

# Social Constructivism, Social Practice Theory and Sociocultural Theory: Relevance and Rationalisations in Mathematics Education. Detail, Reflection and Synthesis

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This paper contains detail from, reflection on and synthesis of the Conference held in Gausdal, Norway, in March, 2000, on 'Social Constructivism, Social Practice Theory and Sociocultural Theory: Relevance and Rationalisations in Mathematics Education'.

The paper is in three parts. Part II contains the main substance, a discussion of themes and issues that emerged related to the three Theories on which the conference was based. Part I sets the scene for Part II by introducing the conference, its purpose and substance; the main themes emerging in discussion; and providing a brief account of papers offered by participants. Part III offers a few final remarks, returning to issues of the theory-practice interface, and the role of theory in our ongoing learning and understanding of mathematical thinking, learning and teaching.

## **Part I – Setting the scene**

### **Preamble**

This conference has been about *Theory*. I write it with a capital T to indicate that we have been focusing on what might be called external or established theory or theories. These are theories that are in the public domain; that have been discussed by scholars, argued over, rationalised; terms have evolved and, in each case, a discourse has emerged which itself conditions thinking and communication.

The three Theories are (Social) Constructivism, Social Practice Theory, and Socio-cultural Theory. This is not the place to expand on

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their detail, nature, claims, warrants etc. To some extent the other three papers in this issue have already done that. Here, my task is to write a commentary on the work at the conference. In doing so, I shall be tackling the Theories through the questions that were raised, issues that were discussed, and ways in which the Theories relate to practices in which conference participants and the wider mathematics education community engage.

Before I start to write this commentary I want to make two sorts of observation:

1. A few further remarks on theory;
2. Some preamble to the task I am undertaking here.

### **1. What do we count as theory?**

The three Theories above relate to knowledge and coming to know. In the mathematics education community this includes mathematical knowledge and how people come to know mathematics, what it means to know mathematics; it includes also knowledge of mathematics learning and teaching. Considerations of such knowledge take us into the realms of history, philosophy, psychology and sociology, and all of these are concerned with forms of knowledge. Thus, these theories are fundamentally epistemological and necessitate the addressing of deep questions about knowledge, its status, truth and validity.

Theory might be seen to explain or idealise practice. In examining these Theories, we may want to explore how they relate to practices fundamental to our professional existence, or even to our personal existence. The practices in which we engage are largely social. They involve interactions with other human beings, either directly or through artifacts that have a social development, tools, books, papers and so on. Thus, we might expect theory to capture elements of the nature of these practices, or to predict elements of practice.

As mathematics educators, practices that we are interested in explaining or predicting relate to students' knowing and coming to know mathematics, and to ways in which teachers work with students to promote or enable mathematical conceptualisation; also to teachers' learning and the development of teaching. We can inspect the three Theories as we perceive them and consider how they contribute to such explanation or prediction. To a great extent, this was our purpose in our conference.

However, as a teacher and an educator, engaging in practice, thinking about that practice, and conceptualising my teaching, I engage in theory in a fundamental way that is not obviously related to the Theories above. This theory (small t) is implicit in my thinking whenever I consider my practice, or think *about* my practice. All of my own work, and my work with other teachers, suggests that we never engage in practice without some theory, implicit, tacit, or even explicit, underpinning what we do. A considerable body of research into teachers' knowledge and beliefs in mathematics and more widely supports this view. We can argue about the nature of such theory and the warrants by which it deserves the name of theory; in some cases we might want to call it *meta-practice*.

Practice is something we do, engage in, get involved in. When we talk together about practice, for example at a conference, or write papers about practice, we are not engaging in the practice, we are talking about it, engaging in *meta-practice*. I could give here examples from my teaching to illustrate theoretical points. However, I can not offer you here my teaching. To do that, you would need to come into the classroom with me. Even then, the extent to which you could enter into *my* practice is doubtful. To address this possibility we should need to start to consider the individual or social nature of the practice, whether it is part of a community of practice, whether we are both practitioners engaging in shared practices, and so on. These are theoretical questions we cannot avoid as soon as we try to engage in a discourse *about* practice.

On the other hand, if we sit down and do some mathematics together, with our focus *in* the mathematics rather than talking *about* the doing of mathematics, we engage together in practice; we are not theorising about practice. At the conference, we engaged in a practice I might call *conferencing*. As part of this practice we talked *about* other practices, such as the teaching and learning of mathematics, the nature of theory and so on. But we did not (mainly) talk *about* conferencing; we did it.

I am making these distinctions here because they seem important to what follows, as I strive to capture elements of our discussion of *theory*. For me an important reason for setting up this conference and engaging in it was to struggle with relationships between theory and practice, and start to understand better how considerations of theory can help us to understand practice and improve practice. To do this, we have to have some clarity of distinction between the two. Of course I realise that *my* clarity of distinction might be opaque or trivial for someone else!

## **2. The task of presenting an account of the work of the conference**

The task I have here, at least as I see it, is to try to present some (coherent?) sense of the working and discussions that took place at this conference, to allow further insights into these theoretical perspectives, their inter-relation, and their importance for mathematics learning and teaching widely.

As I come to this task I feel like a researcher with a vast pile of gathered data. Somehow I have to apply my research skills to emerge with a coherent story – fully rooted and validated in the data itself. I realise, however, that I cannot do the rigorous job that would be required if this was actually a piece of research: (a) I do not have the time it would take to go over and over and over the data, categorising, synthesising, validating and so on; (b) the data lacks a ‘completeness’ (this is of course relative, because how can we ever have complete data). I am relying on session notes from scribes who faithfully and conscientiously kept a record of a session, but who, nevertheless presented their own interpretation of it. I do not have audiotapes of sessions to back up these notes, and coming back to (a) I would not have time to use them effectively if I had.

Jere Confrey asks: What is the warrant behind our research? I am asking here, what is the warrant behind my task?

Well, I believe this warrant embraces (i) my responsibility in this conference: having begun the debate in these theoretical areas, I cannot leave it just as a set of individual stories and a collection of notes; (ii) my responsibility to myself: a lot of work went in to this conference; I know that I shall gain personally from undertaking this synthesis in order to work on my own story; and (iii) my responsibility in helping the thinking of this conference to be available more widely than just to those who participated in it.

Thus, I am going to try to offer an account, which is faithful to the spirit of the conference, but it will nevertheless reflect my own perspectives and values.

### **An overview of themes and issues**

For the main working sessions at the conference, participants formed three groups, each one led by one of the main speakers. These groups remained together throughout. Each day focused on one of the three Theories. This involved a plenary session presented by one of the leaders followed by discussion in the groups, identification of questions to be

addressed and plenary consideration of the questions raised by each group. Scribes made notes in each of these sessions and in each of the groups. The leader each day agreed to try to build on work and discussion of the earlier days.

For the final session of the conference, which had the grand title of “Synthesis”, I scanned the notes prepared up to that point and jotted down what seemed like key ideas and issues. The following themes or categories emerged. I shall give a heading for the theme, followed by some words or phrases that in some way capture the elements of the theme as it was discussed. The purpose of including these categories or themes is to provide an overview of the elements of discussion before moving to an account of that discussion.

### Knowledge

The relationship between knowing and being; knowledgeability; knowledge transfer between contexts; acquisition of knowledge; tacit, incidental and spiritual knowledge; truth; validity; knowing through ‘osmosis’.

### Mathematics

Where is the mathematics? School mathematics; everyday mathematics; mathematics as a discipline? Mathematical processes – generalising, hypothesising, proving.

### Individual/Social

Personal versus contextual; cognitive versus social; intersubjectivity; negotiation of meaning; normativity; social practice shaping knowledge; the role of ‘the other’; communication – access to other perspectives; community; affect of society or culture.

### Power/Authority

Who has the power? ‘Vygotskian theory can be seen to place power with the teacher’, ‘constructivism to place it with the students’. Political issues: democracy, equity, social justice, privileging; demoralisation; deficits.

### Learning

Internalisation versus construction; Consciousness; Intentionality; Affect and Cognition. Where is the learner – e.g. at the core, according to constructivism; shaped by the social situation, history, changing identity; Apprenticeship; Situatedness; Participation; Failure to learn; Where is the teacher?

### Theory-Research-Practice

Recursion; Reflection: “reweaving the fabric of making knowledge secure”: description, explanation, representation and justification (Confrey); Research versus *research*; starting points – problems in the world; dealing with data; methodological implications.

### Teacher learning

“Teacher knowledgeability drives practice” (Adler); Change; PCK

### Practices

Children learning mathematics in classrooms; Teachers learning to teach mathematics; Teacher Educators working with teachers working with children

## Theory and Practice

In accord with what I have written above, discussion at the conference was all theoretical as regards the practices of doing mathematics, learning, teaching and teacher learning. By this I mean there was no *practice* of doing mathematics or of learning mathematics etc. actually at the conference. Thus all our discussions of doing or learning were at a theoretical level. However, there were still two levels of discussion identifiable. The first was where we spoke overtly of theory, especially the big three. So, for example, we might speak of differences between how constructivism regards knowledge versus how it is seen in sociocultural theory. The second was when we spoke of practice. For example, people often illustrated an idea by sharing an anecdote from their own practice. Such anecdotes helped us to ground our theoretical discussions with relation to practice.

One source of accounts of practices was the papers offered by participants other than the main speakers. All these papers were shared in advance of the conference, so that participants would be aware of interests and issues within the community of the conference. Time was given within the groups for reference to the papers. It seems worthwhile here to provide very brief abstracts of these papers, both to offer insights to readers here, and to allow references to the papers in the discussion that follows. I do not include papers of the main speakers, as these are included in their entirety in this issue of NOMAD.

After the paper abstracts, I shall move into Part II of this paper, my account of the discussion in working groups at the conference together with some synthesis of ideas.

## **Papers from participants**

These accounts are very brief abstracts or summaries, provided by the authors of the papers. They give a flavour of the areas of thinking and concern addressed by the papers, and are therefore indicative of the interests and concerns of some of the participants. If you would like to learn more about any of the papers, please contact the author directly by the email address given.

Merethe Anker-Nilssen [merethe.anker-nilssen@hit.no](mailto:merethe.anker-nilssen@hit.no)

How do socio-cultural aspects influence communicative patterns in the mathematics classroom?

Communication between teacher and students in the mathematics classroom seems to follow certain patterns (Alrø & Skovsmose 1993, Alrø 1996, Voigt 1985, Steinbring 1997, Jungwirth 1993, Anker-Nilssen 1998). These patterns affect both each individual student's learning processes and how the teacher assesses the learning outcomes of the students. My aims will be to analyse these patterns or structures, as they constitute a discourse in the mathematics classroom and to get a better understanding of how this discourse relates to certain socio-cultural aspects.

In my project I will investigate socio-cultural aspects on two different levels. In the mathematics classroom a culture will be created consisting of the meanings that are constructed and the ways in which they develop. On the other hand there exists a culture, which represents the tradition of mathematics teaching and learning. This culture consists for instance of the "rules" (not necessarily explicit) for what explanations are accepted and which are not, regulations in the discourse, and the teacher's understanding of students' learning processes and of what mathematical knowledge really is. Both these cultures influence the communication and hence the learning, in the mathematics classroom.

Raymond Bjuland [Raymond.bjuland@c2i.net](mailto:Raymond.bjuland@c2i.net)

Which contributions in social discourse could stimulate the process of mathematics reasoning? A dialogical approach.

Research is reported involving student teachers learning mathematics in small groups through problem solving. The example is given of a geometry problem involving the sum of the distances of a point inside a triangle from the three sides of the triangle. The conjecture is made that this is equal to  $h$ , the altitude of the triangle.

Analysis focuses on the dialogue between the students in the problem solving environment. How do the students come to a mathematical understanding? Part of the answer to this seems to be based in the group nature of interaction – growth of knowledge within the interactions of the group. In what ways might this be seen as a social or a constructive process?

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Triads in Mathematics Education; Semiotic Genre Theory as Theoretical Framework.

The article outlines a theoretical approach to the analysis of the mathematical classroom. Based in semiotic genre theory, taking communication as a basic concept. All aspects of teaching/learning processes can be seen as communicational outcomes. Hence utterance and semiosis (change of meaning) are brought to the centre of the analysis of the teaching and learning of mathematics. Other important framework concepts are: *genre*, *ideology* and *positioning*.

Following Habermas and Bakhtin, the unit of analysis should be the utterance, which can be “any delimitable entity of meaning irrespective of its semiotic medium”. An utterance is seen as triadic; having expressive, addressive and referential elements. The balancing of these elements is in semiotic genre theory *positioning*, and analyses of *self-positioning(s)* can be used to suggest possible *meanings* the activities may have for an acting subject in communication.

Laurinda Brown [laurinda.brown@bristol.ac.uk](mailto:laurinda.brown@bristol.ac.uk)

The role of purposes and basic-level categories in learning to teach mathematics

Two ideas of ‘purposes’ (Brown and Coles, 2000) and ‘basic-level categories’ (Rosch, in Lakoff, 1987) are discussed, illustrating their explicit use in learning to teach mathematics within teacher education. These ideas are used explicitly at the start of courses when learners are entering a new world and the teacher educator is concerned to establish a culture in which student teachers’ behaviours are different to their previous norm. After showing how the idea of ‘purposes’ emerged from observing the work of student teachers of mathematics, an enactivist theoretical frame and methodology provides a link to ‘basic-level categories’ (ibid.). Purposes act as mechanisms for the learner to stay with the complexity of situations providing organising principles, which become linked to action and support rapid decision-making. The implicit use of purposes in the learning of the student teachers is exemplified through the written work of one student. Parallels in the learning of pupils, student teachers, teacher-educators and researchers are indicated. Questions are asked about how a wider sociocultural frame fits this work.



Tony Brown [A.M.Brown@mmu.ac.uk](mailto:A.M.Brown@mmu.ac.uk)

Mathematics or Education? Which is most important?

This paper addresses questions about the fundamentals of research in mathematics and education. What are the bases of our research agendas? Whose interests are being served by our research? Relating to a particular classroom observation, the question might be asked, “In which research agendas would my observation be noticed as being of significance?”

What is the main function of learning theories and how are they associated with revisions in practice? For whose purposes are revisions desired or designed. If research is linked to policy who is being advantaged and who is being disadvantaged by the research outcomes. Research can never be neutral.

“In a post-modernist age, how can we define research questions in mathematics education?”

Anna Chronaki [A.Chronaki@open.ac.uk](mailto:A.Chronaki@open.ac.uk)

School maths in ‘themes’: moving beyond ‘traditional’ and ‘progressive’ pedagogies?

Recently there has been quite a lot of attention on renewing teaching styles and lesson organisation in maths classrooms. In this realm, the use of ‘themes’ in varied forms is often associated with a child-centred ideology and has become largely unpopular with a back-to-basics agenda. As such, it tends to lose grounds in teaching practice. This is not unusual in current school settings where maths teachers are in low supply, teachers are under a continuous pressure for accountability and change and argue ‘..that they have no time for doing interesting mathematics’ (see Lerman, 1998, p.1-18).

This paper discusses the issue of employing a thematic approach to mathematics teaching by using the contrasting cases of traditionally and progressively oriented teachers. The constructs of ‘traditional’ and ‘progressive’ have been widely used to denote diverse teaching styles and pedagogies. This can create polarity when it serves to ‘label’ teachers and to pathologise their practice. Findings in this study support the argument that teachers’ espoused pedagogy influences the mathematics lesson organisation around a ‘theme’, but leaves ineffective the ways teachers address mathematical content.

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Affective representation, belief systems, and values in doing mathematics: Interfaces between the individual and the social

This contribution discusses the concept of affect and its relation to beliefs and value systems, both individual and sociocultural. Some theoretical constructs are reviewed that relate to the emotional domain in mathematical learning, and to how the (internal) cognitive and affective representational systems of individuals interact with socially or culturally shared systems.

The paper highlights the representational function of affect, and interactions between meta-affect and belief systems in sustaining each other, at the social level and in the individual.

Perspectives on affect can possibly help us to think about classroom situations, and to address critical questions about students' responses as they think and solve problems mathematically.

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Reflections on the role of socio-cultural elements in students' understanding of mathematics

This paper presents my reflections on the role of socio-cultural elements in students understanding of mathematics. These deliberations were part of my doctoral research where I studied social interactions both, between students and between students and teacher, as students (11-12 yrs) in small groups learnt mathematics in classrooms in Pakistan.

The rumination raised a number of questions. For example, what are the communities of practice involved as students learn mathematics in small groups? How are these communities linked? Or, how do they incorporate different aspects of society and culture, mathematics itself, construction of meaning? What if classroom practices conflict with social/religious practices or expectations?

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Exploring teachers' construction of pedagogic knowledge through their learning of mathematics.

This paper considers the concept of the teacher-learner: i.e., a teacher who undertakes overt reflection on personal learning processes in order to gain pedagogic awareness of that process. It compares teacher-learners with learner-learners - and shows a reversion to the learner-learner on occasion. An important concept is that of teachers becoming more aware of their interactions with their own students as a result of studying their own learning. Use of the Discipline of Noticing (e.g., Mason, 1997), valuing 'noticing in the moment' is discussed as a device for modifying practice. Gill considers herself as a teacher of teachers learning from the teacher-learners' reflections on their learning. Data was gathered

by written communication rather than interviews using a specially designed form for gathering reflections.

Analysis reflects the developing norms of this course and expectations of reflection. The teacher-learner constructs personal knowledge, but does this in a group situation in which the process is sometimes discussed thus bringing in wider socio-cultural influences.

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The mediation of mathematical learning through the use of pedagogical tools: a sociocultural analysis

A sociocultural analysis suggests that pedagogical artifacts employed in the teaching and learning of mathematics both enable and constrain learning. Five elements of tool mediation (from a sociocultural perspective) are delineated which, in the classroom, are intimately involved in the social interaction that supports learning. Through an analysis of three classroom studies of mathematics learning that have utilised a sociocultural approach, some insight is gained into the ways in which students attempt to make sense of the mathematics they encounter. These include the relation between the characteristics of the device, the students' attempts to see through the device, and the situated processes, discursive interactions, and epistemic considerations involved. Examples include, on the one hand, using a pair of compasses to draw a circle, which imposes consideration of centre and radius of a circle, while, on the other hand, seeing points of intersection, while using dynamic geometry software, as 'gluing' a figure together does not imply a sophisticated notion of invariance.

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School Textbooks and the formation of Epistemological Obstacles: examples from the Early English Mathematics Curriculum.

This paper is a contribution to the study of the social, political and economic development of the mathematics curriculum in England, and focuses on the study of extracts from eighteenth and nineteenth century textbooks. The usual interpretation of the material in textbooks and the ways the books are used is often superficial, portraying only techniques, and giving a 'progressive' view of the skills developed. This is a misrepresentation of the nature of the knowledge enterprises of the time because teaching is influenced by many factors outside the classroom itself. The study of mathematics textbooks in their original social and economic context throws light on the purposes for which they were produced, the mathematical, epistemological and pedagogical beliefs of those who produced them, and also on the origins of beliefs, traditions and techniques, which are still alive today.

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Concepts and symbols – Discourse analysis of knowledge development in mathematics

How we view knowledge and learning in mathematics will influence our choice of theoretical framework for educational research. This paper discusses a discursive approach for studying children's learning of mathematics. Learning is viewed as participation in a sociocultural context. The background for the discussion, is my Ph.D. project. In this project I focus on the pupils as participants in discourses in the mathematics classroom. The classroom study in primary school will last for two years involving the same two classes. Research questions are: How do children create semiotic tools and understanding of mathematical concepts through communicative actions? And how will different educational arrangements influence this process?

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Reflection and Learning

This paper presented empirical data from a research project on elementary teachers' learning to develop their teaching. The data was drawn from the reflective journal comments from teachers viewing videotapes of their mathematics lessons. The teachers have an espoused way of working to meet the reform agenda in the United States that incorporates establishing a culture in which children explain and listen to each others' strategies and ideas – ultimately moving toward students challenging and justifying each others' conjectures. Teachers' responses were compared with researchers' analysis of the interaction and discourse in the classrooms in order to understand teachers' reflective processes in developing their teaching.

## **Part II – Themes and issues arising from work and discussion relating to the three theories**

In this section you will not find any account of the Theories, per se, but rather an account of the questions and issues arising from discussion around the Theories. I have tried to provide a sense of ideas and questions both in the abstract expression of issues and in the words of participants where these are available. It is important to recognise that I will not have captured every issue or idea, and that I will not have credited all participants with their utterances. The notes from scribes were excellent, but they did not always attribute ideas to people, nor could they capture everything that was said.

I have tried to offer a dynamic sense of ideas and debate. What follows is not a seamless well-argued whole, but rather a set of snapshots that will challenge readers to fill in gaps and to ask and answer their own questions.

I have organised this according to chronology in terms of the order in which we took our theoretical topics. The leader each day was asked to try to build on the previous days' work. If you wish you may refer back to the paper that formed the plenary introduction to a session. Each section starts with a brief paragraph to take us into the substance of the section. This is followed by questions which were set by the speaker as starters for group work. The questions are followed by a list of issues arising from group discussion. In each case these were presented and discussed in a plenary session after the separate groups' activity. However, I present them here before offering some synthesis of group discussions as they may be helpful in guiding readers' focus and thinking. I have tried to be faithful to what was said in the presentation of these questions, making only very minor changes for sense-making. In each case, I have ordered the questions as makes sense to me.

### **Constructivism**

In her plenary, Jere Confrey said, "Knowing cannot be considered divorced from what it means to come to know." This is fundamentally an epistemological statement, and fits with Nel Nodding's (1991) proposition (in a critique of an account of Constructivism by Ernst von Glasersfeld (1990)) that constructivism is *post*-epistemological. Thus, constructivism says nothing about the status of knowledge or about 'truth'. Issues of truth and validity were discussed. If knowledge is the product of individual construction, how is it possible to say what is true? Can contradictory assertions be true simultaneously? Jere Confrey suggested that constructivism "removes true by refusing the tautological in it, i.e., it's true if it's true". There is a problem with claims of truth without justification. Is truth a given for which we seek, or is it more helpful to see truth as knowledge that is justified belief? Validity requires justification, and it is only with acceptable justification that knowledge can be judged to be true. Constructivism allows uncertainty and tentativeness of knowledge to be an acceptable position. [See Jere Confrey's paper in this issue of NOMAD]

**Questions offered by Jere Confrey for group discussion:**

1. What are the basics of constructivism for you? How do these relate or conflict with mine?
2. How has your understanding of it changed (over time with your sites of practice)?
3. Have you supplemented/refined/borrowed from your other theories? How do you know they are compatible?
4. Meta question:  
How does this activity/discussion inform (a) full participation; (b) going deeper; (c) cross pollination?

**Issues arising from group discussion of Constructivism**

- What is the nature of truth, tautological thinking, or validity with respect to constructivism?
- Constructivism provides a tool kit to look at problems and issues in the process of learning
- Constructivism opens up mathematics better than other theories. There are problems about where mathematics *is* – in children's heads, or in books on shelves.
- Looking forward – why is constructivism needed for a progressive programme?
- Although its focus is on the learner, individualisation is a mis-reading of constructivism. Social issues are included within construction.
- Can the constructing entity be a group – a dyad? a triad?
- Not an adequate focus on the role of “the other” – intersubjectivity.
- “Learning requires consciousness but not intentionality” – why not intentionality and goals? What about the importance of tacit knowledge and unconscious learning?
- Constructivism places power in the hands of students, whereas other theories place it in the hands of teachers.
- How does the theory talk about *failure* to learn? How do we plan for learning in the classroom?
- How do we organise teaching within this framework?
- How do constructivists think about learning to teach?

- To what extent does constructivism provide a methodological framework?
- Is there anything (e.g. affect) that isn't affected by constructivist theory?
- How is a difference between *Research* and *research* perceived in Constructivist terms?
- What is the virtue of a global system for research over a local one (e.g. this classroom)?

### **Group Discussion relating to the above issues**

These notes from the records of group discussion try to provide some of the substance underpinning the issues raised.

What are the implications of such a theoretical position for practice? Marit Johnsen Høines asked, "We are talking about constructivism as a theory of learning, but what is the implication for how we operationalise the activities to take account of the theory?"<sup>1</sup> This question digs deeply into the relationship between theory and practice where constructivism is concerned. If as practitioners we take a constructivist position, how might this influence our actions? It was noticeable that we often speak as if theories are active – constructivism does this, says that – e.g. "constructivism places power in children's hands". People mentioned the 'usefulness' of theory. Is such usefulness in its explanatory power, or in guiding action? Would such action follow from the theory, or be the responsibility of interpretation by the user? Jere Confrey pointed out "Every statement about constructivism implies an observer". In fact it is human beings who do, say and see, holding and interpreting theory. Theory can be seen as a social construct arising from human thought and rationalisation. This statement by itself can be analysed according to the various Theories. Tony Brown asked, "What is the power of asserting particular models as being the ideal?"

Laurinda Brown said, "What I look for in classrooms is whether there is learning taking place." As experienced professionals how do we recognise mathematical learning? What are the physical manifestations of learning, and how do we associate them? Does such recognising indicate a particular way of seeing, such as through a constructivist lens?

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<sup>1</sup> Words quoted are accurate copies of what the scribe recorded, not necessarily exactly what the person said. Participants have had the opportunity to read what has been attributed to them on the understanding that where there is an objection, the quotation would be withdrawn.

Leone Burton suggested that we cannot differentiate theory from practice. Separation does not work. “As a researcher I am looking for how practices I follow are governed by theory and that brings me back to the practice. How does the relationship between theory and practice expose the set of glasses I use when I look, and what I see when I look?”

If, as according to Jere Confrey, knowledge is “justified belief” then it seems that it would be important to be explicit about our ways of seeing, about the ‘glasses we are using’ to look. Anjum Halai asked whether constructivism explains all knowledge, asking “what about spiritual knowledge?” There was considerable discussion about Jere’s claim that “learning requires consciousness but not intentionality”. Consciousness might imply we do make explicit the way we see things but not necessarily intend to see them in these ways. Yet we know there exists tacit knowledge, where we do not explain (to ourselves or others) what we know, how we know it, or often indeed that we do know it. Janet Ainley pointed out that what we ‘see’ and what sense we make of it, often depends on what we are ‘attending to’ at any time, and this may well have tacit elements that we do not make conscious.

Bjørnar Alseth held up his pen and suggested we take for granted the object and its use within the social setting we are in. However, suppose he was to refer to the pen as a “magic stick”, what would then be our interpretations? Perhaps the reference would make us conscious of the object whereas seeing it as a pen had not been a conscious act. In Janet’s terms, in our familiarity with the pen, we do not attend to its use and function. However, challenged with a ‘magic stick’, we attend to what this might mean in the given context. The use of the pronoun ‘we’ was questioned. Is it possible to make group constructions? Ron Tzur said “I do not see constructivism as relating to the individual but rather to how social knowledge is assimilated.

In her talk, Jere had made the point that Constructivism does not need the appendage “social”, as social interactions are central to human construction and accounted for in the theory. Indeed, other constructivists, Steffe particularly (see for example, Steffe, 2000), have made this point strongly, suggesting, contrary to some criticisms of constructivism that intersubjectivity can be fully accounted for within constructivist theory. Some constructivists have introduced the word ‘social’ for one of at least two reasons: firstly to emphasise the importance of communication and intersubjectivity in individual cognition, and to counter imputations of solipsism; or secondly in



expressing Vygotskian theory as a form of constructivism to distinguish it from Piagetian constructivism.

However, one of the groups felt constructivism does not provide an adequate explanation of intersubjectivity, or the role of “the other”. Is “the other” the teacher? It was felt that the role of the teacher is not clear, that constructivism, in focusing on the learner, ignores the possibility of teaching. This begs questions about “learning to teach”. How can constructivists be teacher-educators?

Gerald Goldin expressed concern about apparent dichotomies between notions of “truth” and “knowledge” as understood in mathematics and mathematics education. Mathematics moves logically from axioms to theorems and proofs and in this sense is essentially tautological. By disposing of “truth” and “correctness” radical constructivism disassociates itself from mathematics. But removing mathematics from mathematics education leads to poor epistemology. Jere suggested that rather than disposing of truth, constructivism focuses on validity and requires justification – although this would not necessarily involve tautological argument. Terry Wood suggested that constructivism promotes the idea that uncertainty and tentativeness of knowledge is positive and does not imply a lack of reasoning or rationality. Reasoning can be an individual process, but it is situated. Making private knowledge public is where validation takes place and thus “truth”. Therefore interaction and negotiation of meaning are essential to knowledge validation. Referring to rationality and reason Bjørnar Alseth asked *whose* rationality is it? Is it locally or culturally situated – an ontological question? The debate here indicates a shift in thinking from an internally-consistent expression of proof and certainty in mathematics, through a recognition of uncertainty and the need for social justification leading to judgments of validity in the social domain, and finally to questions about the social nature of knowledge and learning as opposed to perceptions of individual thinking and rationality perceived in constructivism.

There was debate about the role of reflection in knowledge construction, suggesting that reflection is implicit in describing and justifying, but also that reflection is an inherent part of the abstractive process in mathematical learning. However, as teachers we may wish students to abstract yet recognise their situatedness ‘in the concrete’. Issues of validity of interpretation and common understanding arose again, and the magic stick was again used as an example.

Discussion kept coming back to issues relating the individual and the social. Maria Luiza Cestari felt that “communication is an important element not included enough in constructivism”. For example, through discourse analysis we can explore how children anchor their mathematical learning. Steve Lerman, quoting Jere Confrey, suggested that communication is at the heart of constructivism as it effectively enables perturbation – for constructivism, without perturbation there is no learning. Gard Brekke suggested that Gerald Goldin’s paper on ‘affect’ should be discussed, since affect fundamentally requires a social account. Perhaps affect might be seen as an interface between the individual and the social. How does Gerald Goldin’s analysis of affect relate to theory of constructivism? Perhaps ‘affect’ complements a constructivist analysis? Maria Luiza asked, since we can’t grasp ‘affect’ – as it consists of states of emotion - what theory can we use to relate affect to cognition.

Sigmund Ongstad warned that adding one theory to another is an aspect of modernism that leads to dichotomies – between social and individual, between theory and practice. It is impossible to reconcile theories in such a dualistic analysis.

Should we therefore avoid seeking reconciliation? Would this imply a choice of theory, while recognising its limitations? Or, are we somehow able to site theories alongside each other in a complementary fashion? Are we making mistakes in expecting any theory to explain everything? Can we indeed offer a coherent account of all that requires explanation, or do we necessarily engage a perspective as soon as we pose a question? Which are the questions that constructivism addresses adequately and which ones not? What about the other theories?

### **Social Practice Theory (SPT)**

Jill Adler claimed that “cognition is fundamentally tied to identity”. How we act is not just what we are thinking about but draws on history, context, culture and resources. Learning is seen as increasing participation within some kind of community: practice is constituted by its participants - as we participate we fashion an identity.

A fundamental construct, according to Jill, is teachers’ knowledgeability. Teachers are positioned as both key to change and an obstacle to change, and a deficit discourse is produced. Changing knowledgeability could offer a way forward. However, Jill asks, “Is changing knowledgeability an unproblematic ‘positive’ outcome of participation?” [See Jill Adler’s paper in this issue of NOMAD]

**Questions offered by Jill Adler for group discussion:**

1. Does SPT bring the social and cognitive together?
2. How hard is the boundary between everyday and school mathematics and disciplinary knowledge?
3. What happens or needs to happen as we reconceptualise SPT from apprenticeship, and professional contexts, into school mathematics?

**Issues arising from group discussion of SPT**

- Can SPT (really?) bring together the social and the cognitive? Can we get a theory of everything – that explains everything?
- Is there a tacit assumption that there is *a* learning process applicable to all learning situations? Is it a normative process? Does SPT have normative power, or should it stick with analysis?
- Where *is* the power in SPT? What is meant by power from an SPT perspective? Is power shared in some way?
- ‘Negotiation of meaning’: these words describe a process rather than what is going on in the process. What about power relationships – who is the master in the classroom?
- What does it mean to have mastery of *the* mathematics? What is the meaning of mathematics? What is school mathematics? Who determines it? Is the mathematical focus changing?
- Is the metaphor of apprenticeship appropriate for school mathematics learning? What is the product of the apprenticeship? Does there need to be a product? Are there different forms of apprenticeship?
- How is mathematics handled by teachers? Should we problematise pedagogical content knowledge as obscuring epistemological questions?
- What is mathematics teaching about? In the way that an art teacher is often a practicing artist, why are mathematics teachers not involved with mathematics?
- Knowledgeability – what does this mean? What does it mean in terms of mathematics? Is there continuous change of perception and content? Are changes positive or negative? Does the meaning of knowledgeability change over time?
- How does SPT deal with change?
- How do you apply communities of practice to students: Teacher is master, but students aren’t becoming teachers. Students are masters/novices of what? A community of what?

- Awareness of the individual not being in just one community – belonging to overlapping communities. Are there conflicts between how we want to act in different communities?
- A strength of SPT is that it focuses attention on self-identity of students as *learners of mathematics* – students who do not previously have such a self-identity gaining access to the mathematical community.
- Is identity constructed? *How* is identity constructed?
- Can I be married with a socio-constructivist and have a romance with SPT???

### **Group Discussion relating to the above issues**

These notes from the records of group discussion try to provide some of the substance underpinning the issues raised. Discussion built on that of the previous day.

The first question on which groups were invited to work was “Does SPT bring the social and the cognitive together”. The question itself raises issues about what we mean by these terms (social and cognitive) separately, and whether, indeed, separating them is an unhelpful starting point for discussion.

For many educators and researchers, constructivism does not give an adequate account of the growth of knowledge within social settings. Raymond Bjuland indicated that a question for him in his research related to how people interact together as a group. Such interaction, and the related growth of knowledge, does not seem to fit into SPT, but needs more than a constructivist (individual) account. Discussion explored the role of the individual as part of a social world: for example, “We can never be outside the social world”. However, the degree to which we are absorbed in and by the social is in question. In particular there are questions about human agency; whether we are confined and defined by the (social) structures in which we have being or identity; of what such identity consists. Jill Adler offered an example of the Soweto uprising – without a notion of agency, it is not possible to explain fully how students of apartheid education (as structural oppression) could rise in opposition to it. One suggestion was that they were not acting as individuals but as a community of resistance: there is the constraining of apartheid, but also the constraining of their own social group. It is hard to conceive of the individual because we are all part of our various histories and cultures.

Discussion ranged widely with references to Foucault, Giddens and Bourdieu, (e.g., Bourdieu, 1989; Foucault, 1972; Giddens, 1984) coming back to Raymond's question about the nature of interaction and associated knowledge growth. In research, we need to make focused choices – e.g., to look at influences from outside the classroom from within the classroom. We need to decide what we are looking at before we can decide how to focus. For example, if we are trying to explain performance, we need to consider both the individual and the social: we should not label a child as lacking ability to do mathematics, when other factors may explain poor achievement.

Keith Jones took up Jill's point about looking from two perspectives in a complementary way: at the micro, or psychological level, and at the macro or sociological level; both have their limitations that need to be recognised and built as a critique into research processes. We might be seen currently to be operating at a *meso* level trying to build on the two processes and link them, as has been the case in a number of other areas of scholarship (e.g., human genome project; in physics linking local forces, electricity/magnetism perhaps, with the way the universe works). Alison Price pointed to the two ways of theorising light transfer, wave and particle theories, recognising that each metaphor is useful in some context, and asking whether we need them to come together. Jorunn Spord Borgen suggested it might be worth using two focuses in the same study and asking questions about for example, didactics and pedagogy from the different perspectives. Keith suggested that an alternative is to start from the point of view of a problem, in for example teacher education, and then use different ways of looking at the same problem.

Jorunn acknowledged that in Norway it is traditional to separate out mathematics, didactics and pedagogy in teacher education. Jill indicated that such separation was also true of a South African inservice teacher education programme, and some of the problems in this separation. For example, a focus on pedagogic content knowledge can obscure the needs of the severely disfunctional as described by Jere talking of inner-city educational issues, and also in the South African context – “a culture of immediacy where yesterday and tomorrow are uncertain”. There are often serious social and educational problems that cannot be ignored, yet that are not addressed by particular theoretical perspectives.

Anjum Halai asked, “Where is the power”? In SPT the power is towards understanding the social in order to make sense of the individual. She gave an example, from her research, of a student and his learning

of fractions, indicating the social forces that had to be taken into account before making judgments about his fractional knowledge. Janet Ainley, being deliberately provocative, responded as a school inspector, being only interested in children's understanding of mathematics, "I only want to know what he knows about fractions". A dilemma for the teacher lies in rationalising the needs and demands as the teacher perceives them, and raises difficult questions about power. Joao Felipe Matos suggested that for Lave and Wenger (1991), power is in the community of practice. The inspector has no role in the community of practice of the classroom, whereas the teacher has considerable power.

Notions of community of practice are also problematic. As Keith Jones pointed out, with a subject-based curriculum, secondary students are asked to be apprentices in many different communities of practice, which can be exhausting. Jill acknowledged problems also for a teacher: that the progressive teacher moving towards classroom integration needs to rationalise four (possibly incompatible) 'identities': mathematiser; user of mathematics in life/world; politician, dealing with the power of mathematics in the world; and producer of excellent examination grades so that the school can be high in the league tables. Anna Chronaki suggested the importance of teachers, across schools, forming their own community group to try to resolve issues and protect themselves against external forces. Anjum, who had suggested this to teachers in rural areas of Pakistan, said the physical impossibility – it would take a whole day to get together – stood in the way of such a resolution. Returning to SPT particularly, Janet Ainley asked about the 'product': in Lave's work in tailoring for example, there is always a product to which participants' energy is directed, for example a waistcoat. "In the classroom there is no waistcoat". If we feel that an apprenticeship model can teach us something about school mathematics learning, we need to consider how we can conceptualise the waistcoat. In some situations, getting through the examination becomes the waistcoat. In this discussion the dialectic of theory and practice was starkly laid out. Theory cannot ignore the deep-rooted problems and dilemmas of practice, so where it fails to account for these it fails to be of use.

The possible normativity of SPT was questioned. Is SPT an approach to learning or a theory of learning? Jorunn Spord-Borgen pointed towards the danger of an underlying assumption that one theory will provide the best way of learning everything. One should be aware that learning one thing in one situation through a set of methods does not mean another

thing will be successful. The best way might be a multiplicity of routes, in other words we are neither seeing nor seeking normative theory.

There are nevertheless many questions about particular theories and their relevance to or relationship with practice. Where SPT is concerned, the above discussion highlights a number of issues to do with practices in mathematics education and the main elements of the theory: participation versus peripherality, masters and novices, the nature of apprenticeship and communities of practice. I shall take these one by one and, briefly, highlight the issues:

*Participation versus peripherality:* In any community of practice, how is the move from the periphery achieved? What is the nature of increasing participation? Are there degrees of participation – for example, participation by listening? Does the lack of a clear product (the waistcoat) make it difficult to define effective participation? Are some members of the community less peripheral than others due to the power in their ability to influence better the directions the community takes in its growth?

*Masters and novices:* In a school or classroom community, who are the masters, and what are they masters of? If we describe the classroom as a learning community, novices are learners, growing to being more effective learners. Where does this place the teacher? There is an asymmetry of power between a teacher and students – how does this relate to participation? In a community of practice, do the masters have the greatest power in promoting an effective practice?

*The nature of apprenticeship.* In the learning of mathematics, if students are novices, apprenticed to masters, it would seem that the masters need to be effective learners of mathematics. The product is then effective mathematical learning. What does this have to say about the role of the teacher and what are its implications for teacher education? Does apprenticeship perpetuate ineffective practices?

*Communities of practice:* does legitimate peripheral participation depend on well-defined, distinct, communities? What if participants in any community are participants of a number of overlapping communities – for example, for students, a significant number of

different subject classrooms? Would interference preclude effective increased participation? Have students become effective at separating their different communities, and could this be a reason for perceived lack of knowledge transfer?<sup>2</sup>

Cognition, the nature of knowledge, and knowledgeability kept emerging in different guises throughout discussion. It was suggested that in a community of practice, cognition is located “in the middle of the group”. The need for such ‘location’ is possibly a constructivist phenomenon, deriving from a perception of the location of cognition with the individual – a need to locate; cognition has to be somewhere. An alternative perception of ‘knowledge in practice’ is not one of location. As people participate in the practice, knowledge grows in the practice. Simon Goodchild, responding to these notes, draws attention to Lave’s (1988 page 1) account of cognition as “distributed – stretched over, not divided among – mind, body, activity and culturally organised settings (which include other actors).” This, therefore, begs many questions about the nature of knowledge and its relation to individual belief and identity. Knowledgeability seems to be a measure of what knowledge an individual has, and something about the quality of such knowledge. How does knowledgeability increase? Is this through increased participation? Does participation equate to learning, and what in fact is being learned?

The answers to some of these questions would probably be different if we were talking about tailoring from talking about mathematical learning. Even if we resolve the masters-novices questions, with some clear agreement on the role of the teacher, how do we make judgments about effective mathematical learning? In a changing world of social expectations and values, and educational structures, mathematics has been held as unchanging. However, perceptions of mathematics have changed considerably over recent decades, and these influence value judgments on effective learning. Recognising the complexity here, the product we are seeking has little in common with a simple waistcoat.

The notion of ‘power’ was another recurring concept. In any social setting there is always negotiation of meaning, but this does not imply a legitimacy of participation. In current educational settings external forces and influences often skew the power base. Demands of society,

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<sup>2</sup> In response to these notes, Leone Burton draws our attention to a book by Etienne Wenger (Wenger, 1998) that provides a more recent focus on these issues than references to Lave and Wenger (1991) in the conference sessions. Jill Adler’s paper also makes reference to Wenger (1998).



politicians, inspectors, and indeed educational theorists, all put pressure on the participants of classroom communities. An example was given of the current state of educational interpretation in the UK, where teachers feel impotent in the face of external demands. However, previously, in the UK, there were about 25 years of non-interference by government agencies. This period was not notable for demonstrating increasing knowledgeability of effective mathematical learning, and there was little notable change in practices. Current government initiatives have resulted in clear changes, but we can question their contribution to effective mathematics learning. In SPT, power was seen to lie in transparency, and it was suggested that increasing knowledgeability leads to increasing transparency. However, these are theoretical positions and very hard to interpret in the practices in which we engage.

There is often a tendency for educators to put aside as peripheral the pathological social circumstances that theories fail to address. However, the severe problems of inner-city schools described by Jere, and of the South African educational system described by Jill, will continue to be the major influences on the educational systems they affect. Educators' idealised approaches to conceptualising effective learning of mathematics will always fail while such severity is ignored. Theories that fail to account for the influence of such problems are useless in describing educational settings, let alone in predicting approaches to effective learning.

This takes us to our third area of consideration, that of Sociocultural Theory.

### **Sociocultural Theory (SCT)**

Steve Lerman acknowledged the different focuses of different theories and his concern with “sticking theories together”. As key elements of sociocultural theory he identified *historical and cultural legacy* – we are products of our various histories and cultures involving multiple socio-cultural practices including mathematics; *consciousness* (from psychology) – communication drives consciousness; *our world is mediated by tools and signs* – our necessity to make meaning using language and artifacts; *inseparability of learning from teaching* – wider definition of teaching as learning from others; *internalisation* – the process by which the mental plane is constructed; *redefinition of individuality* – the individual as constituted through social interaction. [See Stephen Lerman's paper in this issue of NOMAD]

### **Questions offered by Steve Lerman for Group Discussion**

1. What tools do we have for bringing sociological and sociocultural perspectives into our micro-analysis of mathematical activity and classrooms?
2. Theories are proliferating. Is this helpful or not?
3. Are there insights from SCT which can help us to do something about the production of failure?
4. What is special about mathematical thinking?

### **Issues arising from group discussion of SCT**

1. Proliferation of theories: how might we explore the balancing of theories, of commonality and diversity from the micro-level of the classroom to the macro-level of diversity?
2. What does it mean to make progress in dealing with theory proliferation?
  - e.g. Unification: resolution of inconsistencies - theory of everything? Hegelian synthesis, antithesis?  
Look for commonalities but respect differences?  
Coordination of theories?
3. Research and practice: As educators, wishing to educate, does the theory support us, e.g., in helping us solve outstanding problems in mathematics teaching and learning? Does it allow us to stabilise our work; build on rather than keep restarting?
4. Why is the empirical field separate from pedagogy and practice? Can we read from the empirical field what is useful for practice?
5. Are theories tools or are tools part of the theory? Any theory tells us something – expanding our perspectives.
6. Discourse analysis and genre theory are useful tools for analysing utterances in mathematics classrooms and helping us to ask questions, e.g., about authority. As they help us do this, do they take us closer to or further away from mathematics? How does the focus of research affect this?
7. If we bring in socio-linguistics with tools developed in other fields, what are we doing when we use these in mathematics?
8. How do various social perspectives provide tools to interrogate the boundaries between mathematical thinking and common sense? How do differing perceptions of these boundaries relate to the production of failure?

9. Perhaps the problem with mathematics is with didactics – if the goal is ‘to get used to it, rather than to understand it’ (attributed to Von Neumann). Why is mathematics special – a tool for sorting people?
- 10 Does becoming a mathematician have to be dehumanising?

### **Group discussion relating to the above issues**

Considerable discussion ranged around the proliferation of theories and the resulting issues, problems or consequences. There were differing views about mixing theories or using more than one theory. Theory can provide a tool which is usable in analysis. There must be good reasons, articulated coherently, for using a range of tools. Some of the views expressed were:

- A theory comes from the voice of a researcher.
- Proliferating theories are useful because they allow a choice of appropriate lenses to view a question.
- When we try to find connections between theories, are we trying to get one unified way of reading reality? Finding the right answer is connected to mathematics. Perhaps we need to go deeper to understand the fundamental issues.
- It is not appropriate to search for the ultimate theory, but to navigate the present with what we have.

The theories of Vygotsky and Piaget, both focusing on children’s learning and development, were contrasted as an example of proliferating theories, and whether it made sense to draw from both in relation to one study. Earlier questions about theory guiding practice arose again. Jere Confrey spoke about her reading of Vygotsky leading valuably to a focus on language and her overt use of terms such as ‘per cent’ as artifacts to evoke elements of students’ cultural lives. However, Gill Hatch pointed towards Vygotsky’s focus on language and instruction leading (valuably or otherwise?) to a legitimization of direct instruction in classrooms. Also, Piaget’s focus on children’s learning outside the direct influence of schooling was used in the UK as an instructional model promoting discovery learning. These two influences can be seen retrospectively to have been ill-judged practical extensions of theoretical perspectives, where practitioners, engaging in direct instruction or discovery learning, may have had little understanding of the theories that spawned these approaches.

The individual researcher making a reasoned use of a theoretical perspective can/should judge critically the consequences and limitations of its use, which is not always straightforward. Theory has often been found inadequate to explain fundamental issues. Discussion focused on the question of how children learn what society has taken so long to create. There is a need to explain internalisation more satisfactorily. Some felt this had not been done adequately by Vygotsky or other socioculturalists. There was also seen to be lack of adequate explanation of the concept of Zone of Proximal Development and how ZPD can be used by teachers. The issue of interpretation of theory by those using the theory can itself be a problem. Discussions of theory by the theorists themselves often lack the clarity that researchers would like in applying those theories to practical contexts.

Nora Linden and Marit Johnsen-Høines (responding to an earlier version of these notes) questioned the issue of power/authority reported as arising from some of the discussion on Vygotskian theory. They say, “Our comments have to do with the theory of Vygotsky. We do not recognise the way the terms, emphasising exclusively the teacher’s power/authority, are used. If the major difference between Activity Theory and Constructivism lies in the question of where authority is “placed”, we regard this as a simplification. Vygotsky’s theory emphasises the teacher as the mediator, with the privilege to define/decide how to organise for learning. In this we see the authority. There is, however, a power aspect in the way the student’s goal and motive for learning is emphasised (Vygotsky, 1978). We remember this as an important aspect in our discussion at Gausdal.”

Jere was asked whether she would incorporate Vygotskian ideas into constructivist theory. She indicated the value of trying to extend, or create new theory, resulting in “genuine diversity”. Gerald Goldin indicated that he had, in earlier writing, proposed an approach to the unification of theories making use of the notions of representation and systems of representation, since keeping different perspectives apart makes the introduction of new constructs extremely difficult (Goldin, 1992, 1998). Questioned about the problem of contradictions among different perspectives, he emphasised the importance of specifying as precisely as possible the nature of the contradictions so that further research can resolve them. Gill Hatch suggested there was actually value (richness) in diversity as well as in commonality.

Eric Love asked “are theories progressive”, do they take us forwards or just offer different descriptions? Bjørnar Alseth pointed out that

‘forward’ as a goal suggests direction, and this in itself is a theoretical position. Laurinda Brown questioned what it would mean, “if we move forward over the years”; who is “we”? “I can’t give my student-teachers my experience.” These interchanges show some of the difficulty of addressing theory separately from situations in which we would use it, since it is necessary to look critically at context in judging theoretical value. However, what do we learn over time from consideration of theory? Is it helpful if our students have to begin again from our own starting points? For example, as we now try to make sense of Piagetian and Vygotskian theories in classroom contexts, can we learn from historical precedents such as those connected with direct instruction or discovery learning?

Someone said, “Even if I’m familiar with a theoretician’s theory, it doesn’t mean that I’ve taken on the theory”. Any individual is the intersection of multiple perspectives and critically conscious control is not always what we might want it to be in theory. For example, the way one brings meaning to SPT might be a function of one’s understanding of the theories around it. “By revisiting Plato in the light of Derrida, Plato means something different.” We need to recognise dialectics and their importance rather than dwelling on the “truth” of each theory. From a perspective of dialectics, in for example the relationship between teaching and learning, the prevention of failure can be understood. We need to examine how theories are a function of each other, and how past theories relate to our current engagement.

The relationship between consciousness and communication (consciousness driven by communication) introduced by Steve Lerman was a focus of discussion. Gunnar Waldermo suggested using “communication” as a vehicle for addressing differences between the three Theories. Terry Wood emphasised the importance of developing shared perspectives through communication, but proposed that communication is itself too broad a term; preferring to talk about *negotiation* for reaching shared meanings.

The nature of research exploring communication and meaning was discussed: is it the discussion or negotiation *as a whole* that we want to analyse or are we interested in its constituent parts? For example, how do we link the macro and the micro in classroom research? The micro might be an utterance, a sign or a concept; the macro might involve discourse, genre, frame, or register, all very fuzzy. The paper from Hans-Jørgen Braathe and colleagues, drawing on the politically focused work, ideology and rationale of Stieg Mellin-Olsen, had discussed “genre” as

a macro focus. Jill Adler asked Hans-Jørgen how social perspectives had helped microanalyses in their project. He commented on the many genres present in a mathematics classroom and the use of utterance as a unit of analysis, placing utterance within genres and analysing in terms of cognitive or affective activity, trying to characterise dominant parts and place into ideologies. A question was raised as to whether “genre” could be described as a tool, and whether other approaches (such as discourse analysis) might also be regarded as tools in this context.

Keith Jones, referring to his paper on use of tools, highlighted the importance of the *historical* development of ‘tools’, pointing out that tools (e.g. from discourse analysis) have not originated in mathematics education research, and simply importing them may affect what we can or can’t do in terms of understanding mathematical thinking. Joao Felipe Matos provided an example from his research in which he is looking at how students and teachers legitimate discourse in mathematics and how emotion constitutes mathematical thinking. Raymond Bjuland added that as we cannot look at thinking we can only look at discourse as an outcome of thinking. The importance of the use of the tool and its relation to the focus and findings of the research is evidenced through these examples, emphasising again the critical use of theory or tools related to a particular focus or context.

The unit of analysis needs to be well defined whether it is a word, an utterance, a group of utterances or whatever. The point was made that the unit needs to be big enough to take into account the context. Simon Goodchild indicated that words, or utterances have to be placed in a social context – who is speaking, listening, beliefs of participants, their perceptions of each other and so on. Raymond Bjuland, drawing on his own research pointed to examples of context: student groups where students work without the teacher, and where power is shared – “no master voice”. However, prior work had been done by the teacher on how to work in groups. Such context is important to analyses of what is actually said by the students. Anjum Halai pointed to differing experiences in her research of observing student groups where children see some individual as having authority, and this influences interaction. Raymond suggested differences in maturity between his student-teachers and Anjum’s students. Jorunn Spord Borgen commented on potential differences in culture between the situation in Pakistan (Anjum’s research) and that in Norway (Raymond’s research). These examples were valuable in making sense of elements of theory and their relating to research – emphasising again the central nature of context in use of theory.

Questions of how a researcher can gain access to pupils' beliefs and values resulted in an anecdote from Alison Price regarding a student who, in discussion about properties of 10 spoke of being 10 (years old), and of his older friend who is 11. What can the teacher say about the child's understanding of 10? The child seemed to be linking mathematical and everyday concepts. However, too often mathematics is done without reference to the everyday – or with inappropriate reference – so that the child forgets to make common-sense links and results in doing incorrect mathematics; a production of failure. The multiple meanings involved in such analysis need to be recognised and critiqued in order to make sense of such classroom situations in a way that is helpful for future practice.

Discussion came around to mathematics, and in particular to “the production of failure” in mathematical learning. Mathematics developed in order to solve problems, but it has been recontextualised for purposes of schooling. So, what is special about mathematical learning? It was suggested that the notion of “understanding” is a problem – although fundamentally a part of our discourse, it limits out thinking. “Understanding is secret and internal – we need to induct people into the genre and discourse that we call either mathematics or mathematical thinking”. It was suggested that there are two different things – the stuff in one's head (thinking) and the stuff in books (mathematics). Wenger (e.g., 1998) was reported as describing the “stuff in books” as the “reification and the thinking emanating from participation”. Steve asked, “is reification a tool or a result? It is both, but phrasing it in this way emphasises the result. I chose to do that because of the problem of internalisation from a Piagetian point of view.”

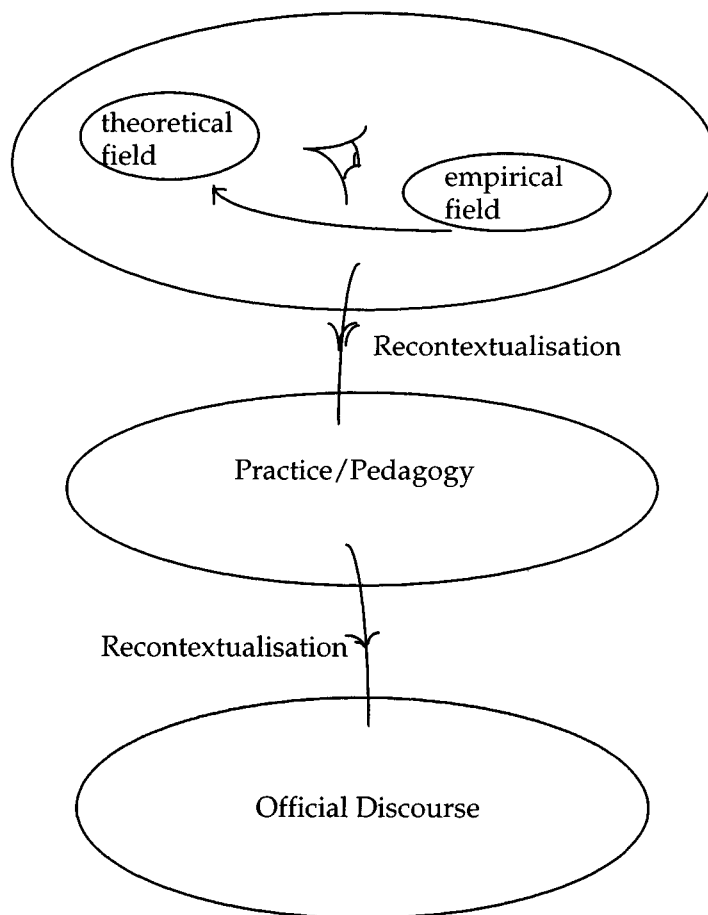
### **PART III– Some final thoughts**

There are no conclusions to this strange paper. Its very nature has been to try to present a set of congruences and counterpoints in the constrained medium of linear prose. If the reader feels frustration because arguments are left in the air, not fully developed, or at times seemingly inconsistent, that is because human discourse works in this way. Each person at the conference developed their own personal perspectives of the theories we negotiated, and perhaps went away with a richer sense if not necessarily a clearer sense of their separateness or inter-relatedness.

In a few final words, I want to come back to notions of the relationships between theory and practice, and ways in which a developing richness

of theoretical understanding might be available to us in coming to know more about teaching and learning mathematics.

At the end of our third session, Stephen Lerman returned to a diagram he had used in his presentation, which I reproduce below. In it we see the “eye” indicating the view or perspective a theoretical lens brings to the empirical field, and the consequent feedback to theory from what we learn in empirical research. The learning that emerges from this research cycle feeds through successive recontextualisations into practice and pedagogy and into official discourses. Steve spoke of a potential gap between the first bubble, and second and third in this diagram, possibly because of the focus in the empirical field. A further consideration of this has potential to explain the (apparent?) theory-practice dichotomies discussed in the last section.





I want to use an example to discuss this briefly.

In the third session, in Jill Adler's group, discussing issues in sociocultural theory, I offered an example from my own recent research (with Despina Potari - see Potari and Jaworski, in preparation). We were discussing tools for analysis of discourse, to aid development of teaching. I asked what is our focus in analysis?

Are we studying teaching? Are we studying mathematical thinking? Such focus is the lens we bring to the empirical field (Steve's "eye"). I also asked, "how do we get to elements of the culture from within the culture"?

The example I gave was of our study of a series of mathematics lessons of one of our partner teachers. We both, teachers and researchers, had a theoretical focus to our research in the form of a construct, called 'the teaching triad' to describe or analyse teaching (Jaworski, 1994). The teacher had used the teaching triad to design her teaching in these lessons, and we (researchers) were using it to produce a microanalysis of the lessons observed. The elements of this triad are management of learning, sensitivity to students, and mathematical challenge. Partially these focused us onto "what is the teacher doing to enable development of students' mathematical thinking?"; partially, again, onto "how is the teacher encouraging different levels of participation, or ways of working in the classroom or on mathematics?" As our analysis focuses more and more on these elements what is being ignored?

The example is a brief account of episodes from lessons which, mathematically, were focusing on relationships between volume and surface area of rectangular solids.

The class had been asked to design a container to hold 48 cubes, each of side 2cm, using a minimum amount of material. Students were working in small groups. Stewart and Tom had used interlocking cubes to make containers of dimensions  $4 \times 8 \times 12$  and  $2 \times 2 \times 96$ . The teacher talked with them about surface areas and which was smaller, encouraging the boys to articulate their thinking. During the conversation, in trying to say why the first one had a smaller area, one boy used the word "compact". This language gave some indication of the boy's appreciation of concepts: "compact" suggesting a linking of dimensions and area. The teacher left them with the questions as to whether it was the most compact they could find.

In the following lesson, the teacher conducted a whole class session inviting boys to offer examples of their containers to fill in a table on the board. During the filling out of the table she asked students to offer explanations for their findings. Some students found it very hard to give explanations. With increasing pressure of time, as the lesson drew to an end and the teacher realised she had other activities to be undertaken next lesson, she shifted gradually into a mode of providing the explanations herself, rather than encouraging students to do so.

In talking about these different parts of the lessons, the teacher felt more satisfied with the first episode and less satisfied with the second in terms of her objectives for students' learning.

Microanalyses of the dialogue between teacher and students in the two episodes suggested that 'sensitivity to students' and 'mathematical challenge' were more in 'harmony' in the first episode than the second. Such harmony related to students' perceived 'understanding' of the concepts on which the activity focused. From a perspective of mathematical learning, the first episode seemed to be a fruitful one, and the teacher was pleased with it. While it happened, there was a relaxed atmosphere; teacher and boys interacted in a friendly way that encouraged articulation and conjecture; all seemed to be giving their attention to the mathematical focus with reference to the physical objects the students had made. In the second episode the atmosphere was much less relaxed. Students had difficulty in offering the explanation for which the teacher asked. Since it seemed that explanations were necessary, in the end the teacher offered them herself. Sensitivity and challenge seemed to be at odds (i.e., not in harmony), and the episode was judged less fruitful in mathematical learning terms. The 'results' that the teacher had wanted from the table did not emerge. In the end she left these results to be returned to at a future date.

Macroanalyses of these episodes looked at factors beyond the particular words in the dialogue to the wider issues affecting these lessons. From the teacher's point of view, factors included demands of curriculum; available time; her planning for future lessons; her knowledge of the students and what she could expect from them, both mathematically and more socially. Students' self-esteem was a concept that was especially important for this teacher, and one of her concerns in the second episode was that it had not contributed to self-esteem; possibly the opposite. Whereas the first episode took place during a time of model-building and talking contextually about the mathematics

of the models, the focus in the second episode was more formal and shifted towards abstraction. We did not interview the students, so do not have data on their own perceptions, but possibly they lost interest when the mathematics became more abstract, they were reluctant to try to explain when they were unsure, they realised the lesson would come to an end providing a release from the requirement to think. Probably they had many more interesting or bothering concerns of their own which took precedence over the move to abstraction in the lesson. Possibly they felt insecure through challenges that were unhelpful to their current state of thinking. Possibly, for some students this episode reinforced feelings of failure in mathematics.

This discussion has been too brief really to examine the wider relationships between the various factors of learning. However, we can see how we might look at it theoretically from various focuses. We can try to analyse the cognitive and affective dimensions of the various episodes from the perspectives of particular students. We can look as portions of dialogue to see how the language used might relate to the mathematical concepts being addressed. We can look closely at the social structures within the classroom, dialogues between various groups of students, with or without the teacher. We can look at the sorts of questions asked, and the ways in which they seem to influence activity and thinking. We can look at the complexity of issues motivating the teachers' actions, and her feelings about the degree of success of the various parts of the lessons. We can look at academic aspirations of individuals, and the academic ethos of the school as a whole. We can explore beyond the classroom doors to the wider social life of the school, relationships between students outside the mathematics class; family issues and concerns; community and cultural influences and demands; students' individual cares and dreams.

As theoreticians, our analyses will draw on a wide range of theories, and the theories we use will both indicate our starting points for observation and analysis and guide our analyses, restricting what we see as they focus our attention. As mathematicians we shall be guided by our particular perspectives of mathematics, and how we judge mathematical thinking and learning, students conceptions of mathematics, teachers' fostering of those conceptions. We might invoke theoretical notions such as ZPD. We might look from perspectives of reducing peripherality to bring students closer to full participation in mathematics. We might invoke genre theory or use tools from sociolinguistics. In these kinds of thinking, we move about in the top bubble in Steve's diagram. Because it is our task as researchers, we formulate

new theories based on a synthesis from our analytical activity. How do/can such theories arising from the research enterprise affect practice and pedagogy and ultimately the official discourse?

Here is the perceived gap of which Steve spoke. If the research we do is more than an esoteric seeking for knowledge, how does it impact on pedagogy and practice? How does recontextualisation take place? These are big questions demanding deeper consideration than I have space for here. However, one of the suggestions made by Steve was that perhaps the gap might be reduced through teachers themselves engaging in the research process. I believe that we have already considerable evidence that this is the case, although the nature of the involvement, the teachers' roles in research, even the definitions of research we use are all part of a complexity we are only just beginning to unravel. In the example I gave, the particular teacher had been involved in research projects with researchers from the university for a number of years. Her use of the teaching triad as a planning device was evidence of this, as was her reflective approach to teaching and her feeding back of observations and analyses into her teaching. One analysis of this process involves looking closely at the teachers developing pedagogic understanding, and its relationship to classroom activity and students engagement in mathematics.

What is clear, where the teacher is concerned, is that she has no option but to engage in practice; that is her job. The nature of this job is that it demands pedagogic thinking. When such pedagogic thinking is more or less directly related to research processes such as the ones mentioned above, the recontextualisation processes become two-way rather than just one, as developments in practice and pedagogic learning feed back into the research process.

I will say nothing here about the official discourse because this is yet another big story!

I want to end by returning to what I started with in my first section above, the nature of theory and its relation to practice. Whenever we (teachers and researchers) talked about what occurred in classrooms we started to engage with theory, or at least meta-practice. As the teachers talked with us about their planning and their reflections on the lessons, we all engaged in a discourse that extracted and abstracted and contributed to theoretical understandings. For us theory was not something divorced from the practices in which we were engaged, learning, teaching and research; it was fundamental to our educational enterprise.

This brings me back to the conference. As we talked about our big Theories, we tried to relate them to practice through examples and illustrations. We were able to consider a very wide-ranging set of ideas and issues, and gain insights into the questions they raise for us in relation to our practice. However, we could not engage in practice, except the practice of conferencing. The gap between the research/theory domain and the practice/pedagogy domain will only reduce when we can discuss our theory alongside our practice in a community that includes practitioners and researchers and theoreticians; indeed where all those involved are seen to take on all three roles, albeit in different ways at different times.

## References

- Alrø, H. & Skovsmose, O. (1993). Det var ikke meningen! – om kommunikation i matematikundervisningen. *Nordisk matematikdidaktikk* 1(2), 6-29.
- Alrø, H. (1996). Hvad er der i vejen med eleven? In S. H. Knudtzon (Ed.) *Kommunikasjon og språk i matematikkundervisningen*. Høgskolen i Vestfold.
- Anker-Nilssen, M. (1998). *Hvorfor forstår de ikke hva jeg sier? En studie av kommunikasjon i matematikklasserommet*. Thesis for Cand. scient in Mathematics Education, University of Oslo, Norway.
- Bourdieu, P. (1989). *Outline of a Theory of Practice*. Cambridge.
- Brown, L., Coles, A. (2000). Complex decision-making in the classroom: the teacher as an intuitive practitioner. In T. Atkinson, & G. Claxton (Eds.), *The intuitive practitioner: on the value of not always knowing what one is doing*. Buckingham: Open University Press
- Foucault, M. (1972/1969). *The Archeology of Knowledge*. London: Tavistock Publ.
- Giddens, A. (1984). *The Constitution of Society: Outline of the Theory of Structuration*. Berkeley CA: University of California Press.
- Goldin, G. A. (1992). On developing a unified model for the psychology of mathematical learning and problem solving. In W. Geeslin & K. Graham (Eds.), *Proceedings of the 16th PME Conference*, (Volume 3 pp. 235-261). Durham, NH, USA.
- Goldin, G. A. (1998). Representational systems, learning and problem solving in mathematics. *Journal of Mathematical Behavior* 17(2), 137-165.
- Jungwirth, H. (1993). Routines in Classroom Discourse: An Ethnomethodological Approach. *European Journal of Psychology of Education* 8(4), 375-387.
- Lakoff, G. (1987). *Women, fire and dangerous things*. Chicago and London: University of Chicago Press.
- Lave, J. (1988). *Cognition in Practice: Mind, Mathematics and Culture in Everyday Life*. Cambridge: Cambridge University Press.
- Lave, J. & Wenger, E. (1991). *Situated Learning: Legitimate Peripheral Participation*. Cambridge: Cambridge University Press.
- Lerman, S. (1998). *The Intension/Intention of Teaching Mathematics*. Centre for Mathematics Education. South Bank University.
- Mason, J. (1997). Recognising a possibility to act. In V. Zack, J. Mousley, & C. Breen (Eds.), (pp 87-101), *Developing practice: Teacher's inquiry and educational change*.
- Noddings, N. (1990). Constructivism in Mathematics Teaching. In R. B. Davis, C. A. Maher, & N. Noddings (Eds), *Constructivist Views on the Learning and Teaching of Mathematics*. *Journal for Research in Mathematics Education*, Monograph Number 4. Reston, Virginia: National Council of Teachers of Mathematics.

- Potari, D. and Jaworski, B. (In preparation). Tackling the Complexity of Mathematics Teaching: using the teaching triad as an analytical and didactic tool, (Working title).
- Steffe, L. (2000). Interaction or Intersubjectivity? A Reply to Lerman. *Journal for Research in Mathematics Education*, 31(2) pp. 191-209.
- Steinbring, H. (1997). Epistemological Investigation of Classroom Interaction in Elementary Classroom Teaching. *Educational Studies in Mathematics* 32(1), pp. 49-92.
- Voigt, J. (1985). Patterns and routines in classroom interaction. *Recherches en Didactique des Mathematique*, 6(1), pp. 69-118.
- von Glasersfeld, E. (1990). An Exposition of Constructivism: Why Some like it Radical. In R. B. Davis, C. A. Maher & N. Noddings, (Eds), *Constructivist Views on the Learning and Teaching of Mathematics*. Journal for Research in Mathematics Education, Monograph Number 4. Reston, Virginia: National Council of Teachers of Mathematics
- Vygotsky, L. (1978). *Mind in Society*. Cambridge, Mass.: Harvard University Press.
- Wenger, E. (1998). *Communities of practice: Learning, meaning and identity*. Cambridge: Cambridge University Press.

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Her research interests include perspectives of theory and practice in mathematics teaching and its development; the professional development of teachers and educators through individual and collaborative critical enquiry; and the constitution of co-learning environments in teaching and research in mathematics education.

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