Leveraging Constructivism to Apply to Systemic Reform[1]

Jere Confrey

The question posed for this conference was "what are our current understandings of the theoretical foundations of mathematics education?" To address this question, I first asked myself, "Why it is important to answer such a question?" Is it primarily an issue of philosophical investigation? Or, is it, rather, a call for some clarification about the proliferation of theories in mathematics education and their interrelationships, reexamining each theory in light of multiple theories. This second query interests me, because increasingly as I extend the scope of my research to see constructivism enacted in classrooms and schools, I need broader theoretical constructs. And, I need to reexamine of my own assumptions and their warrant.

This revision of the question, "what are our current warrants for our claims concerning research in mathematics education and how are they related to theory (ies)" presented even more of a challenge. It led me to the Oxford English Dictionary (1999) to examine the genesis and evolution of the term "warrant." There were three stages of evolution of meaning.

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[1] The Systemic Research Collaborative for Education in Mathematics Science & Technology (SYRCE)

The goal of SYRCE is to investigate and build the knowledge base on new forms of research and forge research methodologies that link classroom-level research with system-level research and policy initiatives. Specific objectives for accomplishing this goal include:
(1) examining relationships among student learning outcomes, professional development, teacher knowledge and community and implementation of standards-based curricula;
(2) building a model of these relationships as they exist in situ to guide decision making and to improve prediction;
(3) building documentable capacity among researchers, teachers, administrators, graduate students and pre-service teachers in urban settings;
(4) improving mathematics, science and technology education for urban students in documentable and convincing ways by building the content and didactic knowledge of teachers;
(5) defining new kinds of research between K-12 schools and university- based researchers.

SYRCE issues papers to facilitate the exchange of ideas among the research and development community in education.

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A first recorded use of warrant was as an "authorized protection" as in a warrant of the Emperor or King (1225). At the word's introduction to the English language, warrant referred to legitimization by virtue of the king or emperor's position of absolute authority. Outside of obedience to a sovereign's authority and one's relative societal position, there was no basis for warrant. By 1460, the meaning of the word evolved to refer to a "public pledge of responsibility on the basis of evidence; a conclusive proof as to take warrant on oneself." Warrant had become based on evidence and proof. By publicly announcing a warrant and describing its basis in judicial authority, logical reasoning and/or perhaps early forms of scientific evidence, an individual could claim warrant. Finally, a third meaning evolved for warrant as the "justifying reason or ground for action, belief or feeling."(1634) such as in "that action was warranted". In this evolution of meaning, warrant extended beyond a pledge of responsibility to become a mandate for action, belief or feeling. In addition, this final stage of definition added the criteria of justification. By adding justification, one subjects the mandate to the scrutiny and assessment of others. Thus due to the last meaning of warrant, a determination of what group is authorized to judge the warrant becomes necessary.

This evolution of warrant is relevant to our deliberations on theory, because it permits one to reject a simplistic idea that one evaluates a theory as to its accuracy in portraying a state of events. Rather a theory is fundamentally a guide to thought, action and feeling; and its validity is negotiated among a group of experts whose expertise should not just include familiarity with the theory, but expertise with the application and use of the theory to solve applied problems and anticipate new ideas. As will be evident by the end of this paper, this expanded view of warrant will prove necessary to leverage constructivist theory to the level of system dynamics.

Education, at its heart, is an applied discipline, a social science. Therefore, one cannot simply construct a theory, argue for internal consistency or coherence, and ignore practice. I view theory as a bridge between research and practice and a guide to methodology. Theory helps me to know what to pay attention to in practice, how the participants and resources are related, and what arrangements are likely to yield improved student learning and engagement. Not only does theory bridge to practice, but it also relates to the activity of creating and modifying applied products for use in that practice. Thus, theory must assist in engaging in the activity of design. Figure one shows the set of relations that surround the activity of theorizing and of engaging theory. Essentially, I consider research, practice and design three types of practices, each of which draws upon the beliefs, actions and feelings of its practitioners. Formal and informal uses of theory are involved in each. A researcher's use of theory is more formal, more principled and most likely to require direct examination, modification and clarification of the theory, therefore I placed theory within research for the analysis in this paper.

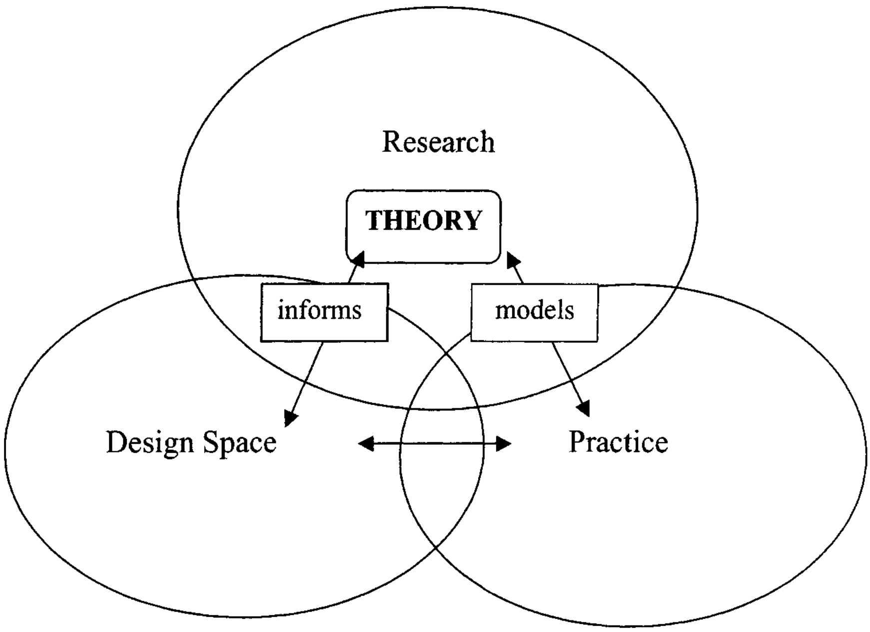

Figure 1

As the field of research has grown, the sites of practice that it seeks to explain have increased. In figure two, I suggest that over the last twenty years, we have increased our scope of examination from an examination of mathematics (to include problem solving and heuristics) to learning (to include individual, peer and group learning) to teaching and learning interactions, to teacher education. Overlaying this array of sites of practice, we have considered issues of curriculum, assessment, and technology and considered their relations to issues of equitable participation, school and district improvement and community involvement. As the sites of practice have become more varied, it is natural that our theories must be revised, modified and that new theories will emerge. To engage with this complex picture, the organizers of this conference have chosen to have three theoretical positions juxtaposed as each addresses the following questions:

- How do we currently use and understand our theory and what do we find compelling about it?
- How have we modified our theory as it is applied to an increasing range of sites of practice?

Although I will not address explicitly the third question of the conference in this paper, it is useful to keep it in one's mind.

- How do we relate our theory to other theories? How do these theories influence our own theory?

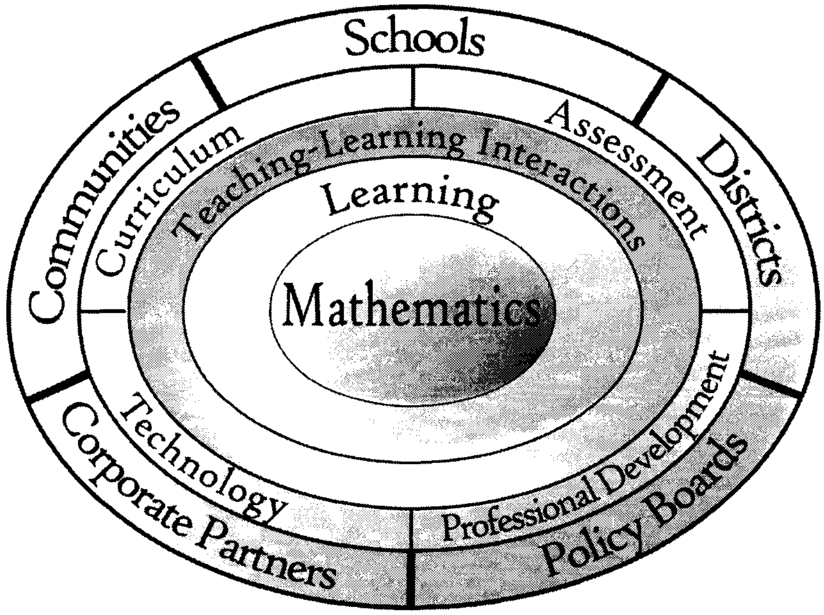

Figure 2

As a further clarification, I wish to stress that the goal of the conference, as a research activity was to explore and juxtapose the three theories. My particular paper serves two goals in that process. It was designed to lead off the conference by setting the agenda, introducing the key aspects of one theory, constructivism, and describing how it has been modified over the last few years to apply to varied settings. This is not a review of the entire theory and related literature. More complete versions of this can be found in the following citations (Tobin, 1993; von Glasersfeld, 1995; Steffe and Gale, 1995; Larochelle, Bednarz, Garrison, 1998). Rather, I will first review a set of central principles of constructivism, describe their modification when applied in the setting of urban education and then describe how we have revised them further to engage in whole school and district level change.

Constructivism Revisited

Theory, in mathematics education, does not evolve on its own. It evolves as one uses it to solve outstanding problems of practice, as one conceives of those problems. Akin to the research program of Lakatos, a research paradigm encounters anomalies or perturbations and must either gain power by accounting for those anomalies, limit its scope to exclude the case, or risk losing power. For me, a major anomaly for constructivism to explain is the intense filtering out of students from engagement with and pursuit of mathematics, as it occurs across the entire population. Moreover, it must explain and lead towards solutions for the disproportionate alienation and filtering out of minorities, people of lower socio-economic class and women, especially those from urban or rural contexts.

I believe that other theories of education may be able to account for the general aspects of this pipeline problem – economic disparities in school resources, bias in the form of racism, sexism and classism in social interactions and so on. Nonetheless, these explanations do not account for the disproportionate way that certain groups are filtered out by instructional practices or experience mathematics as impersonal, insurmountable, or alienating. There is a quality in our construction of the practice and discipline of mathematics, both in engaging in it and in learning it that influences participation patterns profoundly. And because success in mathematics is a ticket to many other opportunities, we must identify and address these issues. In this paper, I begin to explore how such a problematic can lead to a reexamination and modification of constructivism. To do this, I begin with an elaboration of the key elements of constructivist theory.

When I do research, I begin with a problem that I am compelled to understand better or to search for a means to improve. Then I define my theoretical constructs, select an appropriate unit of analysis, and apply the theory to that unit. This preparation permits me to design a research study, conduct the research, analyze it, and consider how it leads me to modify and adapt my theory. No single study will cause me to reject a theory, but over the course of a series of studies, I can evaluate the coherence and validity of a theory. In doing so, I consider its trajectory as well as the results, explanations and design initiatives evolving from it.

Key Concepts of Constructivism

Knowing is fundamentally linked to learning.

This is a statement of genetic epistemology (Piaget, 1972). Only that which one has learned, in the sense of understanding its epistemic trajectory, does one know. This claim is designed to express the fundamental position that a person must construct an idea for him or herself. This does not mean one acts as a lone ranger, building the entire world through one's own efforts, but rather than there is a significant way in which for each idea, a person must act on it and make sense of it for oneself. We cannot give or get knowledge without having engaged with its meaning, and to do so, requires action and activity. Action and activity also provide a purpose to the evolution of an idea, and they supply the constellations of events that provide the elements for its construction.

Knowledge is justified belief.

The second claim is that knowledge, that which we have come to know through acts of learning, only becomes knowledge when it can be justified. This second claim is designed to ensure that the activity described in the previous claim yields knowledge that is subject to examination and evaluation by others. In posing this claim, I am not requiring that all knowledge actually is justified, but rather that if and when challenged, it can be justified. Insisting on only potential challenge is necessary so that in a stable body of knowledge, I can claim such statements as 3+7=10 as knowledge without requiring that I have actually justified it at that time. However, should someone ask me, how do I know that, I am obliged to produce a trajectory of acceptable reasoning and argument. If this does not result in convincing my audience, then the statement's status as knowledge is in question. Its validity will remain in doubt until an appeal to a larger and/or more qualified group of experts can be made successfully or until the previous challenge is resolved.

It is the second claim that leads to the fundamental epistemological issue in constructivism. That is constructivism entails a rejection of assured transcendent truth in our knowledge. Knowledge is inherently pragmatic and is thereby tentative – tied inherently to our human ways of knowing and the evolution of thought. As members of our species and culture and as products of our individual experience, what we know has been necessarily shaped by this trajectory. We can confer, negotiate, investigate, improve, revise, adapt and modify our beliefs, and the best we can hope for is that they become increasingly viable in light of our field of experience and prior knowledge. Many people feel profoundly disappointed or incensed with this element of constructivism, feeling that their assurances of having grasped essential truth is compromised. Rather I find it freeing in that when one encounters anomalies in one's view, one can always seek resolution in changing one's theories or in seeking more accurate and consistent information. Knowledge is not assured; it is a lens for making sense of our experience. When many good minds are put towards understanding the problem or idea, it is more likely that a resolution or insight will be proposed and tested.

It seems obvious to me that since knowledge requires justification the need to differentiate socio-constructivism from constructivism disappears. Although I understand that the term "socio-" was appended by Cobb, Yackel and Wood (1991) to overcome the constant criticism of critics of constructivism that the theory was excessively individualistic, I find their decision unfortunate. The theory of constructivism has always been social – for it has always recognized the importance of social interaction – relative to task construction, to the choice of action or interaction, and to the development and use of systems of representation, description, explanation and justification. The insertion of "socio-" created a distinction within our community that rented the fabric of constructivism unnecessarily. It placed many of the original theorists of constructivism on the other side of an artificial distinction. My criticism doesn't invalidate the considerable accomplishment in Cobb et al.'s (1991) valuable work applying the theory at the classroom level to discuss classroom norms, routines etc. but places it within the constructivist tradition.

Deep learning begins with a perturbation, a felt-need.

A third quality of constructivism is the claim that most learning begins with a perturbation. A perturbation is a "felt-need". It is, as I have expressed before, "a roadblock to where I want to be" (Confrey, 1991). It creates the impetus to want to find a solution to the problematic, and thus creates motivation to solve the problem. In *Personal Knowledge*, Michael Polyani (1958) describes how a mathematician persistently pursues a problem by externalizing that problem and searching the space for its solution. He or she treats it as if its solution were to be found in one's intellectual landscape, as a means of objectifying the problem, assuming possible solutions and moving in pursuit of these. Creating effective tasks that embed significant concepts in a problematic, to explain their genesis, and provide a reason for their birth and endurance is a key concept of constructivism.

Examination of the historical, cross-cultural or interdisciplinary record is a fertile source of examining a concept's genesis.

This search for understanding the genesis of a concept, or for investigating and characterizing the socio/cultural medium for enacting an idea has made historical, cross-cultural and interdisciplinary analyses a crucial part of the constructivist enterprise. My own entry into constructivism followed an extended study of the philosophy of science. Philosophers such as Toulmin, Popper, Lakatos, Kuhn and Feyerabend all strove to explain the course of scientific development. They set out to explain how scientific ideas were invented, what the conditions of their birth were, and how and when they evolved. In doing so, they often combined forms of sociology of science, critical theory and epistemological investigation.

Work in this arena was essential to keep constructivist analysis from becoming too psychological, too mental, rather than situating it in broader intellectual and social trends. In the history of science, researchers have documented pragmatic elements of conceptual development, such as the need for a place value system of numbers to fit on coins, or the possibility of non-perpendicular axes as a means to unpack the proportional character of curve shapes (often built by physical curve drawing devices.) In the sociology of science, researchers have sought to explain such ideas as how one's observations are shaped by one's constructs, and how the distinctions between making measurable observations had to be cleaved away from the constructs as a set of scientists acclimated to various distinctions. For example, Shapin and Schuster (1985) in *Leviathan and the Air Pump* examine the debates between Hobbes and Boyle on the development of the concept of experimentation. Now that experimentation is considered as the basis of "the scientific method", one cannot imagine a time when its use was problematic, but by documenting the debates between these two scientists, Shapin and Schuster "stress the fundamental roles of convention, practical agreement and of labor in the creation and positive evaluation of experimental knowledge" (p. 13). As mathematics becomes increasingly "quasi-empirical" (Lakatos, 1998) such debates can be very informative to mathematics educators. Other rich sources of genesis of concepts have come from interdisciplinary inquiry, where scholars such as Ascher (1991) have illustrated for us the richness of indigenous reasoning, logic and notation.

Mathematics can emerge from the action of a person acting on a physical situation. As such, mathematics is the result of an interaction.

A felt-need for an idea leads to action on the problematic. A key question about mathematics is how it is related to one's experience with a physical world. Constructivism asserts that mathematics is not to be found in the objects of the physical world but rather in one's actions on those objects as described and notated by groups. Simply put, ratio does not lie in a comparison between two numbers but in the action of comparing them such that their relative size is expressed using one of them as the unit of measure of the other. Many claim that mathematics is best taught with manipulatives, however, too often we see this interpreted to mean that the mathematics is "captured" in the manipulative, rather than to view the manipulative creating an opportunity for action, reflective abstraction and hence conceptualization. Because the mathematics is in the action on and with the object, in pursuit of a resolution of a "felt-need", the action itself must be re-presented, both to assess if the need is met, but also to record and retain the action for examination.

Mathematics can emerge from the records of the action, through a process of description, representation, and at later stages, explanation, justification and proof.

In order to signify an act, humans engage in the activity of description, representation and later, explanation. Description is invariably a form of representation. In constructivism, description and representation are typically viewed as a form of expression by the actor, through an engagement in discourse. Mathematics is a means of recording an action or an interaction by inscribing a record of the event and studying that record and its relationship to other records. This leads to two important activities, the use of a system of representation and the contrast of multiple representations.

Building and comparing representations is a critical element of the theory. For instance, early in an algebra one class, we asked students to watch a videotape of a person pouring water into a semi-transparent container. The container had vertical marks on it and a clock in the background. The students were asked to make a representation of the action in the videotape. The students initially recorded three types of representation: qualitative, iconic and tabular. In the qualitative descriptions, the students quantified only the starting time and ending time and then typically listed a sequence of intervals as slow, fast, or pause. Seldom were time periods indicated or measured. In the iconic representations, heights were notated on a picture of the container with times listed besides it. These representations presented a challenge to document the pauses in the action of pouring. Watermarks segmentations were listed by equal intervals of time, equal intervals of vertical marks, or were a sampling, segmented by types of pours (fast or slow). Those students who built tables of data listed two columns or rows and in one recorded the times, and the other the height. In all representations, but particularly in the tables, students struggled over how to record the data, often inconsistently switching between recording the change in time over an interval and the time since the pouring started. This activity marked the beginning of student exploration of the contrast between position time and velocity time graphs.

Working within a representational type, learners typically show progress on refining their representations. This process of constructive activity is aided and enhanced by the systematic introduction of the conventions of the field. By showing students how experts have chosen to build and extend representations, teachers connect student constructions with disciplinary accomplishments. As students enter into a more complex representational system, they will learn to work within that system, constructing representational units that may or may not require them to return to the original action or interaction.

Fundamental to mathematics is also the way in which the coordination of multiple forms of representation weaves a tapestry of understanding around the original conception. A community of learners will stabilize their use of representational forms and will be able to gain insight into a phenomenon more deeply by the coordination and contrast of a variety of representations, such as the set described in the water-pouring activity (qualitative, iconic and tabular).

As these communities interact, they become the ones who examine and evaluate the adequacy of an explanation or justification. Constructivism examines multiple forms of explanation and proof. As an evolutionary theory, a constructivist looks for the increasing adequacy in an explanation or proof, and strives to understand how the student is making sense of the idea. In much of my work, I refer to this as the expression of "student voice" and contrast it with the "perspective of the teacher" (Confrey, 1995) whose knowledge of the field is more expansive and consistent with disciplinary perspective. Even in proof, a constructivist is obliged to seek alternative perspectives and to bring a student towards more sophisticated, authorized and valid views of the field.

Because constructivism views knowledge pragmatically, that is, as tied to action and subsequent systems of representation, explanation and justification of that action, theorists often describe its production of mathematics as the development of tools. In addition to asserting the importance of physical tools in its development (compasses, rulers, manipulatives and technologies), a constructivist's view of conceptual development emphasizes that knowledge must be inherently useful. By knowing its genesis and elaboration, one is more prepared to see the concept's possibilities relative to experience and to phenomena in the world. In general, a constructivist will view mathematics as a model of possible action, representation, explanation and justification.

Reflection of two kinds, immediate awareness and reflective abstraction, are the bootstrap processes that keep the cycle moving and permit it to establish schemes for future use.

In describing the activity of construction, I separate out the idea of reflection. There are at least two kinds of reflection in the theory. There is the direct awareness of having done or recorded something—an action, a representation, a description. This is akin to Piaget's empirical reflection. But then there is the act of reflecting on whether that action has in fact satisfied that felt need that propelled one into action in the first place. Piaget would call this reflective abstraction. (Haarer, 1999; Piaget, 1978)

Engagement with others is critical in all aspects of the process

Although the theory stresses the need for each person to form their own understanding of a concept, it recognizes that this process is embedded in and aided by social and cultural relations. Tasks and problematics are mediated by cultural distinctions and defined through social interactions, both peer-to-peer and teacher-student. Acceptable and likely forms of action on objects are influenced by one's view of those objects as cultural tools. Methods of representation, the language of description, representation are profoundly influenced by the intellectual resources of the environment. Communication with others is essential in explanation and justification. Self-regulation is often achieved through the internalization of a scheme as a result of assessment of one's satisfaction of a felt-need, often governed by interactions with others. In the next two tenets, I focus on the role of the teacher in constructive activity.

Listening to children to see the inventiveness of their perceptions supports diversity of approaches and provides a rich source of contrasting ideas.

Constructivism has little difficulty explaining diversity of approach. Because it advocates for a rich interface with the natural and physical world, the social/cultural world, and the historical world of prior knowledge and experience, constructivism provides and supports a rich media of student activity. In addition, it views a critical role for a teacher as listening to student ideas with an open mind and with the acquired skill of listening to students, not as incomplete adults, but as sense-makers with a different field of reference and intellectual resources. Constructivism obligates teachers to ferret out the possibilities and richness of approach in students' proposals.

At a mid-level, the constructive cycle involves assimilation, accommodation and equilibration as constructive processes of scheme formation.

Piaget has recognized the importance of these cycles of constructive activity. Before focusing on explaining microgenetic instances, Piaget identified a variety of macro processes, he felt were necessary to "factor out" of the puzzle, to permit a deeper engagement in constructive analysis. These larger processes were captured in his use of the terms, assimilation, accommodation and stages. Assimilation referred to cycles of activity in which the learner is more or less adding to his/her store of knowledge by extending the class of application. The schemes needed were like bins to be filled, where no reorganization was necessitated by the new instances.

He also described accommodation as a form of restructuring. When one enters a problematic with a well-formed scheme from previous constructive activity, one can find that a failure in a prediction or an explanation warrants reconsideration of the scheme itself, and not just changes in data. When this happens, a student needs to accommodate the changes, by modifying the scheme.

At the macro-level, across ages and relative to particular cultural practices (including schooling), there is evidence of relatively predictable stages of development. Piaget noted patterns across large numbers of students as a function of age and intellectual experience and maturity Stages for Piaget were broad categories of age during with certain kinds of behaviors could be expected under typical conditions of a population from which one samples. The meaning of typical can vary across ages, across teacher/parent intervention, across culture and across curricular and instructional variation. As a biologist, Piaget probably had a tendency to underestimate the contextual factors, the niche, and to attribute more to biology than we would currently do today (Piaget 1959, 1969, 1977).

Utterances about constructive processes all assume an observer – either the constructive him/herself or an observer who is forming inferences about actions and statements.

Constructivists are obliged to always recognize that knowledge claims entail a claimant – a person who is, in effect, the observer of the constructive cycle. Thus, Cobb and Steffe (1983) wrote of the clinical interviewer as a model builder, not as a documentor of how children think. Their role is through the presentation of planned and revised tasks to challenge the student to reveal his/her approaches and views on the concept, and then to permit the interviewers (or teachers) to use their knowledge of the concept and the student responses to build a model of student thinking.

Feedback, Interaction and Recursion are meta-level processes

A critical element of constructivism is based in the concept of feedback. Because the theory begins with a perturbation, a disequilibration, the concept of feedback provides an essential element to assess the success and viability of the construction. It is this process, undertaken both individually as reflection, and among humans as description, explanation and justification that provides the validity and the exchange of the approach.

Furthermore, feedback when viewed at a broader level, that of examining the process of construction itself, has a fundamental element of recursion. That is, the process to construct the single idea is not unlike the process to construct the complex idea, in a recursive fashion, using similar operations at higher levels of generalization. I believe that this view of knowledge has a normative dimension, expressed in both the decisions to only validate justified belief, and to describe the development of deep learning. By deep learning, I am contrasting it with forms of "operant conditioning" and other "behavioral" theories, which require only the performance of the behavior as the evidence of the acquisition of knowledge. Rather, I am positing that there is a broad concept of "educating" that obligates one to reach this higher standard of learning in mathematics and science, and that will need to be applied recursively to create what I will later discuss as a "learning system".

Multiple meanings of abstraction

The process of abstraction can and should be reexamined in relation to constructivism. In mathematical parlance, abstraction is typically reserved for the mature learner. Even in Piaget's stages, he has reserved abstraction for the highest levels. In contrast, I would argue that abstraction is present early on, and the issue is not the presence or absence of abstraction, but rather what abstraction is made.

In fact, I have argued that the construction of abstraction in mathematics is itself flawed. Generalization is the description of a class of cases, where each case is recognized as sharing common conditions. Thus, when I can say that all lines can be written in the form Ax +By= C, I have described a class and created a generalization.

Abstraction is more complex, and it is the recognition and articulation of the like in apparently unlike situations. This recognition is based on the deep structure of the unlike situations and not the surface, and more visible, characteristics. Hence viewing rate of change as an underlying concept in motion, heart-rate, population growth, banking and an expanding universe represent acts of abstraction. Contrary to common usage in mathematics communities, abstraction does not require the stripping away of meanings, but rather the expression of commonality (within the appearance of difference.)

A Constructivist View of Mathematics

To close this section, I reproduce a definition of mathematics offered by W. Thurston. (1998). He had been challenged as to whether his contributions to mathematics, which has not been formalized into definition, theorem, proof (DPT) should be warranted as mathematics in the historical record. To this, Thurston (1998) responded,

"Mathematics is the smallest subject satisfying the following:

- Mathematics includes the natural numbers and plane and solid geometry.
- Mathematics is that which mathematicians study.
- Mathematicians are those humans who advance human understanding of mathematics." (p. 340)

Thurston's definition captures the heart of constructivist theory as it recognizes knowledge is meant to be learned and understood to serve human purposes. He recognizes knowledge evolves from its origins as scholars pursue ideas He demands that progress be measured by increases in human understanding of those ideas.

Applying Constructivism to School Reform in Urban Settings

In my current work, I am applying constructivism to undertake school improvement in an urban feeder cluster. This is composed of one high school, one middle school and four elementary schools. The students largely come from families with lower socioeconomic status, and the schools have high proportions of minority students, (70% Hispanic, 16% Caucasian, 12% African American).

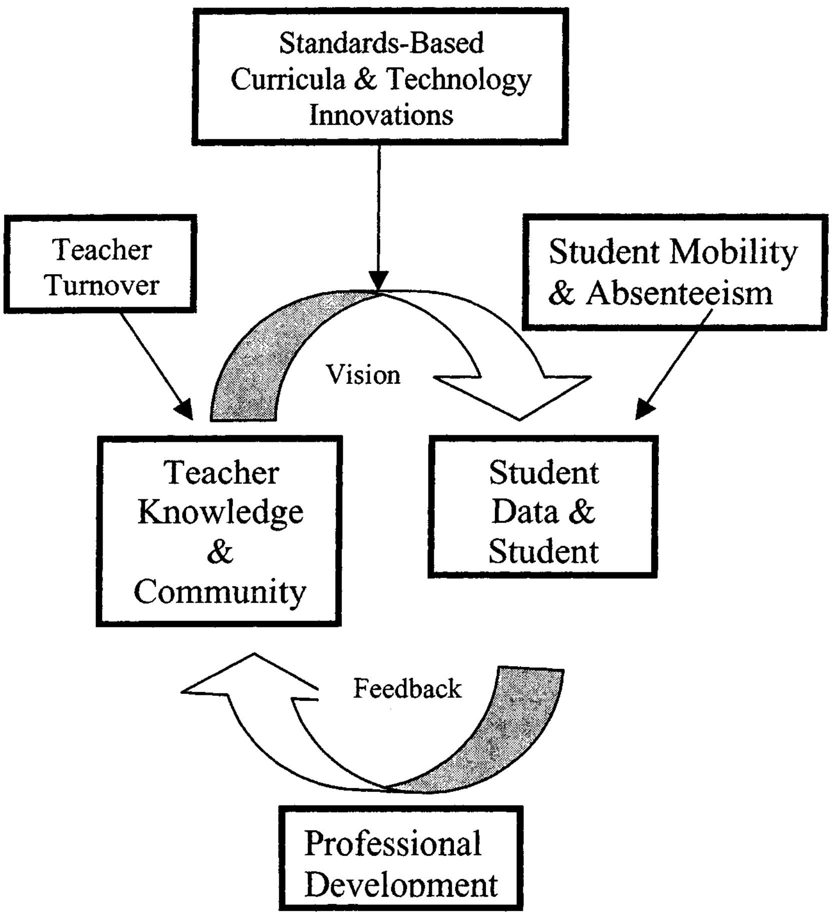

Figure 3

First of all, the project, supported by SYRCE (Systemic Research Collaborative for Education in Mathematics, Science and Technology) has extended the application of constructivism to a cluster by creating the idea of a learning system. Built similarly to the constructive cycle, SYRCE posits a learning system cycle, which is shown in Figure 3. In it, we focus on how to gain school improvement within a feeder cluster by having a vision of mathematics learning that is what is called in the United States as "Standards-based instruction" modeled after the National Council of Teachers of Mathematics Standards. We seek to enhance this vision with a more intensive use of technology. We are also committed to being held to documentable outcomes, not so much as a form of accountability, rather as a measure of feedback on our progress.

In order to engage in this process, we have partnered with the faculty and staffs of the schools. In addition, we are in the process of articulating a kind of research we label as implementation research that seeks through mutual definition of problem, design, implementation and evaluation, to try to understand how to build effective learning systems. (See Confrey, Castro and Wilheim 2000).

As part of this work, last fall, I piloted a set of "replacement unit" materials with a group of Algebra One students every day for five weeks. A replacement unit is a curricular unit that is substituted into a regular course and is innovative and different but last for shorter periods of time than courses. All teachers agree to use it, and it forms the basis for shared professional development and for incrementally building new forms of instruction. Our unit began with descriptions of change, a review and extension of ratio reasoning, qualitative graphing with motion detectors, extensions to bank accounts as accumulation and rate of change and then into slope and linear equations.

In engaging in that work, within the context of "learning systems", many ways of understanding schools and school improvement changed for me. Here I want to report only on some of the changes in my view of students, especially as these interface with constructivist theory. First I will report briefly on the results, and then I will reflect on identifying the implications from the results for modifying or enhancing the theory.

This group of students, in general, experienced little "felt-need" to learn algebra except in order to pass the course for graduation. Since the majority of students were not intending to continue on to college at this point, they saw little need to understand the materials or a strong sense of a need to retain the content for passing subsequent exams.

They were intrigued by the changes to the classroom, and became engaged in the tasks, but easily became bored or discouraged if an activity became too repetitious, difficult or irrelevant.

In the beginning, the students' actions within the task structures were relatively momentary. By this, I mean that they would act on a task with insight and originality and produce robust starting conjectures. But they did not regularly, spontaneously, or habitually record their conjectures or results, and so, asking them to report back in whole group situations was unproductive early in the process.

Their prior knowledge was very weak, so that when establishing their knowledge base prior to activity, such as when an arithmetical skill or insight was needed, it was seldom found adequate. Quick checks on their understanding of percentage, decimals or area concepts showed that considerable prior knowledge was missing. In most areas, they had vague memories of procedures, but the conceptual underpinnings were very weak.

In contrast when given a chance to be relieved of the tedious instruction and drilling to which they had become accustomed and immune, they came alive intellectually, demonstrating curiosity and insight. Attendance improved. Their insights into the problem, once engaged with were easily as rich as from any other group of students I worked with, and not infrequently, as skeptical, practical novices, they often produced observations or asked questions that were delightfully fresh with high potential to be productive.

We found that the students were more proficient when they engaged with relevant tasks. We would contrast the term "relevant" with the widely used term of "concrete" to emphasize that our observations revealed weakness in prior knowledge and in a willingness or interest in obscure ideas, but not in conceptual abstraction. For instance, students' discussion and analysis of rate of change was rich and varied, even when their notation fluency was limited.

As we moved into the latter parts of the constructive activity, argument, justification and reflection, other patterns emerged. At first, engagement in discussion and argument was difficult to achieve. In small groups, students actively discouraged each other from responding to questions on justification, seeing little reason to go beyond giving an answer. However, as the classroom norms shifted to insist on argumentation and justification, the students quickly warmed to it. They began to demand explanation from the teacher, and to banter with each other about whether an argument was justified. After being subject to repeated doses of procedural instruction, they were unaware of the issues of argument and justification – especially of the precision demanded. However once these students were convinced that argument was to be valued, they engaged in it boisterously, especially the males. The females in these cultures tended to be quieter, as they focused on the work, but showed their engagement as they not infrequently observed the males' noisy argumentation and quietly and precisely corrected their errors.

With a large bilingual population, we found that we needed to check more often to see how they were interpreting the meaning of terminology. For instance, in questioning students on the difference between speed and velocity, students suggested that velocity was a description of constant motion where speed was acceleration as they thought of it in the context of "speeding up" (Confrey, Castro and Wilhelm 2000). Small differences in languaging could have significant implications in understanding formal terms.

Throughout our interactions, there were obvious issues of trust. As a teacher, I expressed the opinion regularly that all students could learn these ideas, and furthermore I emphasized that I was committed to all of them learning it. As the class progressed, we observed wider and wider patterns of participation. I sought to find an opportunity to validate each student, relative to some special skill or contribution. As each became a unique character in the classroom activity, they found a way to engage and contribute. It took longer for them to believe in themselves. They were fragile and skeptical in the beginning, demanded considerable levels of emotional support and persistence. In the end, however, when one student failed the exam with a 28 out of 100, the teacher nervously approached him, not wanting to be overly discouraging. Instead he smiled and said; previously his highest grade had been a score of four. He had experienced such extreme failure that even this constituted progress. After this, he became an avid participant in class.

The issues raised by our observations generated elaborated approaches. At the level of the task, we found that we needed to strengthen the context of the task. We revised tasks to draw more on local knowledge (such as sampling salsa for heat intensity and ordering ratios..) We also invited in a Mexican programmer to speak about his career as a programmer.

At the level of activity, we sought more examples from their experiences. Over the course of the intervention, we used more examples from sports and common culture both as metaphors and examples. In addition, we asked more questions of the students to discover what contexts they found interesting and compelling. We believe that an elaborated system of career advisement must be provided along with the instruction to give these students a sense of what mathematical affords.

As they worked, we progressively demanded more and more refined representations and descriptions. The students were surprised to be asked to refine and improve their work, and to be held to an increasing standard of performance. We believed that it was essential to "keep them in the game", and to persuade them that they could succeed, if they persisted and learned the material.

We found that we needed to address weakness in prior knowledge indirectly and in small doses as tools to undertake interesting tasks. We could increase the expectations of practice as the students gained trust that these skills were helpful in understating classroom activities. We also had to provide clear and not unrealistic expectations of out-of-school work and teach them the habits of work that could be assumed in many other places. This meant explicit expectations about attendance, about classroom behaviors, about completing assignments and submitting them. Our preparation for tests was very explicit, and we permitted multiple attempts without lowering the challenge of the items.

A Return to Constructivism

I believe that these tentative results suggest that succeeding in these challenging settings require adaptation of the theory, but not in major ways. In a very short period of time, we saw major gains in conceptual understanding. We attribute this to a) interesting tasks, b) deep belief in the students' potential, c) a respect and valuing of classroom participation and diverse approach, d) a progressive treatment of the quality and consistency of representation and description, e) the development of a culture of argumentation and justification, and f) an array of formative and summative assessment with allowance for multiple tries. With these ingredients, we show distributions of able mathematical behavior that mirror those found in more affluent settings.

We found that in settings in which the prior knowledge is not secure or readily accessible, one needs to weave remediation into regular instruction, viewing prior knowledge as a tool to current activity. In effect, this means that the constructive cycle has to move both forwards and backwards, to build up and modify prior knowledge.

The social/cultural issues were critical. Not only did we have to establish new norms for mathematics learning, and to communicate high expectations, but also we had to reach beyond the classroom in multiple ways. We had to provide a concrete vision of future work possibilities and to use technology as an intriguing lever. We had to seek out convincing and compelling contexts and activities. We also had to convince students that not only did we believe in them, but that they could safely believe in themselves. And we had to understand the array of possible responses and build their knowledge through a sequence of transformations on their intuitive approaches. Building explicit structures for behavior and for gaining more consistent work habits was a constant requirement. The important result was to be more explicit with these structures but to refuse to lower the cognitive level. One can maintain a high cognitive level but design for multiple entries, and make careful and facilitating contexts as catalysts for revealing and promoting abstract thinking.

Constructivism Revised at the Systemic Level

What I have tried to do in this presentation is to describe how a theoretical position can grow and mature as it is applied to more varied settings. In addition, I have tried to describe how a theory can be modified in light of outstanding anomalies, and become elaborated as a result of subjecting to acting as a framework for examining problems in the field. In this case, by working in a high-risk urban setting, one sees ways in which constructivist approaches must be modified in light of a student population with less economic and academic supports.

These modifications to constructivist theory are relatively modest, but essential to adapt it to its use in this challenging and needy urban school setting. The core of the theory held, but there was a need to be much more explicit in establishing a reason to engage in mathematical activity. Potential attendance at college was too distant and novel a source of motivation early in the process. In addition, there were needs to not only avoid one's own tendency to hold low expectations, but to remediate at two levels. The first was to provide extensive scaffolding on necessary but weak prerequisite skills, within the context of high standards instruction and the second was to be consistent about establishing regular habits of academic work. Although these can be cast as classroom norms, they also involved some careful and clear academic management strategies with rewards and sanctions. Finally, issues of trust and fear of abandonment made it imperative that a teacher makes his/her relationship with the students very personal and supportive. One had to build confidence in the class's community as a whole. It was if the core of constructivism held, but to make it applicable, additional elements needed to be enacted.

Perhaps a more significant challenge to constructivism comes as one considers the question of how to scale such innovative practices to the whole school and, in our case, to the feeder cluster. At this level, one has a very different set of audiences and participants in the work. Rather than students as primary clients, one's clients extend to teachers, principals, parents, district administrators, school partners and other departments. Institutional dimensions, such as inadequate teacher preparation content knowledge or planning time, competing forms of assessment (high stakes tests), high absenteeism, teacher turnover, parental alienation from schools, opposition or apathy, are all school and system characteristics. These establish the conditions of work at the school and are typically referred to as issues of policy, accountability, incentives, and punishments. These create the institutional level conditions of instability, resistance, and change.

Theoretically, one can engage with this scaling question as requiring a different theoretical framework – perhaps looking to socio-cultural perspectives or as social practice theory. Rather, I have chosen to modify constructivism to meet the needs of a different order of magnitude. We refer to this change as "creating learning systems" and appeal to many of the same principles only we extend these as cybernetic and ecological qualities, and apply them to larger units of change. My view of constructivism and of learning systems both reject the view that schooling whether at the student or the system level is a production system. Instead, the process is reproductive, with iterative and recursive cycles, and must be designed to increase ecological viability and health, while overcoming low expectations, bureaucratic ineptitude, burn-out and depression.

Our learning systems approach begins to modify constructivism towards a different order of magnitude by analyzing the system in nested levels. Thus there is the classroom level, the department level (at secondary school), the school, the feeder cluster, the district, state and nation. Secondly, we have taken the idea of warrant and extended it to account for a broader nested layer of systems. Now we speak about communication and coherence among the system levels and recognize a much broader participatory structure in our audience (often referred to, as "stakeholders" but we prefer the term partnerships). We recognize ever more fully that warrant must permit activity that involves action, feeling and belief, and must involve justification to the satisfaction and negotiation of the partners. Partnerships in learning systems are difficult to build and maintain as establishing vision, coordinating responsibilities and contributions, and agreeing on plans of action and evaluation takes inordinate amounts of time, commitment and energy. In unstable and under resourced settings, these commitments and agreements are difficult to maintain.

Just as viability was the primary goal of conceptual development, sustainability becomes a similar goal for the overall learning system. Because sustainability is both a pragmatic and a dynamic quality, we approach the problem of school improvement as a task of model building, testing and refining. We prefer models rather than producing program descriptions as models are typically assumed to require adaptability to new settings and thus avoid the naivety of simplistic views of scaling as replication. We have selected a core model as shown in figure 3 for its potential to influence practice as the classroom level, and are elaborating our understanding of the model as we encounter each new challenge.

A focus in our core model is to build capacity in the system by providing all partners with data for decision-making, with research-based frameworks for interpreting data and with professional tools for inquiry. A second focus in the model is to negotiate and establish a vision of standards-based instruction with new technologies. We see a central role for the teacher, as the most directly and consistently charged with instruction. We identify the problem of weak content knowledge, particularly as content knowledge is remade in light of teaching-learning activity, the use of context, and new technologies. Finally, in our model, we link the system up and hold it responsible for obtaining positive student outcomes. In this, we partner with our colleagues in the system to try to make this occur, rather than taking the position of an external observer. This challenges essential notions of hands-off research. We do not call this action research as the models are intended to have generalizability, not just serve as a local solution.

What is particularly interesting in the context of a learning systems approach is one's view of methodology and warrant. Once one appeals to these issues, one moves from distinguishing to stages: theory and implementation to conduct of research in the context of implementation.

We call this "implementation research"(Confrey, Wilhelm and Castro, 2000), (Sabelli and Dede, 2000) and embed our earlier cognitive work within this larger context. Implementation research involves elements of teaching and design experiments, action research and policy studies, but does so in the context of real school change. Primarily, its methodology is based on model building, building partnerships, the multi-directional flow of researchable questions and careful articulation of system indicators especially student performance variables documented over time. The evaluation of research includes such criteria as credibility, feasibility, adaptability, scalability, and content validity. Although we recognize politics, we see our approach as fundamentally educative, suggesting that most solutions to the problems we face entail a dramatic increase in the intelligent and coherent action by the participants in the system. We see this work as essentially consistent with constructivism, but as extending it to the level of system dynamics. As the work is relative new, it is too early to be able to ascertain if it can lead to significant school improvement in urban settings, but this brief introduction to the ideas are at least suggestive of how constructivist theory can evolve to address the pernicious anomalies of systematic filtering and underachievement by large numbers of urban children.

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