Review

Theory of Didactical Situations in Mathematics. Didactique des Mathematiques, 1970-1990.

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Brousseau, Guy (1997) Theory of Didactical Situations in Mathematics. Didactique des Mathematiques, 1970-1990. *Edited and translated by Nicholas Balacheff, Martin Cooper, Rosamund Sutherland and Virginia Warfield. Dordrecht, NL: Kluwer Academic Publishers, xix + 304 pp. ISBN 0-7923-4526-6*

Over the years, many of us have experienced "reflections" of the French *Didactique*, in conferences and publications. We have seen this as reflections of a theoretical framework for the didactics of mathematics, which has been somewhat difficult to absorb, mostly because of the effort needed to really concentrate on the vast material that has mainly been written in French. This is also stated in the preface and is one of the reasons for the book.

The four people – Nicolas Balacheff, Martin Cooper, Rosamund Sutherland and Virginia Warfield - should be recommended to have undertaken the task of translating and editing some of the most fundamental articles from this tradition – written by the perhaps most prominent member of the French didactics of mathematics group – Guy Brousseau. To my knowledge these articles have not been translated before, even if they have had a large influence on the development of the didactics of mathematics also outside of France. *Didactical situations, the didactical contract*s have become well known concepts outside the French tradition – through comments and discussion of other people. It is therefore important that we can get a first hand background and exposition of these concepts.

The book consists of 6 chapters, an introduction as well as an appendix, a short biography of Guy Brousseau, a bibliography and a list of references for all the chapters.

The chapters are as follows:
1 Foundations and methods of didactique
2 Epistemological obstacles, problems and didactical engineering
3 Problems with teaching decimal numbers
4 Didactical problems with decimals
5 The didactical contract: the teacher, the student and the *milieu*
6 *Didactique:* What use is it to a teacher?

The different chapters are based on different sources (articles) that have been written in the period 1970–1990. They have been somewhat edited by the editors, who have also written preludes (there is even a prelude to the introduction) and sometimes interludes and postludes to make it into a coherent exposition of the ideas. The editors have also provided a number of notes that explain the texts, as well as giving further references to other works mainly within the French tradition of didactics of mathematics. The articles (chapters) are not presented chronologically, but so as to give a coherent presentation of the theory.

The chapters are sometimes theoretical, and sometimes a chapter will consider a concrete situation, which then might be linked to the theory.

The main chapter is Chapter 1 which is a synthesis (written by Guy Brousseau in 1984-85) of his dissertation.

> *The purpose of this text was to gather the concepts Brousseau had coined in the course of more than 20 years of research, to formulate them and to organise them in a coherent theoretical framework.*
> *(From Chapter 1 Prelude)*

In the book the word *didactique* (didactics) is used consistently, which perhaps might pose some problems for the Anglo-American tradition, but which is quite natural to our Nordic use of the words.

This chapter introduces the fundamental concepts of the theory. After shortly discussing the work of the mathematician, the student and the teacher – some fundamental questions are stated, relating to the relationship between theory (research) and the teaching learning of individuals.

Concepts like *didactical situation, adidacticl situation, the didactical contract* are introduced and discussed

> *The modern conception of teaching therefore requires the teacher to provoke the expected adaptation in her students by a judicious choice of "problems" that she puts before them. These problems, chosen in such a way that students can accept them, must make the student act, speak and think, and evolve by their own motivation. Between the moment the student accepts the problem as it were her own and the moment she produces the answer, the teacher refrains from interfering and suggesting the knowledge that she wants to see appear. The student knows very well the problem was chosen to help her acquire a new piece of knowledge, but she must also know that this knowledge is entirely justified by the internal logic of the situation and that she can construct it without appealing to didactical reasoning. Not only can she do it, but she must do it because she will have truly acquired this*

knowledge only when she is able to put it to use by herself in situations which she will come across outside any teaching context and in the absence of any intentional directions. Such situation is called an adidactical situation. (p.30)

For this purpose, according to the case, the teacher either communicates or refrains from communicating information, questions, teaching methods, heuristics etc. She is thus involved in a game with the system of interaction of the student with the problems she gives her. This game or broader situation is the didactical situation.

Within the situation, which she is experiencing, the student does not distinguish at once between what is essentially adidactical and, what is of didactical origin. The final adidactical situation of reference, the one that characterises the knowledge, can be studied in a theoretical way, but in the didactical situation, for the teacher as well as for the student, it is a sort of ideal towards which they are trying to converge.

.... The didactical contract is the rule of the game and the strategy of the didactical situation. (p.31)

These concepts are further discussed and expanded. Brousseau draws upon a number of instances relating to the teaching/learning process, e.g. "The paradox of the actor" (from Diderot). This is also related to the situation of the teacher: *As our study shows, Diderot's paradox applies to teacher in an extended way, and it is perhaps more fundamental and more acute than for the actor* (p.47). Ways of modelling didactical and adidcatical situations concludes the first chapter.[1]

The more concrete parts of the book concerns themselves with the teaching of (decimal) numbers. *The didactical study of the teaching of decimal numbers is both the crucible where most of the concepts of the Theory of Didactical Situations have been shaped, and also the more obvious evidence of the power of the theory.* (Chapter 3 Prelude, p.117)

In chapter 3 we find we find an analysis and discussion of the teaching of decimal numbers, related to the French system (curricula and textbooks), but also quite general.

[1] The more the actor feels emotions he wants to display, the less he is able to allow the audience to share the feeling because, being a 'continuous observer of the effects that he produces, the actor becomes a sort of spectator of spectators as well as being what he is himself and can thus perfect his game. (p.46)

Chapter 4 brings together the theory from chapter 1 with the cases presented in chapter 3. Chronologically, however, the content of chapter 4 was published in an article in 1981, well before the thesis "abstract" presented in chapter 1. The editors have somewhat adjusted the terminology so that the chapters fit well together. The chapter is quite long, more than 70 pages, and provides a very detailed study and didactical analysis of decimal numbers.

It also gives an interesting historical background of the development of decimals, but also gives a more extensive scope:

> *With respect to the problem of teaching and learning decimal numbers, the study presented intends to show that conceptions of decimals exist which are real alternatives to the ones classically considered and thus didactical choices that are available to the teacher. (p.147)*

There are also many reflections on method:

> *The classical experimental method consists of organising different choices into an experimental plan and carrying out an inferential statistical test on the data. (p. 184).*

Arguing against the use of such classical methods, Brousseau advocates a deeper analysis:

> *It is necessary, nevertheless, to break away from certain fundamental research routines. Thus, instead of comparing slightly different procedures to observe the effect of a modification of the conditions, holding all the others constant, it is preferable to produce very different processes by varying the conditions, which are judged to be important. (p. 184).*

He then goes on to analyse various such processes.

Chapter 2 is more a discussion of a special topic – *epistemological obstacles* – in light of the theory proposed in the first chapter. With a background mainly relating to the development of the notion of misconception at Shell Centre in Nottingham, it is interesting to read this chapter with the terminology from Brousseau's theory.

> *An obstacle is thus made apparent by errors, but these errors are not due to chance. Fleeting, erratic, they are reproducible, persistent. Also errors made by the same subject are interconnected by a common source: a way of knowing, a characteristic conception, coherent if not correct, an ancient "knowing" that has been successful throughout an action-domain. (p. 84)*

The concept of obstacle is given a thorough analysis with reference to the theory presented in chapter 1. The example(s) discussed are numbers (fractions). The analysis gives a new perspective to the notion of misconception as used in publications from the Shell Centre.

It also opens a window to the French development in this important area of didactics of mathematics.

The didactical contract is an important notion, and the editors have included a recent paper from 1990. The didactical contract was introduced as an important element of the theory in the first chapter, in chapter 5 it is discussed in greater detail.

Through the book there are numerous references to mathematics and mathematicians. We find one reflection in the very beginning of chapter 5:

> *Mathematicians don't communicate their results in the form in which they discover them; they reorganize them, they give them as general a form as possible. Mathematicians perform a "didactical practice" which consists of putting knowledge into a communicable, decontextua, depersonalized, detemporalized form. The teacher first undertakes the opposite action: a recontextualization and a repersonalization of knowledge. She looks for situations which can give meaning to the knowledge to be taught. But when the student has responded to the proposed situation, if the personlization phase has gone well she does not know that she has "produced" a piece of knowledge that she will be able to use on other occasions. In order to transform her answers and knowlede into a body of knowledge, she will, with the assistance of the teacher, have to redepersonalize and redecontextualize the knowledge which she has produced so that she can see that it has a universal character, and that it is re-usable cultural knowledge. (p. 227)* [2]

This sets the stage for the presentation of the concepts in chapter 5. In the teaching-learning situation there is a certain "game", and the didactical contract is the set of rules for this "game". Brousseau discusses the structure of the situations, with reference to observations. The discussion presents a deep and interesting perspective on the teaching-learning process.

The last chapter has the title: *Didactique: What use is it to a teacher?* This very important question concludes the book. It is a question that has troubled researchers in the field, and Brousseau also asks the question why knowledge of didactique spread so slowly to public and to the teachers. The relationship between teachers and researchers is discussed. It is important for Brousseau to discuss this relationship from the theorist's viewpoint, but at the same time have the perspective of the teacher present.

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[2] It is here interesting to note that at the CERME 1 conference in Osnabrück in August 1998 Guy Brousseau gave a lecture in which he stated that 80 % of mathematical research is reorganizing, reformulating, and problematizing work that has already been done. (Authors notes from the lecture)

He sees the effects in the long-term development of the teaching profession:

> *Didactique can, in the end help the teacher change her status, training and relationship with society: by acting directly on the status of the knowledge which she uses, by acting on the knowledge of her professional partners and the general public, in developing better avenues by which the public could use teaching in more satisfying ways, by providing better possibilities for public or private authorities to manage teaching by more appropriate means. (p. 263)*

Concluding remarks

The book starts with a short biography of Guy Brousseau, then follows an example – "setting the scene with an example: the race to 20". A game is presented, and some of the notions to be covered later in the book are introduced. I think it was a good idea not to start directly on chapter 1. These two introductory sections give us some background, so that we feel we know more about the setting and background of the book.

The book is written for researchers in the field. It presents a theory on how to look upon mathematics education. Even if the translators/ editors have done an impressive job in presenting the various articles, it is not an easy book to read. For many it will without doubt be useful to read it more than one time. It contains a wealth of details to be studied. The articles are written in the period from 1970 to 1990. It is remarkable how the text has an up-to-date feel. This can of course be the result of the translation and editing process. However, it shows that there is possible to create more general theories of the didactics of mathematics.

The book's importance lies in the fact that presents a theory, that many of us from time to time have met in the study of didactics of mathematics, but have not seen such a comprehensive presentation. For anyone interesting in learning about the French *Didactique* this is an important and necessary book. It should be in any research library, and we can hope that the editors/translators will go further with more translations.