

# 'Informal activity' in mathematics instruction

Iben Maj Christiansen

*On the basis of an analysis of portions of a modelling course in Danish high school, the existence of 'informal activity' is asserted. This activity can contain reflections on the mathematical activity performed when 'on task', but also allows the students to distance themselves from the content and school-tasks in general. Informal activity may thereby contain a potential for critique which is not realized due to isolation in the informal activity. To give insight on the possibility for change, the formation of discourses and their relation to the setting is discussed.*

We've all been there: a task must be completed, a problem solved, or a product made, and we have gathered a small group to do the work. The task at hand is to plan a party, to collaborate on an article, or to decide whom to lay off. For some time, we are on task, and even if no progress is made, all involved are consciously striving to accomplish the appointed task. Then somebody has an association and soon jokes are flying around the table, perhaps even silly suggestions which no one takes seriously. After a few minutes, one group member usually takes the initiative: "Well! Back to work now!," she says.

These informal situations play an important part in the social atmosphere of group work. Perhaps they are so important that when group work takes place in a more formal setting, room has to be made for the informal situations. In the movies, the men are shown gathering in the sauna after a tedious meeting and in the last decade, professional women have also been depicted engaged in similar activity with their colleagues. But do these situations 'only' mean something in terms of social bonding? No, ideas related to the task may very well be constructed in these situations and reflections on the task as part of a broader context may be undertaken.

Had I realized this at the time, I would perhaps not have been so surprised by how students in a high school class create informal situations which have more potential than just setting the scene for

---

*Iben Maj Christiansen has recently completed her Ph.D. dissertation in mathematics education at the department of mathematics and computer science, Institute for Electronic Systems, Aalborg University, Aalborg, Denmark.*

chit-chat, flirting, or joking.<sup>1</sup> In the following, I will discuss the function of informal situations in a mathematics classroom where the students were working with models.

## Course organization and collection of data

During their three years of high school mathematics instruction, Danish students are required to work with 'mathematical models'. In the guidelines from the Ministry of Education, it is stated that the 'model aspect' can be treated separately or be integrated in the regular instruction. The model aspect

"must give students knowledge of the construction of mathematical models as representations of reality and impressions of the possibilities and limitations of applying mathematical models, as well as equip them to accomplish a modelling process in simple situations"

(Loosely translated from Bekendtgørelse, 1990)<sup>2</sup>

In the freshman classroom I will refer to below, the teacher had decided that the 'model aspect' should be treated separately through having students work on constructing their own population models.<sup>3</sup> First, the students would be introduced to three types of growth (linear, exponential, logistic) in lectures and through exercises. Afterwards, they were to work in smaller groups modelling the growth of some population by expressing relations among concepts in mathematics and implementing these in a computer program.

The course was organized neither to introduce new mathematics concepts to the students, nor to make the students apply particular, previously introduced, concepts to simple examples. Rather, the purpose was twofold: to make students express relations in mathematics, and to give them an opportunity to reflect on the advantages and disadvantages of using mathematical models on a problem. In particular, we wanted to give students some feel for the fact that models are not crystal balls, but tools for hypothetical thinking – that models do not provide us with knowledge of the future, or with

---

<sup>1</sup> This is not to say that the social relations are unimportant, or that they are less important, but as an educator rather than a pedagogue, I am mainly concerned with the cognitive content of students' interactions.

<sup>2</sup> For an English translation of the Danish high school regulations and subject descriptions, see Undervisningsministeriet (1994).

<sup>3</sup> I am most grateful to Bente Jakobsen from Amtsgymnasiet i Hadsten (The County High School in Hadsten), Denmark, for allowing me unconditional access to her classroom and for her help and support in the preparation, evaluation, and analysis of the conducted course. With her consensus, she appears by her own name in this text, but her students are only referred to by pseudonyms.

control over the course of events, but can be used in discussing alternative actions – as long as the (un)certainty of models does not end up being the main point of discussion.

I had helped to arrange the course, functioned as an assistant supervisor during the group work phase, and observed the class as part of my Ph.D.-project (for further details, see Christiansen, 1994). I recorded the class sessions and the interactions in a particular group of five students: Ann, John, Neil, Susan, and Tony. The transcripts reproduced below are excerpts from these recordings. The references indicate from which day, which section of the interaction, and which lines in the original transcript the excerpt was taken. For transcript notation, see the appendix. All transcripts appear in my translation.<sup>4</sup>

### **Distinguishing types of activity**

Being on the lookout for examples of students reflecting on the models they were constructing, there was a particular incident that caught my eye during the first day of the course. One of the female students in the group which I observed, Ann, pointed out that a linear model of world population did not reflect reality, as such a relation could be extended 'back in time' to find the time of zero population. But her observation never seemed to effect the group's work on their task! Indeed, I noticed that the other woman in the group, Susan, blocked Ann's reflections with a call for a return to the exercise:

Su: now, let's get started. .. number two (1,C, 27)

By this statement, Susan indicated that Ann's comment was irrelevant to the task. I decided to see if the students related differently to the content depending on the character of situation, that is, depending on the relation to the task. Since the task at this point in the students' activity was very much synonymous to working on the exercises, I studied the transcripts in order to determine if it was possible to distinguish when students' activity was directed by references to the exercise. It seemed so.

Thus, at certain times, there is a direct reference to the exercise more than once in every ten lines of transcripts. It can be in the form of reading the exercise in order to determine how to pursue the

---

<sup>4</sup> All analyses and interpretation of transcripts have been based on the Danish version, in order to reduce errors derived from the translation. Afterwards, the translations have been performed in such a way that the original analysis could be argued on the basis of the English version. I find this to be the best way to preserve the meaning of the statements as entities – that is, as opposed to the literal meaning of the words in the utterances.

activity<sup>5</sup>, it can be a question about what type of exercise it is<sup>6</sup>, it can be a suggestion to pause the work on one exercise till the teacher has been asked<sup>7</sup>, or it can be a call for the formulation of an answer.<sup>8</sup> The reference can also be to the teacher's statements about the task. In transcript A from the second day, there is a reference to the exercise or the teacher's oral task statement in line 17-8, 21-2, 27-8, 35, 39, 43-4, 59, 65, 80, 90. This tendency continues, but these situations are also interrupted by stretches of communication with no direct reference to the exercises.

## Two kinds of task-oriented activity

Some of the situations lacking direct references to the exercises are still clearly task-oriented, but some conflict has arisen which must be resolved in order for the activity to continue. It can be a question about how to understand some of the concepts involved, or a disagreement concerning how to pursue the task. Here is a short example of how students *negotiate the meaning* of the content – here, specifically the meaning of 'increment' (for *Rules of transcription*, see p. 27):

- Jo: population increment<sup>l</sup>, is that, do they mean .. that, in this, that .. it is people being born, or is it people living on earth right in that<sup>l</sup>,  
 255 Su: increment, it's .. if uh .. there are some that are born, if there are  
 Jo: it's it's it's not just people  
 born, it's that, it's that number, that are on earth,  
 Su: no, population increment it's, if for instance .. there are born 30, 30 people in [our town], right<sup>l</sup>  
 Jo: hmm  
 260 Su: and then there are 20 who dies, then there's a population increment of 10<sub>1</sub>  
 Jo: of 10<sub>1</sub> ...  
 well that's also what I meant, so it's not just those being born, then,  
 (1, D, 253-262)

John and Susan may have had the same understanding of 'population increment' at the start of their dialogue, but John raises the question of the exact meaning of the concept, and it is determined through

<sup>5</sup> As when John says: "yes but try to hear this, [*reading the exercise*:] does the answer correspond with that Paul Ehrlich .. now says [*hesitating*:], that there are 93 millions more per year .." (1, D, 10-11)

<sup>6</sup> Another example with John illustrates: "what, is it just supposed to be such a one uh ... where we say, yes it probably goes together, or, (...) [*Inaudible speech from Neil*] are we to calculate it or what<sup>l</sup> (1, D, 14-16)

<sup>7</sup> "No, we'll wait a little with that one then<sub>1</sub> .. then we'll ask, when she comes<sub>1</sub>" (John in 1, D, 73)

<sup>8</sup> Susan: "and that one<sup>l</sup> what should we answer in that one<sup>l</sup>" (1, D, 109)

a reference to a concrete – though constructed for the occasion – example. They reach a shared understanding which is marked by the simultaneous stating of what the 'population increment' would be in the example (line 260-261).

These meaning-negotiations have been given much attention in mathematics education research. They will not be addressed further in this article.

The next transcript excerpt gives an example of *task-negotiation*, i.e., where the students disagree about how to pursue a task. Such situations may be initiated by a disagreement on an answer which sometimes makes it hard to determine exactly what the discussion is about. In this example, the students have been asked to list which factors may influence population growth. Again, it is Susan and John who engage in negotiation, but this time about how to understand the task:

- Jo: pollution'  
 ...  
 535 Su: but [we know], yes of course John, but pollution it must really kind of make it, smaller'  
 Jo: what' .. oh yes, but it doesn't say whether it, enhances or \  
 To: (diseases?) diseases,  
 Jo: it increases, the (population?) growth, it just says factors, which have  
 539 influence,  
 540 Su: but pollution cannot have influence, .. it must make it  
 Jo: (it does have,?) there will be less then,  
 Su: oh, (...)  
 Ne: the more uhm .. mutants there are, you can't call them people anymore, \  
 545 Jo: (...) pollution,  
 Ne: they are some such  
 To: luminous mutagens,  
 Su: oh yes, pollu, then can't we write, pollution .. reduces' ... or what do you call it'  
 550 Ne: but we won't bother writing about that, it's just which factors have  
 550a influence,

(1, D, 533-550)

The discussion is not about whether pollution influences population growth or even how – though Susan's statement in line 540 could give that impression when viewed in isolation, her objections in line 535-6 and 548-9 both acknowledge that pollution may lead to a decrease in population growth. Instead, they are disagreeing about what to write. First, Susan does not even want to list 'pollution' as a factor influencing population growth – perhaps because 'growth' makes her focus on factors which would increase population growth, or perhaps because she wants to explain why population growth has increased over the years according to their data. When John points

out that he does not disagree with her concerning the direction of the influence (line 541-2), she accepts that 'pollution' should be listed, but she wants to include more information (line 548). This is rejected by Neil in a reference to the task as he perceives it. On this basis I claim that the students are not disagreeing about the content, but about how to pursue the task. The men appear keen on avoiding any extraneous activity, insisting on writing a minimum of text in their answer.

This kind of activity is an example of 'task-negotiation'. As the example shows, there can be some more or less clear reference to the exercise (for instance, John talks about what "it" says in line 537 and 539, where this 'it' is likely to refer to the exercise), or to the task connected to the exercise (as when Susan and Neil talk about what to write, it is implicitly understood that they are talking about what to write in answer to the exercise).

Clearly, there is a vast range of activities which are, in one way or another, task-directed. But these may be interrupted by situations where the students apparently just joke around. In some of these, they talk about the party next Friday, somebody's ugly tie, their pets, etc., which definitely influences the power relations in the group but does not appear related to the content at all. Other situations are harder to classify, as the content may be relevant to the task while other elements of the conversation may indicate that the students are mainly joking. In the following, I will analyze a few of these situations marked by *informal activity*.

### **The exercise as an opportunity to joke: An example of informal activity**

The students were introduced to a computer program as part of the course. Through instruction in the class, exponential growth had been expressed in the computer language and the students were then supposed to enter the program into the computer and do some work with it. Though the purpose was to make the students reflect on the model's reliability, this was not explicitly stated. Instead, they were asked to extend the model backwards and forwards for the time variable, and to see the possible effect of changing parameter values in the model. In the following transcript, they are working on extending the model 'back in time'.

- 10 Ne: (try to go a little further in the program when we have become ...?)  
 To: [*Laughs*] 15 people .. 9 people, *man*, .. we usually end with 2, .. try to press [the button] once more, .. yes, we should, .. (nooh?), that's wrong,

- ...
- 15 Ne: (...) when you've laid eggs,  
To: [Laughs]  
Jo: what, I have a question, what Bente, why are there, why, how can they only  
be be one person and then get to be 2'  
B: [Laughs] that's biologically impossible,  
x: yeah, but that's rather exiting,  
20 To: Neil, he thought it's because  
Ne: it's probably because uh .. that person was pregnant,  
x: yes,  
B: is it, .. yes, .. yes, it has (probably, that's logical?),  
Jo: should we try to increase the step length (...)  
25 ...  
B: but it could well indicate that then (...)  
[The students are busy at the keyboard.]
- (5, D, 10-27)

The students are doing what they have been told to, as they are indeed extending the model to lower values of the time-variable. But this activity 'slips into' joking with the results, when they start to work with populations of a very small size. There is nothing in the students' statements which, taken by itself, indicates that they are no longer working with the exercise, but the interaction as a whole contains signs that this is a borderline case. On one hand, Bente seems to accept the students' activity as relevant to the task as she enters the conversation and tries to draw conclusions based on the discussion (line 26). On the other hand, she laughs at John's question about how one person can become two, and she participates in Neil's joke that the first person must have been pregnant. What definitely marks the activity as non-task-directed is John's call for a return to the exercises in line 24. Since no attempt was made to write down a conclusion concerning the extension of the model, the discussion was not on task, but an example of informal activity.

As I was particularly interested in the students' reflections on models<sup>9</sup>, I focused on this and similar situations, because the students appear to be addressing the lack of correspondence between the calculations and reality – though only implicitly by making a joke of it. Now, the students had been asked to work with the model in order to encourage reflections on such matters, and the interaction reproduced above could therefore be taken as a sign of success. However, it puzzled me that they never engaged in a formulation of such reflections nor did the reflections appear to have any effect on

---

<sup>9</sup> For an extensive discussion of what such reflections may encompass – reflections on a models' validity, reliability, applicability, function when in use, among others – see Christiansen (1994).

their successive activity. This was so, even though the teacher clearly attempted to make the underlying reflections explicit, as seen in line 26 of the excerpt. Instead, the students appear to have ignored the teacher and focused on the keyboard.

Before I discuss the reasons for ignoring the teacher's seeming call for verbalization of the reflections, I will describe one more situation which reflects how the informal activity is separated from the task-directed activity. I also suggest that the informal activity serves a function in the students' way of handling the exercises, but in a perhaps unexpected fashion.

### **Informal and task-directed activity – mutually exclusive**

This particular situation is related to the students' work on the first exercise question, namely: In the article, it says that the world's population was 3.5 billions in 1968, and that there were 70 millions more every year. If that is true, how large should the world's population be in 1992? The students have just completed their calculations and found that with the stated assumptions there should have been 5,18 billion people in 1992. The present situation consists of a discussion originated in their activity with this question.

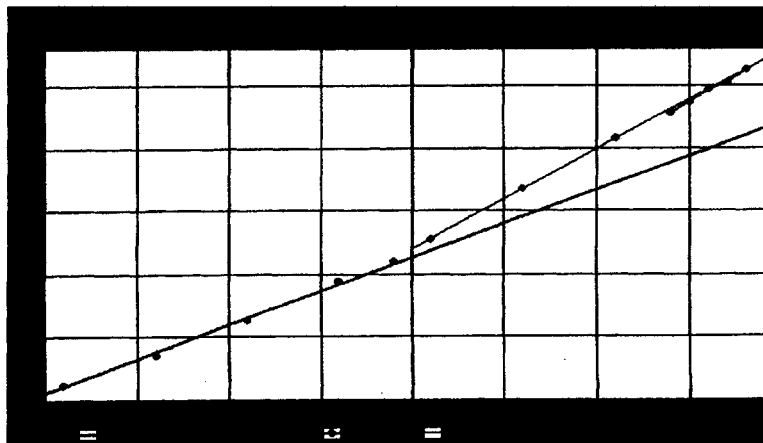
- An: well then we can try to<sup>1</sup> ..  
 Jo: uh, are we just supposed to answer it, are we not supposed to discuss if it makes sense or something<sup>1</sup> .. one apparently is not supposed to<sub>1</sub>  
 Su: yes, that's what comes in the next,  
 5 Ne: [*With a whining voice:*] that's just great  
 Su: no then it says ..  
 An: (...) <sub>1</sub>  
 Su: how large was the world's population  
 An: then one can see when the humans they were .. they were .. born or what  
 9a ever it's called  
 10 x: hey<sup>1</sup> Bente  
 Ne: (...) <sub>1</sub>  
 y: [*Laughing*]  
 Su: [*Laughing:*] what are you saying<sup>1</sup>  
 An: then one can just uh see .. how many times 70 millions go into 3.5, right<sup>1</sup> ..  
 15 and then there are no more people, so at that time (...) <sub>1</sub>  
 Ne: (...) that's the whole  
 16a world<sub>1</sub>  
 Su: I don't understand at all, what Ann is saying. .. try again  
 An: [*Laughing*]  
 Ne: you, you, you shouldn't  
 20 worry about that, none of the rest of us, do<sub>1</sub>  
 An: [*Laughs*] (...) 3.5



- ...
- Ne: what, what exercise are you on now! ..
- An: [Laughing:] one [Laughing]
- 25 Ne: just to ask, a stupid question .. one you made up yourself
- An: [Laughing]
- Su: now, let's get started, .. number two  
(4 sec.)
- Ne: Ann, she has just (consumed?) 50 people,
- 30 An: [Laughing:] no
- Su: how large was the world's population actually in 1992, it
- 31a was 5.4 (4 sec.) 5.5
- (1, C, 1-31)

This short passage seems dominated by a discussion between Susan, Neil, and Ann, initiated by the latter's idea presented in line 9. This discussion interrupts a clearly task-directed activity where John reflects on whether they have performed the exercise sufficiently – as I take his "supposed to" in line 2 to refer to the wording of the exercise. This is supported by his own answer to his question, where he is capable of concluding from the material at hand that they are not "supposed to" reflect on the reliability of their result. Susan also answers his question with a reference to the exercises by saying that the reflections he misses will come in the second one. Not only is their activity determined by the exercises, in the sense that their task itself is to answer the teacher's questions. It also has to take place in the correct order, as signalled by the formulation of the exercises. So Susan is telling John that he will have his expectations fulfilled without having to break with either the content of the exercises or the task-directedness of their activity. Neil's following exclamation – where he in high pitch declares the marvel of "it" (line 5) – may be seen as a rather suitable irony that they can just surrender themselves to the exercises.

Figure 1. The world's population.



The discussion which follows is not directed by the task defined in the exercises. Apparently, their work with the first exercise has given Ann the idea that they can use the assumptions in the exercise to calculate the time of the creation of the human race. In other words, Ann is suggesting that they use the informations in the exercise – that there were 3.5 billion people in 1968 and that the increase is 70 millions per year – to calculate 'backwards' to the time when the census was zero.<sup>10</sup> (With these assumptions, it would be in 1918. See also the approximation to the first half of the data in the graph.)

Ann does not directly compare the model to the actual census, but her laughter signals that she is considering the result of the suggested calculation in relation to her knowledge of the actual world population through time. What else should make the idea so amusing to her that she can hardly express herself for chuckling? In this sense, Ann appears to have realized the deficiency of the linear population model.

We could now imagine that this could lead to a discussion of the model, for instance: under what circumstances is it reasonable to consider the linear model valid? Which factors besides the census and population growth for a certain year could be considered in order to reach a more fitting description of the census over a longer period of time? Is it reasonable to assume that the census can be described by a simple function? Which conditions should be fulfilled if population growth should be constant?

However, these questions are not raised, and Ann's idea does not create any discussion of the model what so ever. Because the other students do not understand? Clearly, Susan does not show any sign of understanding Ann's idea. Neil says that none of the others do either (line 19-20). Still, that does not correspond with his statement in line 16. Though it can not be distinguished fully on the recording, Neil seems to go along with Ann's idea in pointing out that her division would take care of "the whole world". I could speculate on the possible reasons for Neil's shift in attitude, but I will instead focus on the way in which he closes the talk of Ann's idea. First, as I have mentioned, he points out its meaninglessness. But this is honed with a reference to the exercises.

Ne: what, what exercise are you on now! ..

An: [*Laughing*:] one [*Laughing*]

25 Ne: just to ask, a stupid question .. one you made up yourself<sup>f</sup>

(1,C, 23-25)

---

<sup>10</sup> If this idea cannot be constructed with absolute certainty from her explanations in line 9 and 15-16 then it is very clear from a later discussion, where Ann presents her idea in the presence of Bente (1,D, 151-162).

Ann claims to be working on exercise one, but Neil's next comment ignores this answer completely by indicating that she is working on a self-constructed exercise. Right after Neil's comment, Susan calls for turning to the second exercise. What makes the reference to the exercises so powerful that it can stop a potential discussion of the model from exercise one? The answer lies not in what Neil is saying but what he is *doing* with his question.<sup>11</sup> When asking Ann to relate her suggestion to a specific exercise, he is questioning the legitimacy of her activity. Since it cannot be connected directly to an exercise, it is thereby marked as belonging outside the 'official' discourse or activity. It appears that such detours can be brought to a halt by a call for a return to task.

Ann does not object to this rejection of her idea. Thereby she is not only signalling an acceptance of the exercise as the final arbiter of legitimate activity, *she also shows that she did not herself consider her idea relevant to the exercises*. This is further indicated by her laughter with which she marks that her idea is not meant to be an objection to their calculations. She probably intended neither to add anything to the answering of the exercise, nor to improve their insight in the growth of global population. It was simply an opportunity to jest.

### **The exclusion of critique through restriction to informal activity**

The critique of the model is, at this point, excluded from the process of answering the exercise. This is evident from the way in which Ann's informal activity – with no objection – can be interrupted by a call for a return to the exercise. Furthermore, as was also indicated in the previous example, the informal activity does not result in a critique of the exercise, an objection to the regulation of their activity, or a revision of the solution to the exercise.

It is not hard to construct an explanation as to why this must be so. After all, the students are subject to certain obligations – the 'didac-

---

<sup>11</sup> There is a reference to speech act theory implied here (cf. Austin, 1958; Austin, 1961/70). My focus in the present case is not on the act of uttering in itself (the locutionary act), and it is not so much on the act of asking a question with certain intentions (the illocutionary act). Rather, it is on Neil's utterance as a means to create a particular effect (the perlocutionary act). This effect is obtained in an interplay between the 'listener', her experience of the situation (the 'setting'), and the implicit obligation to respond to Neil's utterance in one way or another.

tic contract' – in the classroom (cf. Brousseau, 1984).<sup>12</sup> They appear to perceive their 'job' to be to work the exercises – *even* if the teacher's intention was for students to break the contract and engage in critical reflection. However, the students *do* break the contract and they *do* reflect on the content of the exercises, but only as part of informal activity.

The existence of the informal activity thereby expresses in itself a distance from the 'official' classroom activity. Since certain perspectives are excluded from task-directed situations, the students create room for these perspectives in informal activity. Thus, this activity is in opposition to the task-directed activity in two ways.

First, it allows a completely different relation to the content; it becomes possible to reflect on the mathematical calculations in the light of reality, thereby comprising a discourse<sup>13</sup> parallel to the exercise discourse.

Second, by criticizing – even if only implicitly – the view of the content promoted by the exercise discourse, the informal activity also contains a critique of the task-directedness itself. It is, however, a critique which must not be voiced in the exercise discourse if the students are to succeed within the mathematics classroom.

In other words, an 'official' discourse exists – in this case manifested by what I have called 'task-directed activity'. This implies

---

<sup>12</sup> Brousseau, as a Frenchman, is not using the term 'didactic' with the connotation 'moralizing'. The English speaking reader may think of 'didactic' as what is related to the teaching/learning processes of some particular subject.

<sup>13</sup> 'Discourse' is not only used here as a synonym for conversation or reasoned exposition. Foucault used 'discourse' to refer to the mode of speech within a certain domain of knowledge, but, as does Mellin-Olsen (1991, p. 76), I will use the term in a somewhat broader sense, namely as the (mode of) communication in a particular type of situation. Thus, the discourse is the manner in which a certain structure and regulation of the utterances manifests itself. In this sense, "The discourse, as it can manifest itself, can be said to be nothing more than the presence of what is not being said, and this 'non-spoken' functions as a virus which undermines everything being said." (Mellin-Olsen, 1991, p. 78, my translation).

On this basis, I think of a discourse as characterized by certain (implicit) criteria for what counts as a valid statement (cf. Jensen & Lytje, 1993, p. 14 who use 'language game' with the same meaning). Thus, a discourse is "a set of norms, preferences, and expectations relating language to context, which speaker-hearers draw on and modify in producing and making sense out of language in context." (Ochs, 1990, p. 289)

As the criteria for the discourse must be recognized by the people acting through the discourse, these criteria are continuously (re-)constructed. There is an interaction between agents which simultaneously forms, maintains, constructs, and reconstructs the discourse and familiarizes the agents with it. It is a mutual constitution. However, since this process contains an elements of construction, it also contains a potential for change. This perspective is different from a traditional perspective in socialization theory because it recognizes that in the reproduction process of socialization, the possibility of production is created (cf. Mellin-Olsen, 1992, pp. 2-3).

that to function within the school as an institution, the participants – teacher as well as students – are directed towards expressing themselves within the discourse promoted by the institution's tradition.<sup>14</sup> But this discourse does not always 'have room' for the students' (in this case) experiences. This may lead to the construction of a 'private' discourse which "comments on the expected practice of the institution" (Mellin-Olsen, 1991, p. 80, my translation).

The didactical contract is broken in the situations discussed here, but this breach is separated from the official discourse – keeping the 'contract' intact within the exercise discourse. Thereby, the learning taking place through a breach of the contract (cf. Brousseau, 1984) is disconnected from the official teaching-learning activity.

The example illustrates that a critique of the model based on an (implicit) comparison between calculations and reality *can* take place as a part of the students' activity, but it also illustrates that this is part of an activity which is marked out from the task-orientation.

Thus, the distinction between the informal and the task-directed activity is twofold. On the one hand, it is – as the name says – a distinction between the 'official' classroom activity (task) and the students' 'unofficial' activity. On the other hand, it is a distinction between activity where the students relate the mathematical calculations and modelling to reality and activity where reality is only included to the extent it supports the mathematics on the agenda.<sup>15</sup>

The two types of activity constitute each others' 'other' – but they each also give form to the other. The students' perception that the content in the exercises is meaningless (except as an element of institutionalized schooling) helps to shape the informal situations.



Figure 2. A twofold picture.

---

<sup>14</sup> For a discussion of the 'exercise discourse', see Mellin-Olsen, 1991. Through analyses of some teachers' speech and the institutional conditions, he reconstructs the 'exercise discourse', which has "the journey and the journey's speed as a kernel. The journey was a journey through the lines of exercises with the exam or the next grade level as the goal. The subject related stops were few." (p. 186, my translation)

This is very clearly exemplified in Susan's utterance in 1, C, 4 – see page 10. In general, it is my impression that Susan is highly disciplinary dominant, because she keeps the students on task and direct the progression of the activity. In other words, Susan seems to take responsibility for maintaining the exercise discourse.

<sup>15</sup> Cf. Jungwirth (1991). I have even referred to it as a 'virtual reality' which occurs in the task-directed activity (Christiansen, 1994). Voigt describes it thusly:

"Particular aspects of a task will become relevant for the official classroom discourse only in so far as they serve the goal of constituting certain mathematical concepts in the classroom, or of intuitively familiarizing the pupils with a mathematical view, etc." (1985, p. 95)

Simultaneously, the informal activity – through an implicit rejection of the mathematics in the exercises – contributes to the shaping of the task-directed activity as its opposite.<sup>16</sup>

As long as the critique of the meaningfulness of the calculations is restricted to the informal activity, it will imply a rejection of the exercises' suppression of reality to mathematics in the informal activity combined with a submission to the same suppression of reality in the task-directed activity.

For the critique of the model to be accentuated without a total rejection of the calculations, the task-directed activity must break the informal activity's monopoly on meaningfulness.

It is beyond the scope of the present text to suggest a way to alter the relation between the two types of activity (and modes of discourse) discussed here. But a first step in this direction might be to attempt to understand the (re-)production of the discourses.

The distinction of discourses/activities is maintained by the students and teacher together, as the following example indicates. It is almost as if they refuse to allow the exercise discourse to invade their critical perspectives on the content.

### Maintaining the distinction of activities

After rejecting Ann's idea as irrelevant to the exercise, the students turned to exercise number two. They experienced some problems and called Bente over to help. Susan and Bente engaged in a discussion, which Ann interrupted after some time with her original idea discussed above. They talked about this for a while, mainly joking:

- Su: no, you can see in just 20 years, how much it can suddenly vary from what one really has expected .. it's really (...)
- 175 Jo: we could calculate, when the snake in,  
in the garden of eden, lived
- B: what you're saying  
[Laughter]
- To: to the time and exact minute
- 180 y: it's 1918
- Jo: we could use the carbon 14-method

(1, D, 173-181)

Bente tried to turn this into a discussion concerning the limited domain of the model, thus manifesting the critique implicit in the jokes:

---

<sup>16</sup> A parallel can be seen in the relation between mathematics and ethnomathematics: "... can one not argue that not only is formal mathematics desired as the 'other' of ethnomathematics, but also that it is only through formal mathematics that ethnomathematics can recognize itself?" (Ensor, 1993, p. 138)

- B: yes .. but what you're saying here, it really has something to do with, that one may use such models over a certain number of years, but then, .. there are limits to, how far one (...)
- 185 To: one can use it, one can then  
185a (...) use from here and to 68
- Jo: one couldn't even really do that
- B: no but he really actually uses them up to year 2050 .. and if one then as Ann just tries to calculate 50 years back in time ..
- An: and then there are no people on earth
- 190 B: (...) .. so that says something about, how much one may (...) trust these simple models,  
y: yes
- (1, D, 182-192)

Bente probably left at this point, since she can not be heard on the recording afterwards. Susan turned immediately to the next exercise:

- Su: well .. then that's that one,
- (1, D, 193)

Susan's comment is a call for a return to the exercises. This is accepted by the other students. It marks the entire discussion of Ann's idea as informal activity, which is underlined by the absence of references to the exercise throughout. Thus, Susan's comment serves to dispose of the preceding exercise and move on to the next, but it also serves to dismiss the entire discussion concerning the limitations of the linear model.

As a result, the discussion of the model does not lead to any revisions of the previous activity or the achieved answer, nor does it appear to effect the successive activity in the next exercise – though this would have been most useful. This is similar to the lack of effect Ann's idea had when first mentioned. Still, there is some difference between the two situations in which Ann's idea was discussed.

The first situation (transcript on page10) was task-directed, and Ann's idea was basically dismissed on this account, as it was not considered relevant. As I pointed out earlier, Ann's idea would not be funny or relevant to their discussion if it did not imply a comparison of their calculations to reality. Thus, it is crucial to her idea that the content departs from the limited perspective of the exercise. However, this aspect was discarded together with the dismissal of her implicit call for engaging in informal activity.

The second situation (transcript above) was marked in the fact that the exercise had been solved and that Bente was present. Ann brought up her idea again, and this time Susan accepted this call for informal activity – perhaps because it got her out of her discussion with Bente. At first, Bente treated Ann's explanation as if it belonged

to the task-directed activity, as she took it for a question rather than an assertion: "what is it, you're asking, Ann" (1, D, 157). However, her laughter following Ann's explanation of the idea indicates that she by then had accepted the change in situation.<sup>17</sup> Thereby, a time in between two exercises is constructed, where the informal activity is accepted.

This time, the activity does not appear to have been interrupted by a student's call for returning to some task-directed activity. It was Bente who interrupted the joking by trying to generalize the students' reflections and make them manifest (line 182-184). The students appear to have accepted this inclusion of their reflections in the formal activity (line 185-186), but the separation from the exercises was maintained. This was not the students' sole achievement but was obtained through inarticulately marking the two discourses as separate. Thus, it is not just the students who close off the informal activity by turning to the next exercise (line 193) and thereby returning to the exercise discourse. Instead, Bente's last statement before leaving the group can be seen as a closure of the informal activity.

When Bente formed the conclusion of the limited validity of the linear models (line 190-191) she closed the discussion, which is emphasized by her departure from the group. Her statement is interesting in the light of the circumstances, as it to some extent contradicted the setting. She concluded that the models in the exercises were unrealistic, which in other circumstances would have been a call for revision of the models. But this took place in a mathematics classroom where the students had been requested to work on some exercises assuming the very same models that were criticized. The students *could*, in theory, have pursued the content of Bente's statement and dismissed the exercises' approach, but this would imply opposing the structuralization of the particular setting. This is discussed further below.

It is in this respect that Bente's statement became a closure of the informal activity and thereby also a marking that the discourse where the content of her statement is meaningful is distinct from the exercise discourse.

Her departure from the group underlines this. If she had wanted the students to follow up on her statement as part of their task-directed

---

<sup>17</sup> The interchange (1, D, 158-161) goes as follows:

An: I say, if you divide 3.5 billion by 70 million

B: yes

An: then you'll get the number of years .. there has been people on earth,

B: [laughs] yes .. you do, .. if that model, it



activity, she would have connected it to the exercise formulation. This becomes evident in comparisons with other situations, but will not be further argued in this article.

A similar indication that the reflections are exterior to the task-directed activity can be found in other situations. In the second day of the course, John pointed out to Bente the lack of correspondence between two models:

- 270 B: yes, there is a rather large difference, huh!  
Jo: yes, ugh (...)  
B: yes ... it perhaps doesn't quite fit (all of it?)<sub>1</sub> .. did I get all of your  
272a home-work!  
?: yes [*Bente leaves*]  
An: what do you approximately get in the first one! .. 4.4!  
(2, A, 272)

Here, John's reflections appear to be dismissed with some hand-waving and a very general statement. They are further diminished by the focus on something of great importance in the classroom: the homework. Again, it serves to mark the reflections as being outside of the official activity – which is, in this case, the exercise discourse, with unrefutable clarity.

The point is *not* that the teacher limits the reflections. Indeed, she is an important promoter of making critique 'public' in the classroom<sup>18</sup>, and she initiates the idea that a critique of one model leads to considerations which form the basis for the next model.<sup>19</sup> Rather, the point is that both the teacher and students play their part in maintaining a distinction between task-directed activity and informal activity, and reflections on content belong only in the latter.

---

<sup>18</sup> For instance, she asked the class: "[...] it fits rather well with the years from 1950 and to 86<sub>1</sub> ... how far back, would you actually think it will, perhaps be reasonable to go with this model<sub>1</sub> ... do you have any view on that<sub>1</sub>" (5, G, 3-5).

<sup>19</sup> For instance – in 3, C, 117-136 – Bente asks if the students think that they would obtain a better model than the two linear approximations based on part of the data (see figure on page 11) if they used all the data (implying: in a new linear approximation). David says no because the plotted data cannot be approximated by a straight line. Bente asks if there is another possibility than a linear model. Ursula suggests using exponentially increasing functions instead.

In a later incident, they have worked with an exponentially increasing function which turned out to fit the data extremely well. Here, Bente initiates a progression towards the logistic model by questioning the reasonableness of continued population growth. Ursula has just said that the graph would become steeper and steeper. Bente (indirectly) questions this: "yeah, we can say that<sub>1</sub> .. we'll simply continue that way into all .. future<sub>1</sub>" (5, G, 99). Ian catches on and says that it is not certain that population will continue to increase – it could level out or even decrease (5, G, 100 + 102 + 104 + 107 + 109). This was followed up the next day in the introduction of logistic growth.

Reflections on the reasonableness of the exercises' content are thereby characterized as the 'other' of working on the exercises. If this is not challenged – as indeed it was later in this same course – it also characterizes a formal discourse where exercises are done either for their own sake or for some other opaque reason, and an informal discourse where things make sense but which is to be barred from the school activities. It makes a distinction between what makes sense and what belongs to the school's formal discourse – and thereby hinders critical reflections on the reasonableness of the calculations.

## Discourse formation – and setting

I have stressed how the separation of the two types of activity and corresponding modes of discourse are maintained in an interplay between teacher and students. Since the exercise discourse is not my construction, there may be reason to believe that it is derived from or determined by the framing of the activity by institutionalized schooling. Indeed, Voigt concludes that "mathematics education is stereotyped" (1985, p. 92).

However, it is not so stereotyped that it is impossible to change. In another course on modelling (the 'Øresund course'<sup>20</sup>) – and to some extent also in the latter part of the course referred to in this text – the exercise discourse was far from dominating, and at times even absent.<sup>21</sup> This was the case despite the normal dominance of the exercise discourse in the mathematics lessons I observed before the course, and despite the fact that there was little change in scene. The students worked in the same classroom, their teacher was no longer the only instructor but she still had the power which comes with giving grades, the schedule still said 'mathematics', and so forth.

This challenges a static view of 'context' as "the bowl that contains the soup" (McDermott, 1993, p. 282). In this view, the context is not shaped by the activity, and the two can be viewed separately. Instead, I suggest a dynamic view, where the activity and the *setting* shape each other.

---

<sup>20</sup> The course consisted in the construction of a (simple) model of the water flow in Øresund in order to gain insight in the consequences of building a suggested link from Copenhagen in Denmark to Malmö in Sweden. It was conducted in a sophomore class at Aalborg Katedralskole. I am thankful to Karin Olesen, the teacher, for letting me take part in and observe the course.

<sup>21</sup> It can also be noted that informal activity was close to absent during the Øresund-course. Instead, the students mainly operated within a 'modelling discourse'. I have discussed this in greater detail in Christiansen (1994).

By 'setting', I refer to the environment within which the learners exhibit their activity, but not simply to the scene as much as to the environment as it is known or perceived by the participants (cf. Wyndhamn, 1993, p. 2). Thus, the setting is continuously *produced* by the participants through their meaning-making activity – the activity shapes the setting – while at the same time the activity is constructed within the frames of the setting.

A certain discourse is but one possible way of addressing the content; a way which is shaped by the requirement to establish and reproduce the official knowledge, but none the less a way which cannot solely be deduced from the institutional scene. The discourse is continuously re-constructed, because the participants experience and interpret the content within this discourse.<sup>22</sup>

The freedom to make a discourse different from the rigid exercise discourse active in the 'official' classroom talk may still be expected to increase when the pressure to attain predetermined results is reduced and negotiation of meaning is promoted (cf. Voigt, 1985, p. 109). This may also explain the absence of the exercise discourse in the Øresund course. Here, the students were to perform modelling on an authentic problem, but they were free to make assumptions of their choice – and did indeed end up with very different models.

### **Summary: The function of informal activity**

My analysis indicates that it is possible to distinguish between task-directed activity, where students work with set tasks in one way or another, and what I have referred to as 'informal activity'. The two types of activity are sharply divided, as is evident both in the use of markers to designate an end to informal activity implying a return to task-directed activity, and in the different discourses.

In informal situations, students create a space for reflections – though perhaps most often implicitly in the form of joking – on the meaningfulness of their mathematical activity and the relation between mathematics and reality. In a way, this allows the students to view the mathematical calculations from the outside, from reality as the students know it.

This means that students *do* exercise critical reflections on the mathematics they perform – and this is one of the skills which is

---

<sup>22</sup> Here, I have been inspired by Voigt's description of the 'interaction pattern' in a mathematics class:

"An interaction pattern represents, on the one hand, a certain solution for the joint treatment of the topic in the classroom discourse, while, on the other hand, the participants experience the topic in a way typical for the pattern of interaction." (1985, p. 95)

strongly encouraged in the teaching of applied mathematics in any meaning of the word.<sup>23</sup> On the other hand, they do this by engaging in activity which appears to be excluded from the 'official' classroom activity – where reality is only included to the extent that it supports the mathematics.

At the same time that task-directed situations, with their limited 'respect' for references to reality, help to shape the informal situations, with their implicit rejection of mathematical considerations, help to shape the task-directed situations as their 'other'.

Thus, informal activity contains both a potential critique of the organization and content of the regular mathematics instruction, and plays a role in maintaining the status quo by sustaining and justifying the separation of perspectives. Metaphorically, I could say that the informal activity lets the steam out when the pressure within the task-directed activity gets too big, but thereby also contributes to the prevalence of the task-directed activity by keeping it from 'blowing up'.

If the sharp distinction between types of activity and modes of discourse could be overcome, then perhaps the distinction between the reality perspective and mathematics could also be abolished. This would encourage a dialogue between the two perspectives – which would promote learning in both knowledge areas as well as critical thinking.

One way to break away from the sharp distinction between mathematics and reality, between calculations and reflections, is to make the exercise discourse less dominant. The experiences from the latter part of the course and especially from another modelling course conducted under similar circumstances indicate that this may be approached by promoting the negotiation of meaning and by reducing the insistence on the achievement of predetermined results – something which fits together well with modelling which is directed by an authentic problem rather than by modelling which is directed towards the goal of reconstituting certain mathematical concepts or views.

*Note:* Iben Maj Christiansen has recently completed her Ph.D. dissertation in mathematics education at the department of mathematics and computer science, Institute for Electronic Systems, Aalborg University, Aalborg, Denmark. The article is based on a small part of this work. The dissertation can be obtained from the author: Tel.: +45 98 10 92 57, Telefax: +45 98 15 81 29, Email: hippo@iesd.auc.dk

---

<sup>23</sup> As well as in the notion of a 'critical mathematics education' – see Christiansen (1994) for a discussion.

## Appendix: Rules of transcription

### 1) B interrupts A or speaks at the same time as A:

A: he didn't say so yesterday but maybe we could do it that way anyway ..\

B: yes he did

A: if we use the information from the other article

To mark that A continues speaking even when B interrupts, A's speak continues. If it takes up more than one line of transcript, I've ended the first line of A's speak with a back-slash.

### 2) Pauses:

, very short (NB! When used to indicate a pause, the comma will be followed by a blank. This is to distinguish from comma used in numbers.)

.. short

... medium

(5 sec.) long pause

### 3) Raising/lowering the voice

he left' Raising the voice

he left, Lowering the voice

### 4) Emphasis

*almost* emphasizing

*well* drawling

### 5) Actions

[*smiling*] manner of speaking

[*walking away*] action; descriptive comments

### 6) Unclear utterances

(...) inarticulate utterance

(any? many?) inarticulate, but probable utterance

A? unclear who is speaking, but it's probably A

### 7) Omitted text

[...] Text left on in the excerpts reproduced here

## References

- Austin, J. L. (1958). Performative-Constativ. In J. R. Searle (Ed.), *The Philosophy of Language*. Oxford, UK: Oxford University Press.
- Austin, J. L. (1961/70). *Philosophical Papers*. Oxford, UK: Oxford University Press.
- Bekendtgørelse (1990). nr. 207. *Bekendtgørelse om fagene m.v. i gymnasiet* (Directive no. 207: Directive concerning the subjects etc. in the gymnasium), j. nr. 21240-90 (Record no. 21240-90). Copenhagen, Denmark: Undervisnings- og Forskningsministeriet, DGHF (The Ministry of Education and Research, The Directorate for the Gymnasium and the Higher Preparatory Courses).
- Brousseau, G. (1984). The Crucial Role of the Didactical Contract in the Analysis and Construction of Situations in Teaching and Learning Mathematics. In H-G. Steiner et al. (Eds.), *Theory of mathematics education (TME), ICME 5, Topic area and miniconference*. Occasional paper no. 54, Institut für Didaktik der Mathematik der Universität Bielefeld, Bielefeld, Germany, pp. 110-119.
- Christiansen, I. M. (1994). *Classroom Interactions in Applied Mathematics Courses: Case Studies in Danish High School*. PhD-thesis from Aalborg University, Text no. R-94-2045a.
- Ensor, P. (1993). Plenary Response to Nick Taylor. In C. Julie, D. Angelis, & Z. Davis (Eds.), *Political Dimensions of Mathematics Education 2: Curriculum Reconstruction for Society in Transition*. Proceedings from the Second International Conference on the Political Dimensions of Mathematics Education, National Education Coordinating Committee (NECC). Cape Town, South Africa: Mathematics Commission, Maskew Miller Longman, pp. 138-139.
- Foucault, M. (1972). *The Archaeology of Knowledge*. London, UK: Tavistock. (Originally published in French under the title *L'Archéologie du Savoir* by Éditions Gallimard, 1969).
- Foucault, M. (1984). *Power/Knowledge*. Brighton: The Harvester Press.
- Jensen, J. F., & Lytje, I. (1993). Om tegn og data. In J. F. Jensen, I. Lytje, & P. Øhrstrøm (Eds.), *Tegn & Data*. FISK-serien 2 (Forskning i Informatik, Semiotik & Kulturanalyse [Research in Informatics, Semiotics, and Culture Analysis]). Aalborg, Denmark: Aalborg Universitetsforlag, pp. 9-24.
- Jungwirth, H. (1991). The Real World behind Real-world Problems – some findings of a micro-ethnographical study. In M. Niss, W. Blum & I. Huntley (Eds.), *Teaching of Mathematical Modelling and Applications*. London, UK: Ellis Horwood, pp. 93-102.
- McDermott, R. P. (1993). Acquisition of a Child by a Learning Disability. In S. Chaiklin & J. Lave (Eds.), *Understanding Practice: Perspectives on activity and context*. Cambridge, UK: Cambridge University Press.
- Mellin-Olsen, S. (1991). *Hvordan tenker lærere om matematikundervisning?* Landås, Norway: Bergen Lærerhøgskole.
- Mellin-Olsen, S. (1992). *Tekster om Sosialisering: Til bruk på nordisk forskningsseminar for fagdidaktikk matematikk*. [A short introduction to a collection of texts on socialization distributed to a seminar on this topic]. Bergen, Norway: Institutt for Praktisk Pedagogikk, Universitetet i Bergen.
- Ochs, E. (1990). Indexicality and Socialization. In J.W. Stigler, R.A. Shweder, & G. Herdt (Eds.), *Cultural Psychology: Essays on Comparative Human Development*, Cambridge, UK: Cambridge University Press, pp. 287-308.
- Undervisningsministeriet (1994). Danish Ministry of Education, Department of Upper Secondary Education [Undervisningsministeriet, Gymnasieafdeling]: *The Danish Gymnasium – General Rules and the Subjects: Executive Order on The Upper-Secondary School, Upper-Secondary Level Courses and the Single Subject Upper-Secondary Examination*. Copenhagen, Denmark: Author.

- Voigt, J. (1985). Patterns and Routines in Classroom Interaction. *Recherches en Didactique des Mathématiques* 6(1), pp. 69-118.
- Wyndhamn, J. (1993). *Problem-solving Revisited: On school mathematics as a situated practice*. Linköping Studies in Arts and Science 98. Linköping, Sweden: Department of Communication Studies, Linköping University (PhD-thesis).
- 

## Elevers 'uformelle aktivitet' i matematikundervisningen

### *Resumé*

Hovedpointen i artiklen er, at eleverne har diskussioner i situationer præget af, hvad jeg har kaldt *uformel aktivitet*, og at disse situationer spiller en særlig rolle i forhold til den organiserede undervisning.

Påstandene underbygges med observationer fra et forløb i matematiske modeller, afholdt i en 1. g i den danske gymnasieskole.

Ud fra elevernes kommunikation er det muligt at udskille forskellige typer aktivitet. Jeg beskriver en type aktivitet, hvor eleverne diskuterer indholdet i opgaven, og hvor de er klart styret af at skulle besvare eller løse den pågældende opgave. Dernæst beskriver jeg en tilsvarende opgave-styret type aktivitet, men hvor eleverne diskuterer, hvordan opgaven skal gribes an.

Den uformelle aktivitet har en noget anden karakter, idet den netop ikke er rettet mod opgavebesvarelsen. Dette ses også i elevernes sprogbrug. Det er interessant, at eleverne bruger disse situationer til at snakke om opgavernes indhold, men hvor de tager fat i de ting, der støder mod deres intuition og hverdagsviden.

Man kunne så forestille sig, specielt når der er tale om et projektlignende forløb, at eleverne ville bringe disse kritiske overvejelser i spil, når de vender tilbage til den opgave-rettede aktivitet. Men det er ikke tilfældet. De to typer aktivitet udelukker hinanden. Det betyder, at den kritiske refleksion bliver forbeholdt den uformelle aktivitet og derved er udelukket fra at blive bragt i spil på opgaven.

Derved er den uformelle aktivitet, sit kritiske potentiale til trods, med til at opretholde den organiserede undervisnings opgavediskurs, herunder dennes pseudo-virkelighed, hvor et udsnit af den virkelige verden er underlagt matematikkens regler. Den uformelle aktivitet udgør med andre ord en slags overløbsventil for det tryk, der opstår, når matematikken ikke kan forenes med elevernes hverdagsviden.

Lærer og elever er fælles om at opretholde adskillelsen mellem uformel og opgave-rettet aktivitet. Dette underbygges med nogle konkrete eksempler. Der er således ikke tale om, at læreren forsøger at begrænse elevernes refleksion over opgavens indhold, snarere tvært imod, men det bliver på forunderlig vis både en præmis for og et resultat af kommunikationen mellem lærer og elever, at refleksioner hører den uformelle aktivitet til.

Jeg slutter med at overveje, om det er muligt at bryde dette mønster gennem en anderledes organisering af undervisningen.

***Forfatter***

Iben Maj Christiansen, amanuensis ved afdelingen for Matematik og Datalogi, Institut for Elektroniske Systemer, Aalborg Universitet, Danmark.

***Adresse***

Rum E2-101, Fredrik Bajers vej 7 E, DK-9220 Aalborg Ø  
Email: hippo@iesd.auc.dk

---