Inductively Teaching Quantum Field Theory to Students without Quantum Field Theory

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Introduction and Justification

In this project, I contributed a guest lecture to the course Introduction to Nuclear and Particle Physics, part of the Bachelor's programme in physics at the Niels Bohr Institute. This year's cohort consists of fourteen students. While the Bachelor's programme is in general in Danish, this class is in English, due in part to the lack of Danish-language textbooks for the topic. The primary lecturers for the course have developed it extensively, based in part on their own experiences in UP. However, a persistent problem is that the students report that the material feels too remote, in a way that makes it hard to retain. In general, this type of course is unusually challenging because it involves teaching students material without being able to fully explain why this material is justified. This is because the subject matter of this course, nuclear and particle physics, can to a large extent only be fully explained with the mathematics of quantum field theory, a topic which students are not prepared for until later in their Master's degree at the earliest, and which some students will never study at all. Instead, the course must give intuitive explanations and plausibility arguments, forcing the students to take some number of claims on faith. Such claims are harder to retain, and reduce student feelings of competence, damaging self-efficacy (Bandura 1997).

I was interested in addressing this failure of congruence for two reasons. First, as someone who often writes popularized descriptions of quantum field theory in an outreach context, I felt that I could help provide a more satisfying presentation that would nevertheless be at an appropriate level. Second, the problem is parallel to a similar problem in the US educational system, in which students who do not plan to specialize in physics typically take "non-calculus physics classes", which similarly require students to take claims of things derived from calculus on faith before they have learned the requisite mathematics. As such, I felt that addressing this problem would give me valuable insight into something I might face were I hired in the US after my contract here.

Planned Intervention

My guest lecture was the last lecture of the course, which is typically devoted to Beyond the Standard Model Physics. This informed the topics chosen: my goal was to introduce some of the more common speculations for physics beyond the Standard Model, but to do so in a way that also increased student feelings of competence with respect to the quantum field theory concepts that underlay earlier lectures. I aimed to structure the lecture using principles of active learning, as there is substantial evidence that it aids students in acquiring the conceptual underpinnings of professional scientific thought (Freeman et al. 2014). More specifically, I planned to include elements of inductive teaching (Prince and Felder 2006), which should specifically address student feelings of competence by putting them in a scientific situation.

The structure of the teaching session was a one and a half-hour lecture, followed by a two-hour exercise session. My plan was to begin by reviewing the topic of Feynman diagrams, an aspect of quantum field theory which the students have learned partially, but not in full detail, earlier in the course. I would then have the students complete a short in-class exercise using Socrative on a straightforward aspect of these diagrams, using the conservation of energy and momentum to find what momenta different particles have. After reviewing the results, I would move on to a similar-seeming exercise for a diagram with a "loop", a circular path inside the diagram. Here there is an important subtlety, that there is no one unique answer. Instead, there are many options for the momenta, which is why these diagrams involve integrals. This aspect of the lecture was intended to be inductive: rather than being told what the procedure is to deal with loops in Feynman diagrams, the students would have to try to use their experience and knowledge to figure out what the correct procedure is. My expectation was that more advanced students might notice the issue, while less advanced students would try to solve the problem the same way they solved the earlier one. This would result in many different answers among the different students. By showing the different student answers and comparing them, I planned to illustrate the issue directly, and in the institutionalization step demonstrate the solution, that an integral was required.

With that explained, I could then present the students with a method for estimating the value of these integrals (as actually performing them is a bit beyond what would have made sense to teach in that context). I would walk them through an example of this method. In doing so, I would introduce another inductive question for the students. The diagram that I show them is divergent, that is, it formally gives an infinite result. I would then ask them how to interpret this, giving a multiple-choice question to be answered on Socrative. Finally, I would walk through the different possible answers, discussing each with the students.

Finally, I would use this discussion to lead in topics in physics beyond the Standard Model that are related to the problem of the particular divergent integral I showed the students, then to other topics. Since the class was comparatively small, I planned to have the class discuss some of these topics in plenum, time permitting, to empower the students to consider their own opinions, as several of the topics are still controversial among practicing physicists.

The exercise session would then serve to consolidate and assess the topics covered in lecture. The questions given gave additional practice on the concrete skills introduced in the lecture, and motivated the students to reason about what traits make a physical theory incomplete. The primary structure of the exercise session is of a study café, where the students work on questions in small groups and I answer questions. However, the exercises were designed to include a few more conceptual questions for the students, and for these I planned to encourage different groups to discuss with each other if their answers differ.

Both the Socrative question about the diagram with a loop and the Socrative question about the divergence could be loosely thought of as “productive failure” methods, where the students are expected to attempt a task and fail before instruction. Due to time constraints my plan omitted a few elements that are considered important in the research literature on this, such as group work (see Sinha and Kapur 2019). However, overall I thought that my plan would avoid most of the pitfalls described in that reference, as the lessons would both inspire the students to generate multiple solutions and would quickly afterward provide explanations and analysis of the various “failure modes”. Also, while the exercises during the lecture were not to contain group work, the exercises afterwards are typically addressed in groups.

Applying the Intervention

Lecture attendance had decreased slowly but steadily throughout the course, so that there were only ten students present. Motivation initially seemed low, with the students appearing a bit tired and listless. Some of this seemed to be a general situation (based on observation of a session taught by one of the primary instructors the week before), but some might have been due to a lack of rapport since I was a new guest teacher they weren’t familiar with. This impression continued through the first two Socrative exercises. Nine students logged in to the Socrative, of those one did not do either of the first two Socrative exercises, one skipped the first exercise and another skipped the second. The students were given five minutes for each Socrative exercise. For these first two exercises, roughly four students submitted early, within the first minute or two, while the remainder took most of the five minutes.

In the first exercise, as expected, almost all students who submitted were able to solve the problem (with some minor mistakes). The second exercise unexpectedly spawned three types of answers: two students gave answers that were correct up to the limits of what they had already learned (they attempted to solve the problem but were unable to solve all equations, so they left the equations in the answer box), two gave valid but incomplete answers (they picked a particular solution of the equations), and three gave invalid answers (they did not respect conservation of momentum). Due to the inductive nature of the question a range of answers was intended, but the clustering was unexpected, since they completed the exercise largely silently and thus did not discuss with each other much.

After the second exercise, the students began to ask more questions, though still sporadically. All nine students participated in the third Socrative exercise, and all contributed early in the time period, so that I cut off the exercise after two minutes when all students had submitted. Of the multiple-choice answers, two were correct (with some ambiguity in interpretation) and two were incorrect (one with some ambiguity). Eight of the students chose one particular one of the correct answers, one chose in addition an ambiguously incorrect answer, while the ninth student chose an unambiguously incorrect answer. None chose the other correct answer. This provided a good opportunity to discuss the implications of the different possible answers, underscoring how intuitively surprising the “missed” correct answer is.

The class went comparatively quickly, so that there were roughly fifteen minutes after I finished the lecture. I then offered that the students could ask me questions about other theories of physics beyond the Standard Model that they had heard about. While they took some time to warm up, eventually several ended up asking questions.

There was then a break before the exercise session. The beginning of the exercise session was disrupted a bit. The students had a final project due in two days, and wanted to spend some time asking last-minute questions of the primary instructors and discussing with their project groups. However, the students did eventually address the exercises, some alone and some in small groups. This part of the session “woke the students up” the most. Everyone addressed at least the first few exercises, and several finished all of them.

The students found some ambiguities in how the first two exercises were phrased. I clarified those ambiguities, but the result was still that these exercises were more difficult than I had planned them to be, with one in particular a bit overambitious. Still, all students attempted these exercises.

The later exercises were closer to methods the students had used earlier in the course, but had more open-ended or conceptually inspiring answers. One introduced students to a problem with the quantum theory of electromagnetism, in which the theory gives inconsistent results at very high energies. Students were intrigued that this was a problem they had not heard of before, and impressed at the energy (to quote one, “that’s a fucking big number”). Later problems had students use similar techniques to estimate the mass at which scientists expected to find the Higgs boson, and the lifetime of the proton. The students seemed to enjoy these exercises: one commented that they were much more fun than the course’s usual exercises, and that he had planned to leave the exercise session early but stayed because he found the questions interesting.

Afterwards, I discussed with the primary instructors. They were overall pleased with the lecture, and one in particular commented that it seemed geared at the right level for the students (which was not guaranteed given the advanced subject matter).

Outcomes and Reflection

Overall, the teaching session was a success, though with caveats.

The intervention genuinely did seem to address the core motivation, that of giving the students more of a feeling of self-efficacy with respect to the course content. The material on Feynman diagrams seemed to contribute to this, but a much stronger contribution were the later exercises. This makes sense, as these exercises concretely showed that the skills the students have learned can be used to answer not merely “textbook” questions, but to speculate in an informed way about the future of physics. I do not currently have evidence as to whether this improved attitudes about the course itself (evaluations were due shortly before this report was submitted), but at minimum it seemed to improve the students’ attitudes in general.

Going into more detail about individual moves, the quietness of the students at the beginning was a bit of a negative. Since the students asked more questions after the Socrative exercises, and especially more after the discussion at the end, it might have been better to begin with some kind of verbal “icebreaker” to get the students comfortable talking with me. It also may have improved things to have more closely followed the more successful examples of “productive failure” in the literature, and have the Socrative exercises instead pursued in small groups. My pedagogical supervisor suggested asking students to explain the logic behind their Socrative answers rather than just explaining them myself, this also seems like it would have gotten the students more involved and active earlier in the lesson.

Both the Socrative questions and the exercise session questions ought to have been framed more clearly, as some of their ambiguity was tangential to the teaching goals and slowed the students down. Overall these are the kinds of "ordinary bugs" present in any first version of a lecture that typically get revised in later versions. If elements of this lesson are employed later (more likely by the course primaries than by me, at least in the context of this particular course), then these would be easily fixed.

The inductive activities in particular (the second and third Socrative questions) went particularly well, in that the students had a range of responses that allowed many important conceptual points to be directly linked to their work. This seems like the mark of well-posed inductive questions, and I am happy with how they turned out.

One of the instructors was curious as to why I used a five-minute limit for the Socrative questions, as he usually does Socrative questions without an explicit time limit. I implemented a time limit primarily because I would be more comfortable as a student with an explicit time limit, but I can't tell whether it made a positive or negative impact in this case.

Further Directions

For a variety of reasons, I am unlikely to perform this particular guest-lecture in future, but I will encourage and assist the course primaries to implement aspects of what I introduced if they are interested.

More generally, I can take a variety of lessons from this project to my future teaching in general.

Guest-lecturing itself can be challenging: one has a weaker rapport with the students and doesn't know all of the relevant didactical mores. Some kind of "warmup" where the students get to ask questions seems like it could improve those situations. I don't know how much more guest-lecturing I will be doing (since it tends to be very heavily tied to pedagogy courses like this one), but if others guest-lecture for me this might be a good piece of advice for them.

"Productive failure", and inductive teaching more generally, seems to be a powerful way to get students both to feel like actual scientists and to internalize important conceptual leaps. I observed that these methods work better when students can give answers reflecting several different misunderstandings, and worse when not accompanied by some form of student discussion or group work. More generally, whether students feel like they are learning something from exercises or merely practicing can have an enormous impact on student motivation, so exercises that illustrate unusual examples can be extremely valuable.

Timing is still a tricky question: it is unclear to me how much time is best to allocate to Socrative exercises and how to structure them. This experience at least clarifies the questions I need to answer for myself on that front.

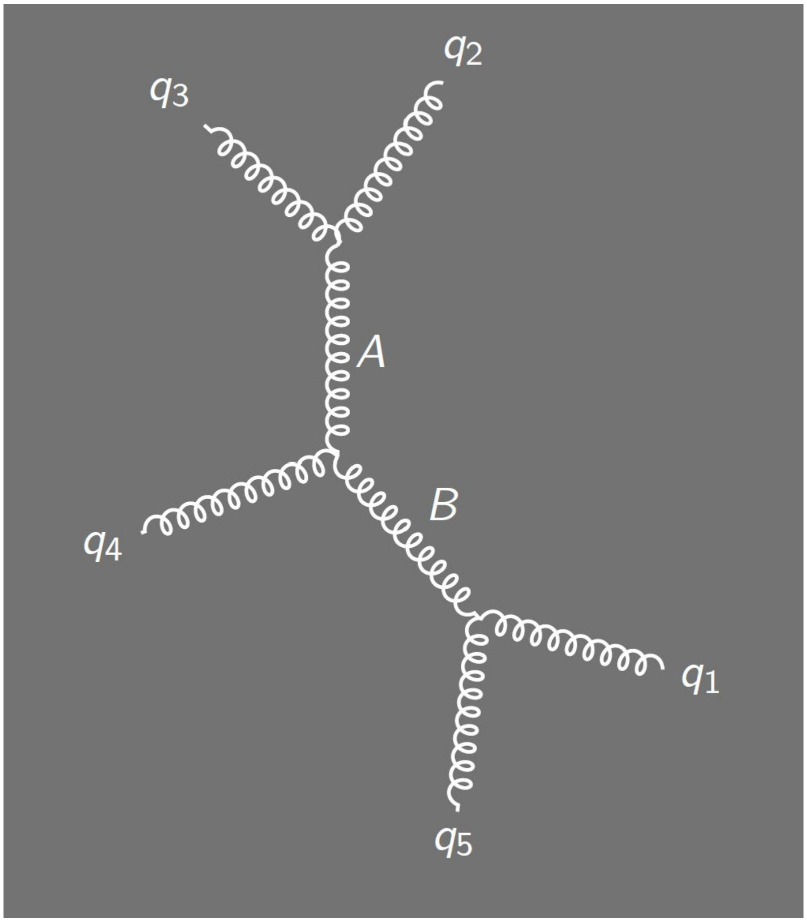
The lecture ended early, so time and resource constraints were not much of an issue. However, this was largely due to the fact that this was a "bonus" topic not on the exam. If I had been required to cover specific material, time constraints may have been an issue, and it might have been substantially more of a problem that students were distracted by their upcoming project deadlines. This is something to pay attention to in my teaching in future.

In general, I intend to use more inductive teaching in future. This may take a form like that in this lecture, with Socrative and PowerPoint slides. With more mathematical courses, blackboard lectures will likely be more appropriate, but Socrative may still be a useful tool. I will have to experiment.

Appendix

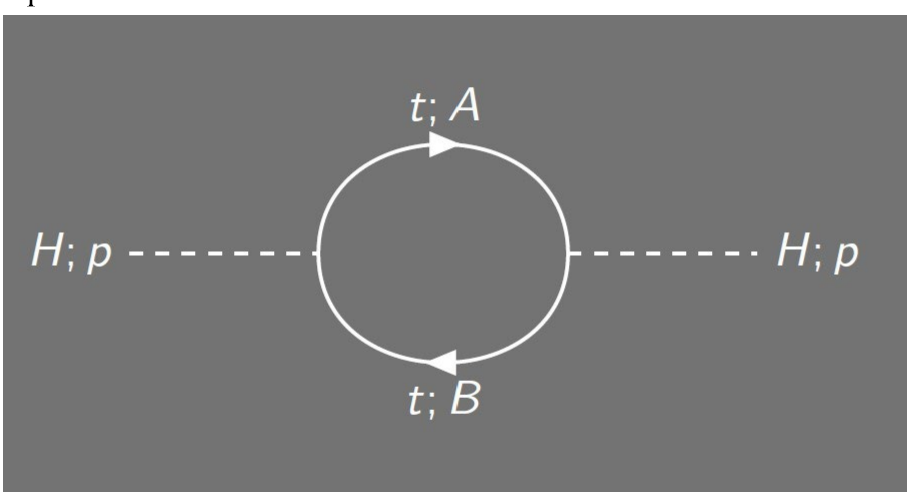
Appendix A: Socrative Questions

This appendix contains the Socrative questions given in the course.
First question:



q_1 and q_2 are incoming, q_3, q_4, and q_5 are outgoing.
Find momenta A and B

Second question:



p is incoming (and $-p$ is outgoing)
Find momenta A and B

Here, as mentioned in the text, there were three clear classes of answers given by the students. Representative examples are given below:

- A=p, B=-A=-p
- A=B=p/2
- A=p+B B=A-p

Third question:
What does it mean that we got an infinite result?

A. The cross-section is infinite

B. The mass of the Higgs is minus infinity

C. We did the wrong calculation

D. Our theory is incomplete

Appendix B: Sample Exercise Questions

This appendix contains two examples of exercise questions that the students found especially engaging. They are reproduced here with the original LaTeX code:

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{\bf Exercise 3}\\
It can be shown that the coupling constant for QED is, to a good approximation, given by
%
\begin{equation}
\alpha_{\textnormal{QED}}(Q^2)=\frac{\alpha_{\textnormal{QED}}(\
mu^2)}{1-
\frac{1}{3\pi}\alpha_{\textnormal{QED}}(\mu^2)\ln\left(\frac{Q^2}{\
mu^2}\right)}\,,
\end{equation}
%
where $\mu^2$ is a low energy scale where we have measured $\alpha$.
Take $\mu^2=1\textnormal{MeV}$ and
$\alpha(1\textnormal{MeV})=1/137$, the fine structure constant.
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What is the maximum value this coupling constant can reach? When does it reach this value? Should you be worried?

{\bf Exercise 4}\\
Recall that W bosons, unlike photons, are massive. While photons, as electromagnetic waves, can have only transverse polarizations, W bosons can be longitudinally polarized, like a sound wave. Without the Higgs boson, the differential cross-section for scattering of longitudinally polarized W bosons would look like,
%
\begin{equation}
\frac{d\sigma}{d\Omega}\sim
\frac{G_F}{\sqrt{2}}E^2(1+\cos(\theta))\,,
\end{equation}
%
at high energies, where G_F is the Fermi constant,
%
\begin{equation}
G_F=1.1663787(6)\times 10^{-5} \textnormal{GeV}^{-2}\,.
\end{equation}
%

Estimate the maximum energy to which this expression is valid, and compare to the energy of collisions at the LHC and the mass of the Higgs boson.