

Understanding constituted by understandings

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While focusing on the function of language as a fundamental tool in the learning process, this article relates to a text-theoretical approach connected to frameworks developed by M.M. Bakhtin (1981, 1998) and J. Lotman (1988), in particular to their shared view of language as gaining meaning in relation to other languages.

Learning is seen as developing text; it is seen as relational – as individual, social and genre-related. The empirical basis for this article emerges from a learning situation in which student teachers are searching for insight into concepts within the theory of functions. The article comprises different texts: the theoretical framework, the empirical text, and a child's beadwork.

The study to which this paper relates (Johnsen Høines, 2002), had the purpose of shedding light on the processes employed by student teachers to communicate mathematical content; on how they gain insight into mathematical notions by exploring different ways of expressing, interpreting and investigating them. I wanted to learn about those processes whereby students engage in learning in ways that can be described as taking ownership of it (Skemp, 1971; Mellin-Olsen, 1989; Goodchild, 2001). Such situations can be characterised by the behaviour exhibited when students are in charge, making decisions about what to do and how to do it. I wanted to study how the students, within the frame of such situations, use different ways of expressing mathematical content as tool for learning. Further, I wanted to explore the fruitfulness of a dialogical perspective on learning and to make such a perspective visible by organising a dialogue between the theoretical texts used in the study and texts that emerged from observations of learning sessions. In

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this respect the concept of learning, in the study this article relates to, is described in terms of *flexible spaces of language* and viewed as dialogical and relational in nature. Thinking is considered to be an inner dialogue where genre-related¹, social and individual voices are brought to the fore when students are working on a subject under study. Understanding is created through the tension and struggle between such different voices as well as between inner and outer social speech. The aim of this article is to present a theoretical perspective as a way to describe learning processes. This is done by elaborating some selected aspects developed in the study (Johnsen Høines, 2002). The article focuses on learning as developing texts. This implies studying how understanding can be described as interaction between different ways of understanding². The differences are seen as important conditions for the processes where different ways of understanding are positioned in opposition to one another. The intentionality³ is discussed in terms of speech plan, and intends to imply methodological challenges.

The empirical basis

The empirical data referred to in this article was gathered during a teaching session in which student teachers were working on calculus and the theory of functions. The study (Johnsen Høines, 2002) focused on the work done by two students, Mette and Kari, as they struggle with a problem that places the mathematical concept of function in an everyday context:

Five 14-year-olds have started a bicycle-club. They repair bicycles, they arrange outings, and they discuss traffic problems. The organisers of the club wish to increase their membership. They set a goal that each member should recruit three new members every quarter of the year until the club reaches a set limit of 1000 members. A waiting list is established from which to pick new members, should someone drop out. Express the desired number of members as a function of time and by using the premises set for how to reach their goal.

Data was gathered through observations of the students' work. This situation was not prepared to be a research-situation beforehand. There were no tape recorder or video camera. However, I had as a teacher developed a practice where I made notes after important sessions had occurred. These notes contained my story about how I remembered the communication, and it contained quotations, how I remembered them. This way of using

such notes was partly developed as a teaching method in the class. The students were used to discuss the notes thoroughly.

The current situation was recorded according to what Kvale (1996) describes as "to be recorded through a reflective use of the research's subjectivity and remembering, relying on his or her empathy and memory and then writing down the main aspects [...] after the session, sometimes assisted by notes taken during the interview" (p. 161). The students were subsequently interviewed and then given the opportunity to read and to comment on the notes made by me as the observer⁴.

I identified the situation to be a complex situation. The students struggled to understand. They investigated different solutions and explanations. I wanted to get insight into the communication that I interpreted was difficult to uncover. The aim of my research developed as to get insight into the students' use of different ways of expressing a mathematical content as tools for learning. This was seen in relation to a text-theoretical and dialogical perspective. To investigate the theoretical perspective and the empirical text developed as the research focus.

A text-theoretical and dialogical approach

As the process in the situation referred to above developed, the students identified mathematical problems. They were searching intensely for insight into these problems by moving between different expressions. Reading of the transcripts indicated a complexity I wanted to learn about. I motivated for close reading. I found the text-theoretical framework related to the work of Bakhtin (1981, 1998) and Lotman (1998) fruitful, in particular their shared view of language as gaining meaning in relation to other languages. It is a perspective where text is understood as dialogical.

Dialogue has a sociological meaning, in everyday language, understood as written or oral communication between individuals. This is referred to by Nyrges (2002, p. 347) as *outer dialogue*. Dialogicity has a philosophical meaning and can be understood as dialogue between a person and a text – as *inner dialogue* – and dialogue between texts or within a text – as *intertextual* processes (Bakhtin, 1998; Dysthe, 1999; Nyrges, 2002).

Learning is accordingly seen in a dialogical and relational perspective. The dialogicity is related to the interaction between utterances. One utterance gets its meaning in the light of other utterances. According to Bakhtin, discourse is dialogical: not because the persons involved take turns speaking, but because the dialogicity is related to how each utterance is dependent on the other and to how the tension between the utterances creates new meaning (Dysthe, 1999, p.81). This dialogical approach is

relevant for analysing communication between people. It is relevant for analysing the process of interpreting a text. It includes the intertextual processes as interaction between utterances within a text⁵. According to Bakhtin the utterance is created in the interaction between the author and the reader (the person that expresses and the interpreter). Additionally, he implies a perspective related to genre. The utterances are formulated in a specific way. This is for instance to be seen in the way mathematical tasks are formulated. The tasks are interpreted according to the perspectives of the authors and the interpreters, and also according to the specific genre the tasks are representing.

These aspects are implied in the intertextual processes. As *thinking* in a Vygotskian perspective is related to inner speech, thinking, according to Bakhtin, is considered to be an inner dialogue where genre-related, social and individual voices are brought together. Understanding is created through the tension and struggle between such different voices and between inner dialogue and outer social speech (Johnsen Høines, 2002; Säljö, Riesbeck & Wyndham, 2002).

The concept of text is related to the dialogical approach referred to above and is underpinning this article's perspective (Bakhtin, 1981; Lotman, 1988; Johnsen Høines, 2002). *Text* is understood in a rather broad way. In the case of written text, text is seen as what is written as well as the reader's interpretation. A text can be oral. It might be a picture. A (learning-) situation can be regarded as text. Text is conceived as that which is being interpreted as well as the interpretation.

Text is understood as being both individual and social. A text becomes individual in the sense that: it is *my* interpretation, consequently it is *my* text. *Your* text will differ with *your* interpretations. Although we might refer to the same situation or the same picture, the texts will always differ. However, developing texts are social processes in the following context. I make my interpretations on the basis of earlier interpretations made relevant in *my* social environment. I interpret what other persons have formulated and what is developed socially. My text includes my interpretations of other persons' interpretations. It is related to *our* interpretations. Consequently, text is situated between the individual and the social and interrelates the individual and the social. According to Wertsch: "Individual consciousness is viewed as being fundamentally situated in the social world." (Wertsch, 1999, p.61).

The individual and social perspective serves as a basis for Bakhtin's dialogical approach. Texts are developed as part of social processes and bear evidence of the dialogical interactions of these processes. "Each word tastes of the content and contexts in which it has lived its socially charged life." (Bakhtin, 1981, p. 293).

Texts are being developed in interaction between texts, by texts confronting each other (Bakhtin, 1981). Learning is, in the context of the study referred to in this article (Johnsen Høines, 2002), seen as developing text. When trying to understand, one tries to interpret; one tries to understand "what it is about" or "what is there". We bring our understandings together to see what is going on in the light of perspectives gained. In the context of this text-theoretical approach, this *bringing together* is seen as organizing the meeting of texts. We bring "our own text" into interaction with the current text that we try to learn about. A staging is taking place; the texts are organized in opposition one to another. Learning is seen as developing texts. It is a text theoretical and a dialogical perspective.

An example: Text as interwoven

Johannes is four years old and has made the beadwork shown in the picture (figure 1). He made his work when I was working on Bakhtin's theoretical approach about how texts are interwoven. I struggled to understand the textual processes, I struggled to understand and articulate the implications in context of learning situations, such as that of Johannes. I do not regard Johannes' work as empirical data. The beadwork work

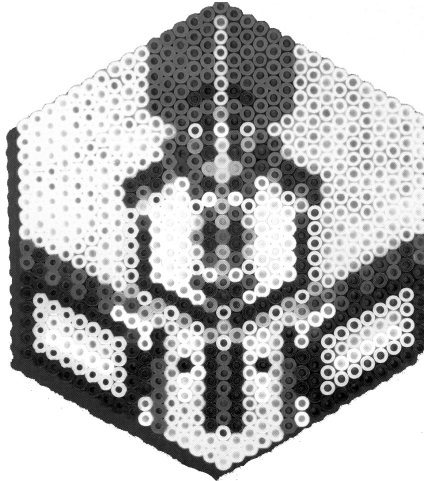


Figure 1. Johannes' beadwork.

helped me to elaborate the theoretical perspective. In this article, I use Johannes' work as a tool to illustrate the theoretical perspective about how texts can be understood as interwoven and to show the relevance when describing learning processes.

Johannes has a frame and uses plastic beads. We can imagine the boy choosing colours and composing a pattern. He considers symmetry, he counts, he reflects on geometrical shapes. The same number of beads at each side. Equal distances. The same colours. "Do I have enough yellow if I choose to go on with yellow pearls here?" We identify one hexagon inside the other and recognise parallelograms. "It looks like an Indian pattern," commented one little girl.

I intend the work by Johannes to serve as an illustration for the diversity and complexity characterizing the learning processes, in particular when focusing on ways in which mathematical creativity involves different languages. Johannes moves between different fields of knowledge (Mellin-Olsen, 1993). The different fields involved are structured in different ways and can be described as different texts. As mentioned earlier, a text has a certain order; it is organized in certain ways. Accordingly we might recognise a numerical text, a geometrical text, etc. We can imagine Johannes as he reflects on what he wants the pattern to be, thinks about symmetrical aspects, looks at the colours available, reflects on earlier pictures, and counts. He moves between different language systems when he counts, constructs shapes, chooses colours, judges symmetrical aspects, makes aesthetic decisions. He moves between different ways of systematizing his work. This can be expressed as: he moves between different texts; as intertextual movements.

Sometimes we can observe such processes as organized sequentially. Johannes makes decisions about colours. Then he decides on the symmetry. He starts counting. Sometimes we see that the processes are organised simultaneously; he combines several ways of thinking at the same time. In no case do the texts implied function independently. They are interwoven also in the respect that they give colour or other qualities to each other. One becomes meaningful in the light of the others. The counting process, for instance, plays its role as a consequence of the symmetry, the colours or aesthetic dimensions. This characterises his way of counting.

The genre-related approach can be described in the context of Johannes work: He moves between different ways of ordering. When he counts, it is his individual work. He creates the system by using a way of ordering that is given by the system of counting he relates to, that is implied in the language. He relates to another way of ordering when he uses a geometrical approach. He constructs individually. His constructions are

socially related. He additionally works in the context of the possibilities and limitations implied by the genre.

Johannes' work illustrates that it is not sufficient that different texts are represented. It is not sufficient that one moves from one text to another. Texts develop as a result of confrontation with one another (Time, 1989). It is the struggle between them, the conflicts, and the changing processes that characterise the way texts develop. It is not easy to create understanding. According to Bakhtin meetings between texts are not described as harmonic meetings. It is not sufficient that different ways of understanding are represented there; it is the tension and the struggle between them that creates meaning (Dysthe, 1999). These aspects are included when I see learning as developing texts.

A learning situation serves as empirical background

Two students, Mette and Kari, are working on the task referred to earlier in this article. They look intensely concentrated and speak to each other in a distanced way: "We don't understand this. It does not fit. We have got different answers and we cannot understand why! [...] We do not understand how we arrived at different answers".

They look at each other's written work. They talk slowly in an investigating way, listening and questioning. Mette's notes show the solution as $f(x) = 5 \cdot 2^x$. Kari shows her result as $f(x) = 5 \cdot 4^x$ and argues that it must be correct. They investigate the calculations done. They try to find explanations that fit as a basis for their respective ways of thinking. They write and then calculate aloud. Kari refers to her solution and claims: "This has to be correct". Mette nods. This suggests that Kari's model seems unproblematic to them. Her notes shows:

$$5 + 5 \cdot 3 = 5 \cdot 4 \qquad f(x) = 5 \cdot 4^x$$

But Mette has something that looks different. Accordingly they focus on Mette's solution $f(x) = 5 \cdot 2^x$. They cannot understand why it is wrong even though they doubt that it can be correct. They are investigating Mette's notes (figure 2).

The students vary between working intensely together and thinking individually through silent questioning. They look into each others writings, they write, make investigative movements shifting between being silent (individually) and interacting in a calm way.

"I have used a month as the variable, Kari has used a quarter of a year", Mette explains. She shows the task in which they are asked to express the desired number of members as a function of time. "We are not asked to express it as a function of a quarter". The explanation is addressed to

$$5 + 5 \cdot \frac{3}{3} = 5 \cdot \frac{3}{3} + 5$$

$$= 5 \left(\frac{3}{3} + 1 \right)$$

$$= 5 \cdot 2^x$$

$$f(x) = 5 \cdot 2^x$$

Figure 2. Mette's notes.

me as the teacher and observer. The two students are cooperating. Mette tells me that they have reached a sort of common clarification of this, and Kari asks: "Is it possible that both solutions are correct? Could it be two ways of describing the same development?" They try several numbers in the two formulas and get different answers. They now have documentation that the models are different and are convinced that Kari's model is correct. The two students study the calculations for a while; they are silent and thoughtful.

Then Mette makes a distinct communicative turn by challenging Kari. She points at the notes in Kari's book. She sits up, gets more energy in her voice: "But if we think quarter, why is it $f(x) = 5 \cdot 4^x$? I cannot understand why it has to be $f(x) = 5 \cdot 4^x$!"

Kari writes: $5 + 5 \cdot 3$ and she talks when she writes: "They were five and by the end of a period of one quarter each of them has recruited three". "Yes, it corresponds with my thinking", Mette says and points at the notes in her book (figure 2).

Kari writes: $5 + 5 \cdot 3 = 5 + 5 + 5 + 5 = 5 \cdot 4^1$. "This makes five plus five plus five plus five; that means five multiplied by four ... raised to the first, it makes five multiplied by four to the x^{th} ."

Mette looks at Kari and says: "Yes ... I see that it is correct, but ...". They look at each other. They both seem confused. I interfere by commenting: "You see that it is correct, but you do not understand why this is so?"

Mette looks gratefully at me. "But," Kari says and (nearly) repeats her explanation. She writes nearly the same as she did earlier:

A five and three more	$5 + 5 \cdot 3$
it makes four fives	$= 5 + 5 + 5 + 5$
consequently it makes five multiplied by four	$= 5 \cdot 4$
that means	$= 5 \cdot 4^1$
that means	$f(x) = 5 \cdot 4^x$

Mette gets energy in her voice. She attacks: "Then it could be $4 \cdot 5$ as well, and then $4 \cdot 5^x$ ". Kari looks at her, paralyzed: "Yes ... but ... ". They stop, looking at each other, are silent.

I interfere: "Can you explain what you do not understand?" Mette, without hesitation: "That it is four times, that it is four times every time." I say: "Make a drawing. See if that helps."

Mette makes a drawing (fig. 3). She works slowly. Kari leans back in her chair. She observes Mette in a brooding and distanced way.

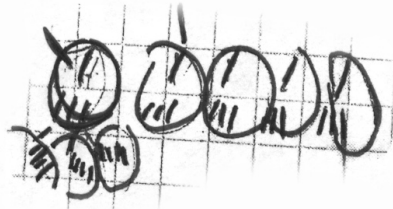


Figure 3. *Mette's drawing*

Thoughtful and pondering Mette says: "Oh ... yes ... one ... becomes ... four ... that's ... why ... it ... has ... to ... be ... four ... times ... (long pause) ... yes, I understand". Mette seems satisfied. They put their things together and leave the room.

The differences as conditions for the process

The different solutions of Mette and Kari can be regarded as different texts confronting each other (Time, 1989). The students have developed their solutions individually, sitting side by side. When they discover that they have got different answers, they get eager to understand how they differ. By testing different values they became sure that Mette's solutions had to be wrong. However, they cannot see how Mette's calculations were wrong. They investigate the written and oral argumentations, for instance by looking into how they use variables differently. The students tried to gain insight into the differences by investigating them, one in light of other. The differences were in themselves conditions for the process (Renshaw & Brown, 1999). The confrontations were visible; they provoked the process.

The differences may be in terms of how the solutions are connected to different mathematical models. When Mette turned by questioning:

"But if we think a quarter, why is it $5 \cdot 4^x$?" She referred to her own solution. She positioned the two texts in opposition one to the other. This is shown in her attitude and for instance in her comment: "It corresponds with my thinking". The students were moving between the different solutions. Their arguments did not exist independently; they got coloured by each other. They got their meaning from the other. This happened because of the differences. The perspective of seeing text as interwoven is underpinning my analyses. Mette's attack, when she stated that "It could as well be $5 \cdot 4^x$ ", is for instance to be seen in relation to her satisfaction when she arrived at "oh ... yes ... one becomes four". It might be that she had the insight to question *why one becomes four, and not two* when she asked: "But if we think a quarter, why is it $5 \cdot 4^x$?" – She worked on how the mathematical models were different.

One could see the interactions as conflicts between the students' personal understandings. They could be seen as personal contributions and one could reflect on the process as conflicts between their individual perspectives. It seems however more appropriate to focus on the different ways of understanding that are brought forth. When Mette said "it corresponds with my thinking", it appears more as her interest to compare ways of understanding than to prove *her* argumentation as a good one or as the best. The approach is characterized by utterances such as "What does it mean if we do this?" or as "If we try this, what would we get?" The two students positioned understandings in opposition to each other, eager to get the insight. They were not defending *my model* or eager to document that *my solution was the best one*. Close reading helped me to understand the communication as conflicts between understandings more than conflicts between the two students' *personal* understandings.

The textual differences became important also in terms of the structure, in terms of how the argumentations were expressed. Kari's text became gradually more visible. It was structured in an algebraic way. The structure appeared predictable and linear. The words were following the written symbols in a rhythmic way. She repeated the oral and written explanations several times. She spoke and wrote slowly and distinctly. The fact that she tended to repeat herself underlined the way of structuring. Her voice became clear and predictable. When Kari let her word follow the written symbols (Pimm 1987, p. 20), we might recognize a voice we have heard earlier. It could be interpreted as *a teacher's voice*. The word she chose, the way she put them, her modes of expressing them, and the way she followed the symbols by pointing with her finger showed a picture of *the authoritative teacher*. It sounded like a teacher telling *how it is*.

I interpret Mette's text to be an investigative text. She moved in a less predictable way. She was questioning, was patient and tentative. She

used time for thinking. Her sentences were often unfinished. I got the impression that she was thinking when she expressed herself. She varied between drawing and traditional written calculations. Her language gave the impression of being questioning. Even when she in the end said: "oh ... yes ... one becomes four" it was articulated in a slow and wondering way. In the context of this, her question seems like an attack: "But if we think a quarter, why is it ...?". Here, her voice sounded powerful and distinct.

The students developed their argumentation in interaction with each other. The differences provoked the process. It is likely that Mette identified the teacher's voice. She had heard it before. Mette's positioning was done in relation to her interpretation of the way Kari was putting her arguments. She moved in relation to the movements she observed that were present. Flexibility is, in the context of this, seen as a quality.

It could be that Kari did not identify the voice of the authoritative teacher, but the comment she gave afterwards can indicate that she did: "This happens over and over again. We do not listen well enough. I did not understand what she did not understand. I wanted her to understand it my way!" (Johnsen Høines, 2002, p. 146).

It seemed, however, that Kari did not find any better way of explaining herself. It was as if she was caught in the genre. She offered a text and Mette seemed to refuse. Mette moved within another structure and had different opportunities. The ways of structuring imply different possibilities and limitations. The interplay, is decided individually, socially and it is also related to a perspective of genre⁷.

Speech plan as a methodological challenge

According to Bakhtin every utterance is connected to chains of utterances, and there is a speech plan for every utterance. When he claims that every utterance is *addressive*, he emphasizes that it is directed towards the answers; it is directed towards the continuations (Bakhtin, 1998). The meaning of the utterance is interpreted in context of earlier utterances and the continuations.

How an utterance is to be seen as connected to chains of utterances, and its relation to the addressivity, can be shown in Johannes' beadwork when, for instance, he starts counting. He might organise his action as: "3 this way, 4 this way; the same on the other side". Choices are made in the context of what he has done, he relates to what is available, what he wants to make, and what he sees as possible to make. He moves between being a reader and an author of his own work. The steps he takes are directed towards the continuation. He is aiming for how he wants the work to become. The aims develop as he progresses. We can see his

steps as utterances. Every utterance is directed towards the continuation. (It is addressive.) The development is seen as inter-textual processes. It has elements of 'answering' even in the absence of any other person in the dialogues.

To analyse an utterance implies to interpret it in context of its' continuations. Different continuations make different utterances. This implies interpreting what a person says in the context of what the person is aiming for. I wanted to get insight into how the students' explanations were different and into how the students used the differences as tools for learning. It became important to see how explanations were different in the context of how they were directed towards continuations.

However, the dilemma is obvious: one cannot *know* a person's intentions or interpretations. It is difficult to identify the correct interpretations. The challenge becomes to show the plausible interpretations and try them for justification (Dysthe, 1993; Johnsen Høines, 2002, p.78).

This is for instance a challenge when analysing the steps where Mette made a distinct turn and questioned: "But if we think quarter, why is it $f(x) = 5 \cdot 4^x$? I cannot understand why it has to be $f(x) = 5 \cdot 4^x$!" Her questioning was not arbitrary. Her questioning has to be seen in the light of intentions that could be relevant:

- a) It could be the case that Mette conceives the investigation on the basis for the solution $f(x) = 5 \cdot 2^x$ as finished. She might be convinced that it was wrong, and that it thereby was no longer of any interest. She changed course to gain insight into the other one.
- b) It could be that she considered the possibility that $f(x) = 5 \cdot 4^x$ was wrong, and that she searched for proof for that it was wrong.
- c) It could be that she accepted $f(x) = 5 \cdot 2^x$ as wrong. But that she, in spite of or because of this, wanted to gain insight into the field connected to it. She wanted to understand the logical argument, also connected to what was wrong. She turned to $f(x) = 5 \cdot 4^x$ in order to relate insight from $f(x) = 5 \cdot 4^x$ to $f(x) = 5 \cdot 2^x$. She wanted to understand arguments as basis for the models.

Her words before and after she took her turn argues for the last point as the most relevant. Her patience, the quiet concentration she showed sitting bent over the calculations that had been done earlier, argues for c. Pencil in hand, she pointed from symbol to symbol. She talked slowly, almost soundlessly to herself, checked, over and over again. She leant back in her chair, looked into space, and then she switched attention to Kari's

book and asked her question. The energy that followed was strong. Her voice was powerful and distinct, she sat straight up, and her gestures were "precise". I interpreted that she attacked Kari's solution as a consequence of the concentration connected with her own solution; the intensity from the investigations done was brought further into the next step. She had not forsaken the arguments for $f(x) = 5 \cdot 2^x$.

The following analyses will, confirm or contrast the interpretation made for the relevance for c. Developing text is seen as an ongoing process. The perspective of continuation is emphasized by Lotman (1988, p. 36) where he regards text as something that is steadily built up along the time axis. According to Bakhtin (1998), text is "in between", there will never be the last and we cannot talk about the first, a text is in itself an ongoing process.

When Mette later said: "Oh yes, one becomes four, that's why it has to be four times ... yes, I understand.", this is to be interpreted in the light of c), and c) is still to be evaluated in relation to this utterance.

In an interview held later, Mette referred to what she thought was the most important aspect within the situation. Without hesitation or comment, she referred directly to the problem she had been struggling with: "I remember I could not understand why it was not 2^x . (Pause) Why is it that it is not correct? (Pause) Oh, it means that we could have thought one more every month. No, it gets more in that way. (Pause) I understand." (Johnsen Høines, 2002, p. 146).

Mette told about her intentions, when she reflected afterwards. She knew what she had been trying to understand, and what kind of understanding she had been searching for. Her earlier steps, when organising the different texts in opposition to one another, was to be understood in the light of this. The text was developed as a consequence of the continuations. The text does not develop chronologically; the future is an argument for the development.

However we have the danger of simplifying. It is not as easy as: Ask the informant and you will know the intentions. The processes are complex. It might be that all the intentions a), b) and c) are active implications. Sequentially or simultaneously. It might likely be that new intentions get revealed. It is a challenge to uncover important parts of the kind of interplay I have tried to exemplify⁸.

Flexible spaces of language

The study this article refers to offers *Flexible spaces of languages* as a way to describe and grasp the complexity of learning processes (Johnsen Høines, 2002). The investigative movement is characterized by how different ways of understanding are organized in opposition to one another, in order to see them one in light of the other. Consequently *understanding* is constituted by *different "understandings"*. The concept of learning is viewed as dialogical and relational in nature. Thinking is considered to be an inner dialogue where genre-related; social and individual voices are brought to the fore. Understanding is created through the tension and struggle between such different voices as well as between inner dialogue and outer social talk. The intention of this article has been to make visible some aspects in this framework, to offer it as a way of describing learning processes.

By using Johannes beadwork as an illustration, and by using empirical data from a learning session as a basis, I argue for the importance of textual differences. The differences are described on two levels: a) on how to understand the expressions related to different mathematical models, and b) on how to understand the differences related to the meaning of the utterances. The focus has been to elaborate how the interactions between the texts make changes, on how the different ways of understanding (texts) give meaning to one another. They get their identity partly because of the other. The intentionality is discussed in terms of speech plan. When one tries to get insight into a problem, one organises for interaction between different ways of understanding (texts). This is done intentionally, by for instance Kari, when she offers her explanations. Her action is seen in the context of the other actions within the communication.

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Notes

- 1 Each text is ordered in a specific way. Genre refers to the way of ordering. When we learn to use language, we learn how to form the language according to different genres. When we listen to another person, we guess the genre even after we have heard a few words, we get an impression about the length of the utterance, how it is built up, and we have some ideas about how it ends. From the beginning we have a kind of overview of what is going to be differentiated through the process of talking (or writing, or using other ways of expressing). If genres function to mould or shape our utterances, they will carry forms of understanding. Our understanding of an utterance is reflected by our knowledge about genre that is actualised. We interpret what is told *and* the way it is told. The genre gives directions for content. When I work on identifying the genre, I try to characterize a text by identifying the specific ways of ordering that is offered by this certain kind of text. Mathematical proof, mathematical exercises, assessments tasks and problem solving activity can be characterised as different genres. The texts are ordered by specific rules, and appear different from each other.

- 2 I use *understanding* and *ways of understanding* nearly as synonyms. Ways of understanding reflects more the characteristics or descriptions in relation to contexts or perspectives. To understand multiplication as repeated addition is seen as *a way of understanding* multiplication. However this does not mean that *understanding* reflects the whole idea. We always understand or reflect fragments of the whole idea. The impact of seeing *understanding as constituted by understandings* would be the same as to see *understanding as constituted by ways of understanding*. Understanding is seen as a process.
- 3 The intentionality is discussed in context of what the students are aiming for, what they intend to achieve when they express themselves, when they investigate, when they ask question and when they explain their understanding of the relationships they have chosen to focus on. It is discussed in context of their basic argumentations on why they are entering the communication the way they do. The intentionality is further on discussed relationally: the students develop their intentions according to the intentions they interpret from fellow students, as a social process. They develop the intentions by interpreting which possibilities they find as available when they study the mathematical area. According to the theoretical perspective in this article, genre gives directions for content. The intentionality is discussed as the basic argumentation for the communication that develops relationally between the individual and social and in relation to genre.
- 4 The observer, researcher and teacher is one person.
- 5 In the study referred to, the struggle between Kari's and Mette's text was identified. The analyses additionally identified that Mette "discussed with herself". The interaction between the different explanations that was identified within Mette's text, can explain what is referred to as intertextual processes. Interaction within a text, can also be applied to the process when we, for instance, read a novel or a mathematical text. It describes how we move between different texts in the process of our reading.
- 6 Mette's solution shown in fig. 2 is in itself a complex text that is analysed in the study (Johnsen Høines, 2002). However this is not an issue in this article.
- 7 As referred to in note 5, further analyses in the study (Johnsen Høines, 2002) show intertextual interactions within Mette's text. I identified two texts, Mette1 and Mette2, where Mette1 includes aspects common to Kari's text. The analyses argue that when Mette attacks Kari's text, this is also to be seen as confrontations within her own text, and this is seen as important conditions for the process. These aspects are not elaborated further in this article.
- 8 Another danger of simplifying is related to the fact that Mette and Kari brought different perspectives into the communication. Their different texts imply intentions developing in the interaction with each other. This is a perspective the analyses have to include. It is not an issue in this article, but it is a central issue in the study *Flexible Spaces of Language* (Johnsen

Høines, 2002) where it is developed further in terms of moving between *a field of problem and a field of solution*, and also in terms of *authoritative texts* and *investigative texts*.

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Sammendrag

Forståelse – som interaksjon mellom forståelsesmåter.

Matematikklæring beskrives som tekstutvikling i denne artikkelen. Gjennom et tekst-teoretisk og dialogisk perspektiv relatert til Bakhtin og Lotman, søker forfatteren å beskrive læring gjennom hvordan den som lærer beveger seg mellom ulike forståelsesmåter. Språket har en ordnende funksjon. Det former vår forståelse. Språk får betydning ved at en ser det i lys av et annet språk. Forståelse fremtrer i interaksjon mellom forståelser. Artikkelen drøfter hvordan slike prosesser foregår individuelt og sosialt. Den beskriver også hvordan forståelse utvikles i relasjon til språkets struktur. (Matematikk-) språket ordner på en særlig måte. I perspektivet som her legges til grunn fremheves det at innhold er inkludert i formen. Særlig vektlegger artikkelen betydningen av hvordan forskjeller mellom forståelsesmåtene er drivkraft i læringsprosessen. Videre søker den å tydeliggjøre betydningen av at alle ytringer har en historie og en fortsettelse, en taleplan eller talevilje. Fortsettelsen og forhistorien er innskrevet i ytringen, har mening i lys av det. Tre tekster møtes i denne teksten: en tekstteoretisk tilnærming, empiri fra lærerstudenters arbeider med funksjonsbegrepet, og fireåringens perling.