On pupils' reactions to the use of open-ended problems in mathematics

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This article will focus on the preliminary results of one experimental class from the three-year research project "Open tasks in mathematics", which has been carried out in junior high schools during the years 1989–92 in Helsinki (Finland). The experiment groups used open-ended problems (problem fields) regularly, i.e. once a month, within their normal teaching. The main results from one class (N = 18) were as follows: The pupils liked most of the problem fields used. Their mathematical views did not change statistically significantly, but the non-significant changes in questionnaire ratings, classroom observations and the teacher's evaluations indicate that there was a change, and the change was in most cases positive.

The common way of reforming school teaching has been to introduce new regulations (curricula, etc.). For example, the change in the school system in Finland in the early 1970s from a parallel school system to a comprehensive school was performed as an administrative renewal in which individual teachers would have no possibility to influence anything.

Teaching can also be influenced through new textbooks. This method was practised in Finland in the 1980s. However, the books were so elaborate that teachers lost the opportunity to control their own work and apply their teaching skills. Today, such developmental solutions are being imposed where the initiatives for change, at least to some extent, are in the hands of practicing teachers – the so-called "bottom-up method" (e.g. Nohda, 1991). Teachers are encouraged to develop such learning environments where their pupils have more time and room to reflect, to discuss and to investigate on their own.

1 On open-ended problems

One promising method to develop mathematics teaching in school seems to be the so-called "open approach". There the teacher offers the class an open learning environment, in the form of an open-ended problem. His aim is to develop, among his pupils, mathematical

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problem-solving and communication skills, and to give them an opportunity to learn on their own way and at their own rate.

What are open-ended problems?

In the conference on the Psychology of Mathematics Education (PME-17, Japan) in July 1993, there was a discussion group on the use of open-ended problems in mathematics classrooms. One aim of the discussion was to find answers to the question What are "open-ended problems"? since open-ended problems do not seem to be well defined. During the discussion, several types of problems were put forward: investigations (a starting point given), problem posing (or problem finding or problem formulating), real-life situations, projects (larger study entities, requiring independent working), problem fields (or problem sequences or problem domains; a collection of contextually connected problems), problems without a question, and problem variations ("what-if"-method). Some mathematics educators use the word "exploratory" as a synonym for "open" (e.g. Avital, 1992), in order to prevent confusion with the unsolved problems of mathematics (cf. also Silver, 1995).

For more about the concept of open-ended problems see e.g. Pehkonen (1995). Furthermore, many examples of different types of open-ended problems can be found, e.g. in the papers of Nohda (1995), Stacey (1995), and Silver (1995). Here, we will deal with one realisation of the open-ended problems – the use of problem fields; and some preliminary results are discussed.

On the idea of using open-ended problems

The method of using open-ended problems in the classroom to promote mathematical discussion, the so-called "open-approach" method, was developed in Japan in the 1970s (Shimada, 1977; Nohda, 1988). At about the same time in England, the use of investigations, a category of open-ended problems, became popular in mathematics teaching (Wiliam, 1994); and the idea was further spread by the Cockcroft report (1982). In the 1980s, the idea to use some form of open-ended problems in the classroom spread all over the world, and research on its possibilities is very vivid in many countries (e.g. Nohda, 1988; Pehkonen, 1989; Silver & Mamona, 1989; Williams, 1989; Mason, 1991; Nohda, 1991; Stacey, 1991; Zimmermann, 1991; Clarke & Sullivan, 1992; Pehkonen, 1992a; Silver, 1993). In some countries, they use a different name for open-ended problems; for example in the Netherlands, they are using real-life situations, and call their method "realistic mathematics" (Treffers, 1991). In Nor-way, problem solving was the central topic of the curriculum in the 1980s, and today its realisation is tried through open-ended tasks (Borgensen, 1994; Solvang, 1994; Alseth, 1995).

The idea of using open-ended problems in school mathematics appears in the curriculum of some countries, in a form that there is left freedom for teachers' choice and decisions. For example, in the mathematics curriculum for the comprehensive school (Gesamtschule) of Hamburg in Germany, about one fifth of the teaching time is left content-free, in order to encourage the teachers to use mathematical activities (Anon, 1990). In California, they are suggesting open-ended problems to be used in assessment, in addition to the ordinary multiple-choice tests (Anon, 1991). Within the developmental work done today in the area of school assessment in the United States, the researchers are inquiring into the possibilities of using open-ended problems, especially problem variations (Cooney et al, 1993). In Australia, some open problems (e.g. investigative projects) have been used in the final assessment since the late eighties (Stacey, 1995).

2 On the research project

Here, we will briefly introduce the three-year research project "Open tasks in mathematics" (Pehkonen & Zimmermann, 1990), which was carried out during the years 1989–1992 in Helsinki (Finland) and sponsored by the Finnish Academy. This project was realized in grades 7–9, and it concentrated on the use of problem fields.

The aim of the project

The purpose of the project was to improve mathematics teaching in junior high schools, especially to develop and foster methods for teaching problem solving. Problem fields were used as a method for change in mathematics teaching. Thus, the objectives of the project were to clarify the effect of open-ended problems (problem fields) on pupils' motivation, the methods and how to use them. In conducting the experiment, we tried to stay within the frame of "normal" teaching, i.e. in the frame of the valid curriculum, and to take into account the teaching style of the teachers when using problem fields.

The theoretical framework for the research project was the constructivist understanding of learning (e.g. Davis et al, 1990; Ahtee & Pehkonen, 1994). In the understanding of learning which is compatible with constructivism, it is essential that a learner is actively working, in order to be able to elaborate his knowledge structure. Thus, the meaning of pupils' beliefs (subjective knowledge) concerning mathematics and its learning is emphasized as a regulating system of his knowledge structure.

In order to see whether there are any changes in mathematics classes, one should try to see and understand mathematics lessons from "inside". This framework implies use of interpretative research methodology which here means interviews, classroom observations and careful interpretation of questionnaire results.

On the realization of the research project

The project started in the autumn 1987, and ended in the summer 1992. In the pilot study of the project during 1987–89, the research design was tested, measurement instruments were developed and the problem material was worked into its final form. The main experiment was realized in the Greater Helsinki Area with about ten grade 7 classes, and continued with those classes throughout the Finnish junior high school (up to grade 9), i.e. until the summer of 1992.

Both in the beginning and at the end of the experimental phase, teachers' and pupils' conceptions of mathematics teaching were gathered using questionnaires and interviews. In the main experiment, experimental group 1 (nine teachers with 157 pupils) and experimental group 2 (six teachers with 103 pupils) differed in the point that from the mathematics lessons of experimental group 1 about 20 % (i.e. once a month about 2–3 lessons) was reserved for dealing with problem fields. For each problem field, there was a questionnaire in which the pupils' evaluations of using that problem field were ascertained. The teachers' evaluations of using problem fields were obtained with short interviews after each term.

The teachers in experimental group 2 were told that they were participating in an experiment, whose aim was to investigate the development of pupils' problem solving skills. They were told nothing about problem fields nor the experimental group 1. Pupils in both experimental groups solved in their classwork some problem fields which were the same for both groups.

In the following, we will name group 1 the experimental group, and group 2 the control group.

The focus of this article

Here we will deal with some partial results of the research project. The focus will be to find answers to the following question: How did the pupils in junior high school react to the use of open-ended problems (problem fields) in their mathematics teaching? This main question will be considered from different viewpoints, and thus is split into more detailed questions:

(1) How did the teacher evaluate her pupils' motivatedness when using the problem fields?

(2) How did the outsider (observer) describe the pupils' readiness to work on the problem fields?

(3) How did the pupils themselves evaluate the problem fields used?

(4) How did the pupils' mathematical views change during the experiment?

These questions will be answered with the aid of preliminary results from one class (18 pupils) of the experimental group. For a description of the teachers' reactions see Pehkonen (1993).

3. Data collection

In the research project, there were a total of 20 problem fields which were used during three years in the experimental classes. Thus, for each grade (7–9) of junior high school there were about seven problem fields.

One example of the problem fields used

From all 20 problem fields used, tangram was selected as a representative example, since it is an old chinese seven-piece puzzle which is dealt with in the literature very much. Furthermore, tangram was most liked in all pupils' and teachers' responses gathered during the research project. For teachers, it seemed to be easy to use in classroom, and pupils are fascinated by its simplicity which, however, contains many possibilities for creative pupils.

The tangram activity was carried out within the research project in all experimental classes (grade 7) in October 1989. After school began in August, the teachers were given all the material for the grade 7 problems, with each problem field being dealt with on the average of four pages. The tangram task was divided into four segments on two pages, as follows:

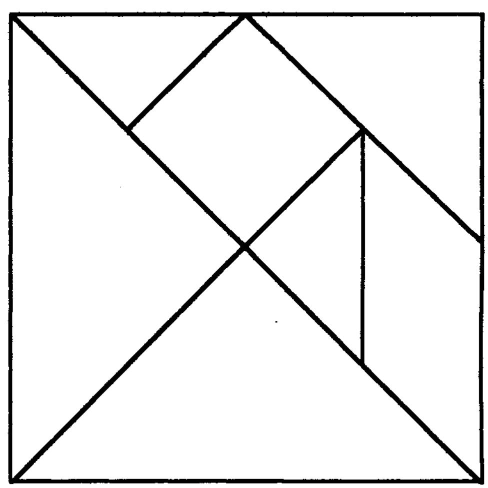

Figure 1. The Tangram puzzle.

(1) Make, e.g. from cardboard, a tangram-puzzle according to the figure given (the side length of the square is e.g. 10 cm)! Which bigger pieces can you make with the smaller ones (e.g. two triangles make together a square)?

(2) The area of the whole tangram puzzle is 100 cm^2, if its side length is 10 cm. Calculate the area of each piece! Let us say that the area of the whole tangram puzzle is a unit. Which fractions do the different pieces comprise?

(3) Which "well-known" polygons (triangle, square, rectangle, parallelogram, trapezium) can you make, if you are using only a) two pieces, b) three pieces, c) four pieces, ... etc.? Using all seven pieces, try to make a square, a triangle, a rectangle, a parallelogram, and a trapezium. Can you find other solutions? Which other polygons can you make using all pieces?

(4) The tangram puzzle was made by using all seven pieces to construct a square with a side length of 10 cm. Calculate the area of each polygon (triangle, square, rectangle, parallelogram, trapezium) that you have found in question (3)!

Usually, the teachers used a part of each lesson, for 2–3 lessons, for dealing with tangrams, typically in a time slot of 15–20 minutes. Some teachers used ready-made tangram puzzles, and the other ones allowed pupils to make their own tangrams, according to the figure given.

More examples of the use of problem fields may be found in Pehkonen (1992a).

The test subjects

The research project was carried out in junior high schools in Helsinki and its surroundings. In this paper, we will restrict ourselves to results concerning one experimenting teacher and her class.

The school in question is an ordinary suburban junior high school in Helsinki, with three parallel classes. The pupils were from the school's surroundings, i.e. its pupil population was unselected. The number of pupils in the class in question varied slightly over the 3 year period, but was always about 20 pupils. The same teacher (Terry) taught the class through the whole lower secondary level, i.e. grades 7–9. Only 18 pupils were present from the beginning to the end of the experiment. The distribution of the grade 7 notes (autumn 1989) of these 18 pupils was as follows: 4 pupils were above average, 12 pupils average, and 2 pupils below average. And the distribution of their final notes in grade 9 (spring 1992) was respectively 7–7–4.

This kind of change in pupils' notes is rather usual in junior high school, since teachers tend to give somewhat higher final grades.

Methods of data collection

In our description of the pupils' and the teacher's reactions, we used the information obtained by questionnaires, interviews, classroom observations and field notes gathered during the experiment has been used.

In the data collection, the variety of data was an aim: After the use of each problem field in the class, the teacher asked the pupils to fill in a short structured questionnaire concerning the pupils' evaluation of the problem field used. Once a month, the so-called project group, which was composed of the experimenting teachers and the researcher, had a business meeting. In the meeting, the use of the last problem field was discussed, with special emphasis placed on the teachers' realization and experiences of the problem field used. The data gathered from the teachers in these meetings is in the researcher's field notes. After each half year, the experimenting teachers were interviewed, concerning their experiences when using problem fields during that half year. In autumn 1991, there was an intensive observation period of one week in all classes. In the beginning and at the end of the experiment, the pupils' mathematical views were extracted through a questionnaire, and furthermore, two pupils were interviewed from each class.

Statistics used

When considering the results of the questionnaires, the statistics used were mainly percentage tables, and means and standard deviations of items. The responses to the open questions, as well as the field notes, were classified according to a proper category system. In the interview and observation data, interpretative methodology was used.

4 Some preliminary results

In the classroom, the teacher dealt with each problem field during 2-3 lessons. Here we will concentrate on the pupils' reactions on using problem fields from different view points: the teacher's evaluation, the observer's perceptions, and the pupils' own view of the problem fields used and, more generally, of mathematics. The main question is how Terry's pupils experienced the use of the problem fields. The teachers' reactions on the use of problem fields are here put aside, since preliminary results from the teachers' view point were discussed in Pehkonen (1993).

Terry was an experienced teacher, and as a background information her view of mathematics is described. In the initial interview, her answer to the question about mathematics was that "it means primarly calculations". Then she continued that "it is a logical matter which is based on certain facts, and ... then you use your own brain". In the final interview, she responded to the same question with the following words: "Calculations and calculation and thinking. Using of your brain." Thus, she stressed consistently the two main components of mathematics: calculations and thinking.

Terry's evaluation on her pupils' motivation

Here, the data source is Terry's interviews. First we will look at some of Terry's expressions concerning the whole class: The group is "great, no more disturbing", eager to work (after grade 7). When "you are teaching a new topic, they are very pertinent and very quiet", "they are working quite well", and "if they do not understand, they will ask immediately" (after grade 8). "It is a nice group, speedy, uninhibited, full of ideas and nice" (after grade 9). From Terry's wording, one may read that the class was very active, but she had managed to tame it and to guide their activity to the study of mathematics.

Furthermore, some of Terry's comments concerning a couple of individual pupils are given: The weakest pupil in mathematics is Bob, but he tries hard and "now likes mathematics because of these experiments" (after grade 7). Tom has raised his level, he is today among the best three pupils. He "has got involved because of these [problem fields], and with hard work" (after grade 8). Terry seemed to evaluate highly the experiment and its alternative form of teaching.

According to Terry, the problem fields in which all pupils, independent of their ability, were highly motivated, were as follows:

- Tangram (a seven-piece puzzle),
- Tower of Hanoi (a puzzle of moving discs from a pile to another),
- Steeple Chase (a board game in algebra),
- Functions with Matches (to find a pattern in match configurations),
- Noughts and Crosses (a multiplication variant of the well-known game "tic-tac-toe").

When working with them, the main point lies in activity, creativity and insight which pertain to the area of mathematical thinking, and not so much in mathematical topics. Only in the case of two problem fields all pupils were unmotivated.

- Package Problem (to plan a variant of the given package net),
- Repair of Lodgings (to plan a repair of lodgings with the help of a floor plan),

These two problem fields represented activities which were developed from real-life situations, and therefore, meant tedious thinking and long calculations. The class was impulsive, and not interested in such long-term work, and Terry's emphasis on calculations seemed not to be sufficient.

Some classroom observations

In October 1991 (in grade 9), there was an intensive observation period in Terry's class. The researcher wanted to have a snapshot of Terry's teaching style and of the general learning atmoshere in her classes. The period was compounded of eleven observed lessons within three days. In addition to the structured observation scheme of each lesson, the observer wrote at the end of each day one page in free form about her perceptions and experiences during the observation.

The main result of observation was that Terry's style of handling pupils was not unique in the experiment class. In the words of the observer: "Pupils' high motivation and activity were common features for all of her [Terry's] lessons." The teacher emphasized in her teaching "the meaning of one's own thinking", and she used a variety of practical examples. Terry's experiment group seemed to be more impulsive than her other classes. On the other hand, Terry's belief about the two components of mathematics (calculations and thinking) was not only superficial, since this belief was seen in her teaching practice.

The pupils' liking of the problem fields used

After the use of each problem field, the pupils were asked to fill in a questionnaire which explored their liking of the problem field used, and their reasons.

In the questionnaire, the first question was a simple multiple choice about the problem field used: liked – neutral – disliked. In Terry's group, the most favoured problem fields were as follows: Tower of Hanoi (95 %), Noughts and Crosses (95 %), Tangram (94 %), Structure of Packages (89 %), and Frogs (87 %). And in all experimental classes, the corresponding percentages of liking were respectively: 78 %, 75 %, 80 %, 74 %, and 62 %. Thus, the group of the "top-five" was almost the same for both.

The percentages of liking in the pupils' responses of Terry's group were as a rule higher than those in all experimental pupils. But there were two problem fields (Package Problem, Repair of Lodgings) which Terry's pupils liked remarkably less than the other pupils. The percentages of liking were in these about 25 %, whereas in the group of all experimental pupils, they were about 40 %.

Furthermore, the problem fields seemed to be much more favoured on grade 7 than later on: In Terry's group, the means for percentages of liking for grades 7, 8 and 9 were 82 %, 60 % and 59 %, and in all experimental groups respectively 73 %, 53 % and 58 %. One explanation could be that the different type of mathematics teaching with problem fields in grade 7 was very new, and therefore of interest. In later grade levels, the pupils were already accustomed with them and with the teaching style and, therefore the interest was no longer so great. Another explanation could lie in the pupils' age: They were in early adolescence (13 – 16 years).

Changes in the pupils' mathematical views

The same questionnaire with 32 structured statements to be answered on a five-step scale (1 = fully agree, ..., 5 = fully disagree) was administrated in the beginning and at the end of the experiment (the questionnaire used was the same one as in the paper of Pehkonen (1992b)). Changes in the pupils' views were checked with the help of a t-test comparing the pupils' initial and final responses.

In five of 32 statements, the difference of means was statistically significant in the (unpaired) t-test. The introduction for all statements was "Good mathematics teaching includes ...". In Table 1, these five statements are given with their value and level of significance in the (unpaired) t-test.

In all experimental classes together, there were statistically significant differences in nine statements, whereas in the control group, only five differences of means were statistically significant. In the statement concerning paper-and-pencil computations (item 3), the statistically significant change has been noticed both in the experimental and in the control group. The ratings changed there from agreement to neutral. An explanation for this phenomena could be that on the elementary level (grades 1–6), calculations are stressed, and in junior high school, the pupils have experienced that mathematics is not mere calculations.

The pupils' attitude to the demand of exactness in mathematics changed from agreement to neutral (item 5) or almost neutral (item 16). And the same happened to the pupils' desire for help (item 15). Whereas the favoring of the idea that studying mathematics requires a lot of effort by pupils (item 23) changed from neutral to agreement.

		p	sign.
3	doing computations with paper and pencil	2 %	(*)
5	the idea that everything ought to be expressed always as exactly as possible	0.01 %	(***)
15	the idea that the teacher helps as soon as possible when there are difficulties	0.7 %	(**)
16	the idea that everything will always be reasoned exactly	5 %	(*)
23	the idea that studying mathematics requires a lot of effort by pupils	4 %	(*)

Table 1. The statements with statistically significant differences in the (unpaired) t-test.

With the aid of the paired t-test, it was found that the pupils' ratings changed very much during three years. In most items, the change was statistically significant – only in seven of 32 items was it non-significant. The only statement in which the pupils' ratings remained rather similar was item 19 (the idea that studying mathematics has practical benefits): They were in both cases about 1.5, i.e. between agreed and fully agreed. Thus, one may state that the changes obtained in the pupils' mathematical views only show the direction of change.

5 Discussion

From all different methods of data collection (teacher's interviews, classroom observations, questionnaires), one could get a unified view about what was happening in Terry's class. In the following, there is a summary of results, and some discussion of them.

Summary of results

The observations made in Terry's class support the picture she gave about her pupils in the interviews. The experimental class was more impulsive than usual, but she had tamed it with her superior teaching style and personality.

In the case of problem fields, Terry seemed to know her class. In the top-five list of problem fields, they both had three of them in common: Tower of Hanoi, Noughts and Crosses, Tangram. It is interesting to notice that these most liked problem fields could all be classified as some kind of game. Most people like to play, also with numbers, if they feel themselves confident enough with the situation, i.a. the task is not too demanding for them. And in the question of the least liked problem fields, Terry and her pupils fully agreed: Package Problem, Repair of Lodgings. They did not like application-oriented situations.

There were some changes to be noticed in the pupils' mathematical views. In five items, the change in the period of three years was statistically significant. One of these changes could also be seen in the pupils' views of the control group: The pupils valued less paper-and-pencil computations; they saw that mathematics is more than mere calculations. This could be a general difference between the primary school and the secondary school mathematics. At the primary level, the development of computational skills are stressed, whereas at the secondary level other components of mathematics, such as reasoning, are given emphasis.

The other statistically significant changes seen in the pupils' views in Terry's class might be due to the use of problem fields. The pupils placed less emphasis on the demand of exactness, and desired less help from the teacher. At the same time, they saw that studying mathematics requires a lot of effort by pupils. All these aspects could be explained by the use of problem fields. Open-ended problems offer an open learning environment to pupils where one needs in the first place creative ideas, and where the mathematical rigor and exactness only comes in the second place. The starting point of the problem fields was usually some problems with concrete materials (cf. the tangram activity). And the first problems were so easy that everybody could solve them, and so get involved in the independent attempt to solve the following problems.

Another explanation might be that the teacher (Terry) was the main source for the change of the pupils' view point. The influence of the teacher from that of the teaching material is very difficult, perhaps impossible, to distinguish. But she was also affected by the problem fields.

Summarizing the results, one could state that the pupils experienced the problem fields used as an interesting form of learning mathematics. They liked most of them very much, and were also motivated and activated during other parts of mathematics lessons. Their mathematical views did not change statistically significantly. But the non-significant changes in the questionnaires, classroom observations and the teacher's evaluations overall indicate that a change had occured, and the change was in most cases positive.

Furthermore, when the experiment was finished, the teacher (Terry) wanted to continue with similar materials later on. She has adopted problem fields as an integral part of her teaching, and has independently been developing new ones, too.

Conclusion

When interpreting the results, there are many critical points to be observed. For example, in measuring pupils' mathematical views, there seem to be some problems. The use of the paired t-test shows a variety of changes in the pupils' responses (the initial–final comparison), also in those statements where no statistically significant difference between the initial and final testing was found in the unpaired t-test. Thus, the pupils' answers to the statements seem to change greatly in three years, but not in the same direction each time. So the question arose as to whether the pupils have any fixed views to measure at that age. Is it sensible to try to determine pupils' mathematical views with a questionnaire?

The results suggest that the open-ended approach, when used parallel to the conventional teaching methods, seems to be a promising one. The pupils preferred this kind of mathematics teaching where one important factor was the freedom alloted to pupils to determine their learning rate. However, the method of using open-ended problems (problem fields) is not sufficient to keep the level of motivation high. The pupils' ratings for their liking of the problem fields used diminished during the years of the junior high school. Could it be that there does not exist any solution with material for the motivation problem in the junior high school? And is the teacher the most important factor for motivation?

But the use of open-ended problems, in the form of problem fields, was felt to be such a promising approach that some teachers have written a textbook series for the teaching of mathematics in grades 7–9 which uses the idea of problem field. The textbook has been accepted by the National Board of Education for a larger survey, with the help of which they are trying to determine the extent of its possibilities at large. The survey will be conducted during the years 1994–97, and will be reported later on.

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Elevers reaktioner på öppna problem

Denna artikel har fokus på preliminära resultat från en försöksklass som deltagit i ett treårigt forskningsprojekt, "Öppna uppgifter i matematik". Det har genomförts på högstadiet under åren 1989-1992 i Helsingfors, Finland. Experimentgruppen använde öppna uppgifter (problemområden) regelbundet under en månad in ramen för den vanliga matematikundervisningen. Huvudresultatet från en av klasserna (N = 18) var följande: Eleverna uppskattade de flesta problemområden som användes. Deras syn på matematik ändrades inte på ett statistiskt signifikant sätt. Men de icke signifikanta förändringarna i resultaten från enkäter, klassrumsobservationer och lärarnas utvärdering indikerar en förändring och att denna i de flesta fall var positiv.

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