Staking Claims

Jeremy Kilpatrick

The field of mathematics education has both scholarly and professional aspects. On the scholarly side, the question of what counts as research is still being debated. An examination of two proposed sets of criteria for evaluating the quality of research in mathematics education reveals that, interpreted appropriately, criteria borrowed from the natural and social sciences are relevant to a field that is attempting to be scientific. On the professional side, mathematics education must inevitably be concerned with the application of specialized knowledge to assist the students and teachers who are its clients. Teacher education remains a major function of mathematics education, parallel to the search for reliable knowledge to be applied. University mathematics educators need to work closely with mathematicians and with classroom teachers in developing both theory and practice. Mathematics education has flourished in countries in which institutional structures have supported it as an identifiable academic field.

When gold was discovered in California in the middle of the last century, miners rushed to stake their claims. A claim was a tract of public land, and by marking its boundaries with wooden stakes, the miners indicated that they had claimed the right to extract valuable minerals from it. Their stake was their interest in the mineral rights; their claim represented their entitlement to exercise those rights.

I have chosen the title "Staking Claims" because I want to discuss not only the claims made for research in mathematics education but also the claim that mathematics education itself is a scholarly field. I start with a discussion of research in mathematics education. By examining criteria for the merit of a research study, one can begin to understand what research in a field is and what it can become. Various criteria for judging research have been proposed over the years; I consider two recent proposals.

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As a scholarly field, mathematics education has a short history, one that differs from country to country. I discuss some contrasts between its development in the United States and in Sweden, offering thoughts on issues these contrasts raise and suggesting how the field might develop further.

Before discussing research in mathematics education, let me consider the name we give to our field. I use the term *mathematics education*. In some countries, the preferred term translates as *didactics of mathematics*, and it is often contrasted with a more general *pedagogy* (Kilpatrick, 1992, p. 4; 1994, p. 86). For example, in France, *pédagogie* as a tertiary subject seems to have become strongly associated with courses in teaching methodology that draw upon practical experience but not a body of scholarly knowledge. *Didactique* seems to have been taken up to express a particular scientific approach to our field. In Germany, there is a similar usage: *Mathematikdidaktik* refers to mathematics education considered as an academic field.

In American English, however, both *didactics* and *pedagogy* have taken on negative connotations. To call teaching *didactic* is to suggest that it has been not merely instructive but moralistically so. A *pedagogue* is not just a teacher; the term implies that the teacher is long-winded and boring – a pedant. Americans use *education* and *educator* to avoid these unpleasant connotations. They view the educational studies as a domain that, though relatively low in status, has established its place in academe. Americans use *mathematics education* to refer to both the activity and the field. It seems, however, that *mathematics education* does not mean the same thing as, for example, *didactique des mathématiques* (Balacheff et al., 1993, p. 179); the field is still constructing its identity.

Research claims

Criteria revisited

Several years ago, at a symposium entitled "Criteria for Scientific Quality and Relevance in the Didactics of Mathematics" in Gilleleje, Denmark, Anna Sierpinska and I each took the same list of criteria for judging research in mathematics education and developed complementary arguments as to how these might be understood and applied (Kilpatrick, 1994; Sierpinska, 1994). I would like to revisit these criteria, restating and summarizing my main contentions and adding a few remarks.

The eight criteria we chose were the following:

- **relevance**
- **validity**
- **objectivity**
- **originality**
- **rigor and precision**
- **predictability**
- **reproducibility**
- **relatedness**

The questions I asked were "What criteria dominate current research in mathematics education? And what criteria should be used in selecting problems and methodology so that the research will be high in quality?" My intention was to show how some of the criteria that are often applied to the natural sciences and that are now seen as inappropriate for educational research might be refreshed and reclaimed if approached somewhat differently.

Research in mathematics education has largely ceased emulating natural science and is increasingly adopting methods used in social science. Until the 1970s, much mathematics education research, and especially that done in North America, attempted to specify the behavior of pupils or teachers and to analyze that behavior into components. Researchers attempted to follow what they understood as the positivistic approach employed by physicists and chemists. "The world of mathematics teaching and learning was seen as a system of interacting variables. The purpose of research was to describe those variables, discover their intercorrelations, and attempt to manipulate certain variables to achieve changes in others" (Kilpatrick, 1994, p. 16). Although some researchers in mathematics education still have this orientation, most have followed other researchers in education who have borrowed theoretical frameworks and techniques from the social sciences. Approaches seen as phenomenological, interpretivistic, social constructivistic, or ethnographic have become especially popular among researchers in mathematics education.

At times, some of these researchers talk as though there were only one way to do research – the way they are doing it now. In my view, it is perfectly reasonable for an individual researcher to concentrate on a single path. It is not reasonable, however, for the whole field to adhere to one and only one research paradigm. Just as genetic diversity helps insure the health of future populations, so diversity in the way research is done helps keep the field alive and growing. Mathematics education needs the multiple perspectives that different approaches bring to the study of teaching and learning.

Salomon (1991) claimed that one ought to distinguish between two approaches. In the so-called analytic approach, external events such as teaching are manipulated so as to permit inferences about internal events such as learning. In the systemic approach, internal and external events are studied as they interact and overlap. Salomon viewed the two approaches as complementary: "The analytic approach capitalizes on precision while the systemic approach capitalizes on authenticity" (p. 16).

Citing Salomon's article, I contended at Gilleleje that

> The systemic approach currently dominates research in mathematics education. Naturalistic settings are favored over environments in which events are manipulated. Authenticity is at a premium. But researchers in mathematics education should never become wedded to a single approach, epistemology, paradigm, means of representation, or method. All are partial and provision; none can tell the whole story. In particular, no single research method can address the full range of questions of interest to mathematics educators. Although an individual researcher may stick with one method, the field as a whole needs to encourage multiple methods. Moreover, researchers must look beyond the face value of a study to ask whether other criteria of good research have been met. Some methods yield research that satisfies criteria others do not; multiple methods will yield a body of research that collectively can be of high quality even when individual studies are deficient.
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> (Kilpatrick, 1994, pp. 17-18)

I drew a distinction between the report of a study and the study itself, which the reader can only know through the report. In attempting to apply criteria to a research study, the reader often has difficulty disentangling the two and determining where a fault may lie. Given another report, the reader might decide that the criteria were met. Consequently, one should view any set of criteria as rather blunt instruments for assessing quality.

Although some critics might say that the criteria are outdated and unusable, I claimed, "they remain useful – when interpreted appropriately – not only because they represent important insights by our intellectual forebears but also because they help us see the range of what should count as good research" (Kilpatrick, 1994, p. 18). I have discovered, in retrospect, that this point was not made as forcefully as it might have been and seemed to be lost on many readers of the published paper. The question I apparently neglected in discussing these criteria was, What is "appropriate interpretation"?

Relevance

The first, and probably the most important, of the eight criteria of quality is relevance. In Freudenthal's (1991) China Lectures, he commented that the more pretentiously something is presented as educational research, the less useful it is likely to be (p. 149). Freudenthal rightly observed that research in mathematics education lacks any criterion of truth, but that does not necessarily make the research irrelevant or valueless. One can still ask how useful, relevant, or meritorious a study is and to whom. I do not share Freudenthal's apparent mistrust of the capacity of educational research, presented as research, to be relevant to teachers or to other researchers.

I suppose that here an appropriate interpretation of the criterion would note that the relevance of an isolated study is exceedingly difficult to ascertain. The criterion works better when applied instead to research that is set within a corpus of similar studies. No one should expect to draw strong implications for practice from the results of a single research study. The results of a study may be its least important part. Research in mathematics education gains its relevance to practice or to further research by its power to cause us to stop and think. It equips us not with results we can apply but rather with tools for thinking about our work. It supplies concepts and techniques, not recipes.

The relevance of research is entwined with its usefulness and quality. Relevant research is research whose quality is high (because it meets other criteria) and that can be used by others. Use, in this case, does not mean wholesale expropriation. Instead, a useful research study is more like a term that has entered professional discourse: It helps us reflect on and express what we know.

Validity

Validity also relates to the question of use. A research study is not valid in itself but only with respect to the uses to which it is put. What are the claims we want to make for our research? A research study is weak in validity when it yields spurious claims; the validity is attached not to the study but to the conclusions drawn from it. An appropriate interpretation of *validity* requires attention to potential interpretations of the research and to their potential consequences. As Kvale (1993) noted, when we move to a conception of knowledge as the social constitution of reality, we encounter "a communicative and a pragmatic concept of validity. Communicative validity implies testing the validity of knowledge claims in a dialogue. . . . Pragmatic validation . . . focuses on whether the new interpretations lead to changes in behavior" (pp. 192-193). A researcher in mathematics education cannot make a study valid, but he or she can anticipate readers who will interpret and use the study by beginning the dialogue and foreshadowing the consequences of various interpretations and uses.

Objectivity

Objectivity has turned out to be the most contentious of the criteria that we proposed for consideration in Gilleleje. I attempted to argue that "even though absolute objectivity is ultimately unattainable, one can still view it as an ideal worth working toward" (Kilpatrick, 1993, p. 23). The choice of objectivity as an ideal, however, is rejected by some researchers today who argue that even to speak of objectivity is to raise a false banner. Today all of us are subjectivists and relativists, goes the argument. All knowledge is restricted to our consciousness and our senses, and the validity of that knowledge is relative to the people having the knowledge.

Why does all research in mathematics education need to be viewed from this perspective? Is there only one correct epistemology? (If so, how could we know that? Have we reached the end of philosophy?) I contend that we need to interpret *objectivity*, whatever our view of knowledge, as involving the effort to clarify our own biases and their potential effect on our work, as well as the effort to refute our own conclusions as a means of examining our subjective view of them.

Originality

Originality is a criterion that professors apply to doctoral work and that editors and reviewers ordinarily apply to manuscripts submitted for publication. I argued in Gilleleje that replication studies can be original, quoting Freudenthal's (1991) words approvingly: "Reproducing does not mean parroting" (p. 161). Since advancing that argument, I have come to have an even greater respect for the value of replication studies, perhaps especially in helping novice researchers orient themselves in the field but also as a contribution to the field itself. As an anonymous researcher put it:

> In replication you learn a lot about what is still needed. That is not understood. In mathematics, there is no replication. When you prove it, it's proved. But we are not mathematicians; we are a human science. And so when somebody has shown something, we have to try to do it again to figure out what the critical variables were that determined it and what might possibly affect the result. Because the result might be an artifact. (quoted in Silver & Kilpatrick, 1994, p. 738)

An appropriate interpretation of *originality* would allow for replication. Whatever their source, original studies have an element of surprise that both causes and equips us to see mathematics teaching and learning in a new light.

Rigor and precision

Like objectivity, the criterion of rigor and precision needs to be interpreted as relative, not absolute. Rigor is connected to objectivity because the researcher attempts to refine his or her research methods so as to see the phenomena of interest as accurately as possible. Precision needs to be interpreted as precision of meaning and not, as it has so often been seen, as precision of measurement (Kilpatrick, 1993, p. 26). Again, anticipation of possible misinterpretation by the reader drives the effort to make the research rigorous and precise.

Predictability

No one today assumes that we can use research, either a single study or a body of related studies, to predict what a student will do when learning mathematics or what a teacher will do when teaching it. It is not clear that such a goal was ever considered appropriate for research in mathematics education, but it has become common to stereotype behaviorist researchers as having sought such goals. In the present context, I offer predictability as a worthwhile criterion for research when it is understood as involving the search for regularities and patterns of behavior. Students and teachers do not act at random. Research does not attempt to specify what they will do next, but it can attempt to ferret out common predispositions and conceptions that guide their behavior. Prediction, in this case, is not stipulating what will happen in a given situation; it is understanding the events that are likely to occur under circumstances similar to those studied in the research.

Reproducibility

If we are to use research, if we are to draw valid consequences from it, it must be reported in a fashion that would allow it, in principle, to be reproduced as conducted. If we cannot discern, as Freudenthal (1991) requested, "the process that brought [research knowledge] about" (p. 161), we will not be able to separate authentic knowledge from dogma. Research must be public; it must be shared. Researchers should interpret reproducibility as a call to accountability.

Relatedness

Although relatedness can be seen as connected to the criterion of relevance, it is meant to have a slightly different interpretation. If mathematics is being used in a research study as a vehicle rather an ingredient – if, in other words, it is a placeholder for any school subject – one can question the contribution the study might make to mathematics education. As I noted in Gilleleje, even when a study has used mathematics in an unenlightened way, it can still be useful to mathematics educators. But how much better it would have been if mathematics had been made an integral part of the study and its facets explored. Relatedness as a criterion should be interpreted to mean that the study sheds light on mathematics education in a way that illuminates the mathematics being taught and learned. Mathematics education is a multidisciplinary field, but researchers need to take mathematics as problematic and not as given when drawing on those disciplines.

Alternate criteria

At the annual meeting of the National Council of Teachers of Mathematics (NCTM) in 1994, a research presession was co-sponsored by the NCTM Research Advisory Committee and the Special Interest Group for Research in Mathematics Education of the American Educational Research Association. One of the plenary sessions was organized by the editorial panel of the Journal for Research in Mathematics Education (JRME). It was entitled "Evolving Standards for Judging the Quality of Research Reports." The purpose of the session was to propose a set of standards (criteria) for judging the quality of research and to engage the audience in a discussion of those standards and others. The panel was concerned about the reviewing of manuscripts – How do you decide which manuscripts deserve publication? – and they wanted help from the research community in determining what those standards might be. They presented a list of 10 standards, which the large audience reviewed, revised, and supplemented.[1]

The JRME editor and associate editor (Lester & Lambdin, 1994) subsequently proposed a list of seven general criteria for research in mathematics education:

- **worthwhileness**
- **coherence**
- **competence**
- **openness**
- **ethics**
- **credibility**
- **intangible qualities**

Perhaps the first thing to notice about the list is that none of the terms is the same as those in the Kilpatrick and Sierpinska list. There are some connections, however. It is important to note that Lester and Lambdin prefaced their list with some underlying assumptions.

Their first assumption was that the research has mathematics education as its primary focus. This assumption is essentially the relatedness criterion from the previous list. The second assumption was that the criteria must necessarily be general and rather abstract if they are to be applied to the whole spectrum of research studies in our field. This assumption is connected to the idea that the criteria on both lists need to be interpreted appropriately. The third assumption was that there is a need for open, public criteria for judging the quality of research. I agree that criteria need to be part of the critical discourse that should mark our field. The fourth assumption was that the criteria are not immutable and should not be applied in a mechanical way. Again, I agree: My purpose in discussing all of these criteria is, in part, to show how provisional and partial any such set of criteria must inevitably be.

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[1] As an indication of how difficult it is to achieve consensus on these matters, although the session was to have led to an article in which the *JRME* editorial panel would offer a refined list of standards, the panel members have apparently not been able to agree on a list. As of this writing, no article is forthcoming.

Worthwhileness

The criterion of worthwhileness appears to be just another way of phrasing the criterion of relevance. Contending that this is the most important criterion of all, Lester and Lambdin (1994) argued that research should be judged according to its potential contribution to understanding mathematics teaching and learning. If a study is worthwhile, it will generate research questions, contribute to the development of theory, be situated in a body of related research, and contribute to practice. Lester and Lambdin made one point that I find difficult to accept: They contended that the worth of a study is a function of the time when it was conducted and that, consequently, a study once thought excellent may be considered of little value at a later date when the issues it addressed are no longer deemed important. I suppose worth is relative, and certainly there are fashions in research just as in other intellectual matters. But I would like to hold open the prospect that the worthwhileness, or relevance, of a research study or of a body of research can be seen to reside above and apart from those fashions.

Coherence

A key consideration in evaluating any report of research is the harmony between the various components of the study: the questions posed, the methods used to investigate the questions, the techniques used to analyze the data produced by the research methods, and the evidence used to address the research questions. I wish I had thought to include something like this criterion in the earlier list, for the question of how well the various parts of a research study fit together has for years been one of the hallmarks of my graduate course on research. Clearly, research cannot be of high quality if it is not well articulated.

Competence

Lester and Lambdin (1994) maintained that research needs to be evaluated according to how competently appropriate techniques of data collection, analysis, and interpretation have been applied in conducting the research. They argued that research training needs to introduce researchers in mathematics education to the research traditions of several disciplines and that expertise may need to come by means of collaboration with researchers in other fields. I question how this criterion can be applied, given that it refers to researchers and not research, and given its clear overlap with every other criterion. I do not see what it adds. If some part of a research study has been done incompetently, that will infect the entire study and cause it to fail other criteria.

Openness

Researchers need to acknowledge and make public their biases and assumptions. They also need to describe their research methods and techniques completely enough to allow for scrutiny by the research community. These qualities relate to the criteria of objectivity and reproducibility. In fact, *openness* may well be a better term to cover the interpretations I have attempted to make of those criteria.

Ethics

Researchers should get the informed consent of students and teachers who participate in their work; they should respect the confidentiality of their sources; and they should attempt an accurate portrayal of the situations and people in the study. Moreover, they should acknowledge all who contributed to or influenced the research. I regret that something like this criterion did not appear in the Gilleleje list.

Credibility

A study is credible if its findings are grounded in evidence and not merely in rhetorical eloquence. The report of the study should allow the conclusions to be tested. Lester and Lambdin (1994) noted that credibility and openness "intersect." I claim that credibility intersects with validity as well.

Intangible qualities

Some qualities of a research report seem intangible: lucidity, clarity, organization, terseness, candor, originality. Lester and Lambdin (1994) lumped these together as one criterion; we (Kilpatrick, 1993; Sierpinska, 1993) mentioned only the last of these. We did not see originality as any more intangible than the other criteria we proposed, which indicates once more how provisional any of these lists must necessarily be.

Claims of scientific quality

Why should researchers in mathematics education want or need criteria for judging the research they are doing? Why should the mathematics education community pay attention to any list of criteria if the list clearly can never be "fixed, exhaustive, special, or definitive" (Kilpatrick, 1993, p. 31)?

> Although journal editors, editorial boards, and researchers themselves may find such a list helpful, the value of any set of criteria lies in its ability to provoke discussion and the exchange of ideas. These criteria offer us tools for thinking, templates against which to place the problems research has addressed, the means used to investigate those problems, the results thereby obtained, and the uses to which the results have been or could be put. To use another image, the criteria are lenses through which the research landscape can be viewed. (p. 31)

The exercise of examining criteria advanced for judging the quality of research in mathematics education allows us to rethink what we are doing when we do research. Is our research scientific? That question has lurked below the surface of discourse in our field since it began.

Claims for the field

As a field of activity, mathematics education is ancient (Kilpatrick, 1992, pp. 11-12). Mathematics has been taught as long as it has existed. As a field of scholarship, however, the roots of mathematics education go back little more than a century (pp. 4-11). Although by the turn of the 18th century, chairs of education had already been established in several European universities, mathematics education was slow to follow. Eventually, near the end of the 19th century, when (secondary) teacher education was becoming an increasingly important function of universities, mathematics education began to become recognized as a university subject. The early mathematics educators were mathematicians who took an interest in how their subject was being taught. Occasionally, they conducted research, but more often they taught and wrote about methods of teaching mathematics. Meanwhile, psychology was becoming the "master science" of the school, and university students preparing to teach were studying how children learned. Mathematics and psychology became the seminal disciplines supporting the new field of mathematics education; eventually, they were joined by other disciplines such as anthropology, sociology, epistemology, cognitive science, semiotics, and economics (Balacheff et al., 1993, p. 183).

Schubring (1983) argued that mathematics education is both a professional and a scientific field. The two aspects, though linked, are not the same, and in a given country both may be underdeveloped. Schubring contended that a profession requires the following:

(a) specialized knowledge,

(b) a corporate character,

(c) self-determination and autonomy, and most important

(d) a clientele.

The last quality means that the profession is concerned with the application of knowledge.

A scientific field (or scientific discipline, in Schubring's terminology) is marked by

(a) a community,

(b) a corpus of theoretical knowledge codified in textbooks,

(c) unresolved questions,

(d) research methods together with a set of paradigmatic problem solutions, and

(e) specific career patterns and institutionalized socialization processes for selecting and educating candidates according to accepted paradigms.

There is a necessary interconnection between the two aspects of mathematics education: The scientific side cannot develop very far unless it is somehow applied to professional practice, and professional development requires the specialized knowledge that only scientific inquiry can provide.

When mathematics education was taken up by universities in Great Britain, Germany, Belgium, and the United States at the turn of the century, its function was seen as professional. Lectures were organized that supplemented the lectures in mathematics that secondary teachers were receiving and to prepare them for teaching practice. At the same time, as Schubring (1993) pointed out, the first chairs of mathematics education were established, and the first doctoral degrees in mathematics education were awarded, which signified that mathematics education was emerging as an independent scientific field. The problem was that not everyone saw it that way. Many, perhaps most, scholars regarded mathematics education as little more than a craft or trade, with essentially no body of theoretical knowledge to be applied in educating teachers.

Not until the 1960s, amid a growing professionalization of teacher education, did mathematics education in many countries begin to acquire professional status by offering prospective teachers more than the usual courses in curriculum and teaching methodology could provide. New courses appeared in subject-specific departments that went beyond questions of content and methods to consider how theory, research, and practice could be combined productively. As mathematics education became more professional, therefore, it was also becoming more scientific, although clearly it would inevitably be an applied human science.

The mid-1950s to the mid-1970s were a time of enormous expansion in mathematics education research around the world. The "new math" movement that touched many countries left in its wake new journals, new professional organizations, new research institutes for mathematics education, and a host of young researchers. Although data on this growth are difficult to obtain, surveys done in North America and the United Kingdom of the number of articles and dissertations appearing each year showed exponential growth in the 1960s and 1970s (Kilpatrick, 1992, pp. 27-28). After that time, the growth slowed substantially. The number of dissertations listed in *Dissertation Abstracts International*, for example, seems to have leveled off at something between 200 and 300 a year; the number of research articles in journals searched as part of the Educational Resources Information Center (ERIC) system currently runs just under 200 a year.

The United States has long accounted for the lion's share of research in the field. A survey of some 3,000 studies of mathematics learning, for example, found that 85% of them had been conducted in the U.S. (Bauersfeld, 1979, pp. 203, 210). Nonetheless, other countries are beginning to contribute in greater numbers. In a recent book synthesizing research on the psychology of mathematics education, all of the principal authors, and most of the references cited, were from outside the U.S. (Nesher & Kilpatrick, 1990). An examination of the affiliations of authors publishing in two of the most prestigious journals of mathematics education, the *Journal for Research in Mathematics Education* and *Educational Studies in Mathematics*, showed that, whereas only about one fourth of the *JRME* authors were from outside North America, only about one fourth of the *ESM* articles were from inside North America (Silver & Kilpatrick, 1994, pp. 746-747).

Mathematics education in the United States

The establishment of mathematics education in the United States was stimulated by an emerging community of mathematicians as well as by a huge expansion in secondary education that produced a shortage of mathematics teachers. Assisted by foreign mathematicians such as Felix Klein and by local leaders such as Eliakim Hastings Moore and David Eugene Smith, the U.S. mathematics community began to get involved at the turn of the century in promoting the advancement of mathematics teaching in the schools (Stanic & Kilpatrick, 1992). The mathematicians and the school mathematics teachers eventually formed separate professional organizations – the American Mathematical Society, the Mathematical Association of America, the National Council of Teachers of Mathematics – but individuals within these organizations were able to maintain close ties, with many belonging to more than one organization. The organizations themselves participated in many joint efforts. As the number of professional organizations in the mathematics community has grown, the community has continued to find ways to collaborate and cooperate. The United States has been fortunate over the past century in having had a continually replenished cadre of mathematicians who have supported and encouraged the development of mathematics education as a scholarly field.

The 1960s and 1970s were, as noted above, a time of especially rapid growth for mathematics education in the United States. New graduate programs were established at many institutions that were moving from state college to state university status and instituting advanced degree programs in mathematics education. In some instances, these programs developed within mathematics departments, but the great majority were in departments, colleges, or schools of education. In the latter case, their colleagues in education may not have studied much mathematics, but they ordinarily were able to evaluate something of the quality of the mathematics educators' research and scholarly writing. In the former case, however, mathematicians were often at a loss to understand, let alone evaluate, work that was so far removed from research in mathematics. The incorporation of mathematics education into departments of mathematical sciences was seldom a smooth process, primarily because of differing conceptions of the research needed to establish one's scholarly credentials.

At the University of Georgia, we are fortunate in having one of the few departments of mathematics education in the United States, if not the world. Certainly, we are one of the oldest and largest such departments. Our department has been in existence for just over 30 years, and we currently have 13 faculty members. Although located within a college of education, we have many associations with the department of mathematics. The Department of Mathematics Education at Georgia is the beneficiary of a long history of efforts by our professional forebears to stake out a claim for mathematics education in the American university. The turn-of-the-century establishment of mathematics education at institutions like Teachers College of Columbia University and the University of Chicago blazed a trail for us to follow; we did not have to convince anyone that subject-matter didactics has a place in a university in which education itself is seen as both a professional and a scientific field. We are also the fortunate heirs of decisions by the College of Education at the University of Georgia to create departments for the various school subjects and to maintain them in the face of a movement in the 1970s and 1980s when many institutions across the country did away with subject-specific professorships in education.

A major engine of our growing reputation and status internationally has without question been the large number of our doctoral students who come from outside the United States. These students not only have returned to their home countries to develop the field there, but have helped us raise the level of our courses by bringing new perspectives and fresh ideas. They, together with a continuing stream of visiting faculty from abroad, have also assisted us in forging links to universities and institutes in other countries, all of which have helped us put our programs on a more scientific and professional footing. What we have done with our programs is being repeated in other countries as students increasingly cross national boundaries seeking advanced educational opportunities in our field.

Mathematics education in Sweden

As the mathematics has developed in other countries, the Nordic countries – and especially Sweden – seem to have lagged behind. If the Nordic countries were contributing to mathematics education in proportion to their numbers, for example, one might expect something on the order of 20 doctoral dissertations a year. The actual number falls far short of that. Fortunately, the number of new programs, journals, conferences, and organizations devoted to mathematics education seems to have risen sharply in recent years.

A curious fact is that although lectures in education were given as early as 1804 at the University of Uppsala, a chair in education was not established there until 1910 (Kilpatrick, 1992, p. 4). Moreover, although most countries can identify mathematicians and psychologists who early in the century took an interest in mathematics education and attempted to promote the development of the field, Swedish mathematics educators do not have many professional forebears they can claim. One of the few was Frits Wigforss (Kilpatrick & Johansson, 1994), and his work is largely forgotten. Swedish mathematics education, as a professional and scientific field, appears to have had a difficult time getting established. Courses in mathematics education have been offered at universities such as Göteborg and Linköping for a decade now, but there is still no chair in mathematics education in Sweden.

This is not to say that the mathematics education community in Sweden is not alive and well. On the contrary, individuals, organizations, and journals appear to be thriving. The mathematics department at Umeå University has initiated a doctoral program with a specialization in mathematics education. The department of mathematics education at Göteborg University, of which I am a proud member, is almost as large as its sister department at the University of Georgia. It publishes two major journals for mathematics educators and offers a variety of courses to preservice and inservice teachers. Its members are active nationally and internationally, organizing conferences and teachers' groups, consulting with governmental and school officials, speaking at international meetings, and conducting research. The department does not, however, have any graduate programs to renew the community and advance the field.

There is no lack of talent among Swedish mathematics educators. What is missing are the institutional structures that would recognize, value, and support mathematics education as a scholarly field in which advanced professional and scientific work is not only possible but desirable. To develop such structures requires a recognition by Swedish educationists and mathematicians alike that mathematics education has come of age internationally and ought to claim its place in the Swedish university.

Raising the stakes

Mathematics education has never been healthier as a profession and as a scholarly field. The past few decades have seen the increased professionalization of mathematics teaching, with mathematics teacher education recognized as a legitimate function of the university in a growing number of countries. Simultaneously,

> an international community of researchers [in mathematics education has formed] that holds meetings, publishes journals and newsletters, promotes collaboration within and across disciplines in doing and critiquing research studies, and attempts to keep a research consciousness alive in the councils of the mathematics education organizations in which members of the research community participate. (Kilpatrick, 1992, p. 3)

Nonetheless, the field still faces serious problems of status and identity. The International Commission on Mathematical Instruction, a commission of the International Mathematical Union, recently undertook a study entitled "What Is Research in Mathematics Education, and What Are Its Results?" (Balacheff et al., 1993). Part of the impetus for the study came from a feeling that mathematicians do not understand the field of mathematics education. There was also the feeling that mathematics educators themselves "often talk past one another. There seems to be a lack of consensus on what it means to be a mathematics educator" (p. 179).

A clear status differential between mathematics and mathematics education continues to exist, as can be seen in the following thought experiment. Imagine a research mathematician saying, "I am growing old and can no longer do original mathematics. I find my grandchildren's efforts to learn mathematics fascinating and have been visiting some school classrooms. I have decided that mathematics education is a field I would like to join because I think I can make a contribution." Does that sound plausible?

Now imagine a mathematics educator saying, "I am growing old and can no longer do original work in mathematics education. I find my grandchildren's efforts to do mathematics fascinating and have been visiting some university mathematics departments. I have decided that research mathematics is a field I would like to join because I think I can make a contribution." Does that sound plausible?

To the extent that these two assertions do not sound equally plausible there is an imbalance in status. To many people, mathematics is a field one joins by taking advanced courses and seminars and by demonstrating one's competence through the publication of original research. Mathematics education, in contrast, is a field one joins simply by declaring one's interest.

I have not made a systematic study of how mathematics education has developed around the world. I have, however, taught courses in Spain, Italy, and Colombia, and have spent time at universities in other countries in Europe, the Middle East, Australasia, and South America. I have three firm opinions about how the field might be strengthened.

The first is that mathematics educators everywhere need to form and maintain stronger ties to mathematicians. Our field grew out of mathematics, and to drift away from it is to descend into a sterile preoccupation with method over content. Each school subject has its own structure, psychology, and social context. When the special features of mathematics are submerged in general curriculum or pedagogical studies, teacher education loses its power to ensure that teachers not only know mathematics but have reflected on its learning and teaching. Convincing mathematicians that they have a stake in how mathematics is taught in schools and in how mathematics teachers are educated is ordinarily not very difficult. Identifying mathematicians who are willing to devote time to working with mathematics educators on improving the teaching and learning of mathematics, however, is far from trivial. Building a climate of mutual trust and respect between mathematicians, university mathematics educators, and mathematics teachers demands much effort and is not accomplished overnight.

The second opinion is that researchers in mathematics education need to form and maintain stronger ties to practicing mathematics teachers. Education is a profession in which the gulf between research and practice is especially wide, so researchers have a particular responsibility to ensure that the work they are doing is connected to and informed by practice. Individual researchers need not attempt to do work that a teacher will be able to apply immediately, but unless the field as a whole is doing research that has practical value, it will be seen as not only irrelevant but unproductive. Fortunately, in many countries, the concept of "teacher as researcher" is being explored, and teachers are increasingly members of research teams rather than simply subjects of research.

The third opinion is that, although university mathematics educators can certainly thrive in mathematics faculties, mathematics education as a field progresses more rapidly when mathematics education is a distinct program or department within the faculty of education. The profession of mathematics teaching is ordinarily the province of the education faculty, and mathematics education as a scholarly field fits better among the social sciences than among the natural sciences.

Mathematicians and mathematics educators have essentially different orientations to research and scholarship. Research in mathematics involves abstractions and generalizations that can be handled by means of deduction. Although specific cases are often studied inductively as a means of supporting conjectures and suggesting lines of investigation, the machinery of deductive proof is used to sanction claims and ensure validity. Research in mathematics education is another matter, as the discussion of criteria for research quality in the first part of this paper was meant to show. To the extent that mathematics education is a science, it is a human science. If it is seen as a scholarly field rather than a discipline, it is a field that rests on a variety of other disciplines, most of which are social sciences. Researchers in mathematics education do not prove theorems. The claims they make are conditional, tentative, and deeply embedded in a context. When university mathematics educators do work within a department of mathematics, everyone in the department needs to understand that, although one can justifiably view mathematics education as one among several mathematical sciences, the criteria for quality scholarship are not the same as for the others.

Mathematics education is a university subject and a profession. It is a field of scholarship, research, and practice. More than mere craft or technology, it has aspects of both art and science. In every institution or country, however, it is bound by its history. How far it develops and is able to influence teachers and learners in positive ways depends heavily on whether educational policymakers can find ways to recognize, institutionalize, and support it.

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Att muta in ett område

Matematikdidaktik [Mathematics Education] som kunskapsområde omfattar både vetenskaplig och yrkesmässig verksamhet. På den vetenskapliga sidan diskuteras fortfarande frågan om vad som skall räknas som forskning. En granskning av två föreslagna uppsättningar kriterier för kvalitetsbedömning av forskning i matematididaktik visar att kriterier lånade från både naturvetenskap och samhällsvetenskap, tolkade på ett rimligt sätt, är relevanta för ett fält som försöker bli vetenskapligt. På yrkessidan måste matematikdidaktik oundvikligen syssla med tillämpningar av specialistkompetens för att stödja studenter och lärare som är områdets klienter. Lärarutbildning kommer även fortsättningsvis att ha en viktig funktion för matematikämnets didaktik, parallellt med sökandet efter tillförlitlig kunskap att tillämpa.

Det är viktigt att matematikdidaktiker vid våra universitet får arbeta nära och tillsammans med matematiker och yrkesverksamma lärare i utvecklingen av både teori och praktik. Matematikämnets

didaktik har blomstrat i länder där institutionella strukturer stött ämnet som ett identifierbart akademiskt kunskapsområde.

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