to make an *a priori* analysis of the problem he gives to the students. An a priori analysis of the actual problem might have pointed at the necessity (for the students) of mastering a technique of symbolizing a general odd number. In addition it might have pointed at different interpretations of the request to make a mathematical statement based on the figurate numbers in the pattern (exposing that an implicit formula would be in conformity with an explicit formula).

Negotiation of a didactical contract takes place in a metadidactical situation (see Brousseau, 1997, p. 248), outside the didactical situation, in which the teacher reflects on and prepares the sequence (lesson) he must construct, and the student looks at the teaching situation from the outside. In teacher education the actual episode could be used at a metadidactical level to reflect on the didactical situation; the devolution of the learning responsibility to the students, and the validation and institutionalization of knowing and meaning. Reflection on the didactical situation would contribute to a better understanding of the didactical phenomena (Brousseau, p. 247) related to the didactical contract (e.g. the Topaze effect), and reflection at the metadidactical level could predict expected outcomes of the didactical contract.

The paper also points at the necessity of paying attention to the fact that a point of reference is to be chosen when handling generalizing problems in algebra. Awareness about and the dealing with the referential problem would probably have improved the negotiation, in the sense that it could have been more productive from a mathematical point of view. Thus, the paper indicates the importance of implementing considerations about the referential problem in an a priori analysis of generalizing problems in algebra.

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Notes

1 I will refer to the student teachers as 'students', and the teacher educator as the 'teacher'.

2 Devolution is the act by which the teacher makes the student accept the responsibility for an adidactical learning situation or for a problem, and accepts the transfer of this responsibility (Brousseau, 1997, p. 230).

3 The transcripts have been translated from Norwegian by the author.

4 *Reformulated* denotes that the task is narrowed to be what the teacher originally had in mind (an explicit formula), excluding the possibility of working on an implicit relationship.

Appendix 1

Transcription codes

[]	inarticulate utterance
...	pause
italics	emphasis
(text in brackets)	representation of action, explanation of nonverbal action, or comment on utterance or action
T:	the teacher

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Sammendrag

Artikkelen presenterer en analyse av en observasjon av tre lærerstudenters gruppearbeid med en generaliseringsoppgave i algebra. Oppgaven er formulert av en lærerutdanner i matematikk, som også er med i den beskrevne episoden. Analysen begynner med å se på lærerstudentenes oppmerksomhet overfor lærerutdannerens tenkning, noe som går på bekost-ning av deres egen tolkning av problemet. Videre analyse omhandler vanskelighetene ved å skifte representasjon fra naturlig språk til matematisk symbolspråk. Analysen tar utgangspunkt i Brousseaus teori for didaktiske sitasjoner i matematikk, og en semiotisk tilnærming til problemet med algebraisk referanse, støttet av Radford.