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Constructive Alignment in the Course "Mathematics and Optimization"

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The present manuscript, written in July 2011, is my final pedagogical project for the *Higher Education Teaching Programme* ("Adjunktpædagogikum") 2010/2011, which is a one-year programme for assistant professors and postdocs organized by the Department of Science Education, Faculty of Science, University of Copenhagen.

The project is aimed at reporting on, analyzing, and developing my own teaching in *Mathematics and Optimization*, which is a 7.5 ECTS course for students at the Faculty of Life Sciences (abbreviated LIFE) that was held (in Danish) for the first time[1] in block 3, 2010/2011.

Mathematics at LIFE

I imagine that all teachers, no matter the subject which they teach, encounter many of the same problems and challenges in their teaching. One might think that mathematics, being so logically structured as it is, would require a minimal effort to teach. Indeed, mathematical textbooks are renowned for proving and explaining every little detail. However, it seems to be an unfortunate fact that many students have difficulties comprehend-

[1] While it is true that Mathematics and Optimization is officially a new course, it is based on the predecessor *Mathematics and Planning*. This course was held for the first time in 2007/2008.

ing mathematics. This seeming contradiction was a topic of reflection as early as in 1908 in Henri Poincaré's[2] book on scientific methodology:

> *"One first fact must astonish us, or rather would astonish us if we were not too much accustomed to it. How does it happen that there are people who do not understand mathematics? If the science invokes only the rules of logic, those accepted by all well-formed minds, if its evidence is founded on principles that are common to all men, and that none but a madman would attempt to deny, how does it happen that there are so many people who are entirely impervious to it?"*
>
> Henri Poincaré (Poincaré 1908, p. 47)

I believe that mathematics is as easy, or as difficult, as any other subject, but it certainly depends on the way it is presented and taught.

Until 2007, I was employed at mathematics departments at the universities of Aarhus and Copenhagen, where I taught courses in abstract mathematics primarily for math students. Teaching mathematics to math students is, in some sense, easy since they are usually quite motivated, and since the material is expected to be presented in the same traditional and rigid way, going through definitions, lemmas, propositions, and theorems (with rigorous proofs). In my experience, the challenge of teaching math students tends to be more mathematical (e.g. answering perceptive questions) than pedagogical (e.g. figure out how to present a topic).

Since 2007, I have been employed at LIFE. The students at this faculty are focused on topics such as animals, environment, health, agriculture, economics, forests, and biotechnology, and certainly *not* abstract mathematics. Such students see mathematics only as a tool, and their motivation (if any) for learning it lies in the desire, or need, for a better understanding of the more technical aspects of their main subject. Teaching mathematics to this kind of students is, in my opinion, a considerable, important, and interesting pedagogical challenge.

In order for a mathematics course at LIFE to be successful it must, in my experience, be based on concrete examples which relates mathematics to real life problems of relevance for the students in the class. The course Mathematics and Optimization is developed with this is mind, for example, a typical problem of interest would be as in figure 5.1.

[2] Jules Henri Poincaré was a French mathematician and a philosopher of science who lived 1854–1912.

A farmer wants to grow potatoes (P) and tomatoes (T) in some combination on his field, which has an area of 10 acres. He must consider the following restraints:

Profit:	1 acre of potatoes $\sim$ 3000 DKK 1 acre of tomatoes $\sim$ 2500 DKK
Contract:	Must produce at least 2 acres of tomatoes
Workload:	1 acre of potatoes $\sim$ 2 hours/week 1 acre of tomatoes $\sim$ 0.5 hours/week

The farmer can spend up to 12 hours per week on cultivating his land. Thus, in order to optimize his profit, the farmer needs to solve the following problem:

$$\begin{cases} Q(P,T) = 3000P + 2500T = \text{Max!} \\ P + T \leqslant 10 \\ T \geqslant 2 \\ 2P + 0.5T \leqslant 12 \\ P,T \geqslant 0 \end{cases}$$

Fig. 5.1. A typical problem of interest in the course Mathematics and Optimization.

To actually solve this problem, the students need to learn about Dantzig's simplex algorithm.

Organization of the Course

Mathematics and Optimization is a small course at LIFE with 10–15 students; in 2011, twelve people signed up. The course is organized somewhat traditionally. Lectures, problem sessions etc. were divided between the following three teachers, according to a detailed teaching plan.

- Henrik Holm (HH) $\sim 50\%$,
- Henrik Laurberg Pedersen (HLP) $\sim 25\%$,
- Thomas Vils Pedersen (TVP) $\sim 25\%$.

The weekly course activities were as follows:

Time	Tuesday	Thursday
8–10	Lecture	Lecture
10–12	Problem session	Problem session
13–17		Project

Each week the students were given a detailed work sheet, like the one in Appendix A, containing, among other things, a reader's guide to the textbook and a list of exercises to be solved. In the next section, we shall describe how the students worked, and how they were intended to learn, in the various types of course activities.

Course materials (slides, projects, solutions to exercises, work sheets, syllabus etc.) were distributed via the course's homepage.

After the course had ended, the students evaluated various aspects of it via LIFE's standard online questionnaire. I followed up on this evaluation with an interview of the class which took place on 5 April 2011 from 9–11 am.

Planning for Constructive Alignment

Constructive alignment – which is devised by Biggs and described in Biggs & Tang (2007) – is a principle used for devising teaching and learning activities, and assessment tasks, that directly address the students learning outcomes. I strove to make sure that the course Mathematics and Optimization was constructively aligned; below is explained how.

Intended Learning Outcomes (ILOs)

When I wrote the outline for Mathematics and Optimization, I made sure that it included the intended learning outcomes for the course[3]. Here are three concrete examples of ILOs:

> After completing Mathematics and Optimization, the student is expected to be able to do the following (within the scope of the course):
> 1. Selecting between optimization methods to find the one which is relevant for solving a given problem.
> 2. Solve concrete optimization problems.
> 3. Give mathematical descriptions of linguistically formulated (simple) real life optimization problems.

[3] As written in the guidelines from the Danish Ministry of Education, I divided the intended learning outcomes into knowledge, skills and competences.

I tried to think about the level of complexity of the ILOs in terms of the SOLO taxonomy[4], and this was actually quite helpful when preparing lectures, exercises etc. For example, the first ILO described above is SOLO 4 (comparing), the second is SOLO 3 (doing algorithms), while the third is more like SOLO 4–5 (analysing and reflecting).

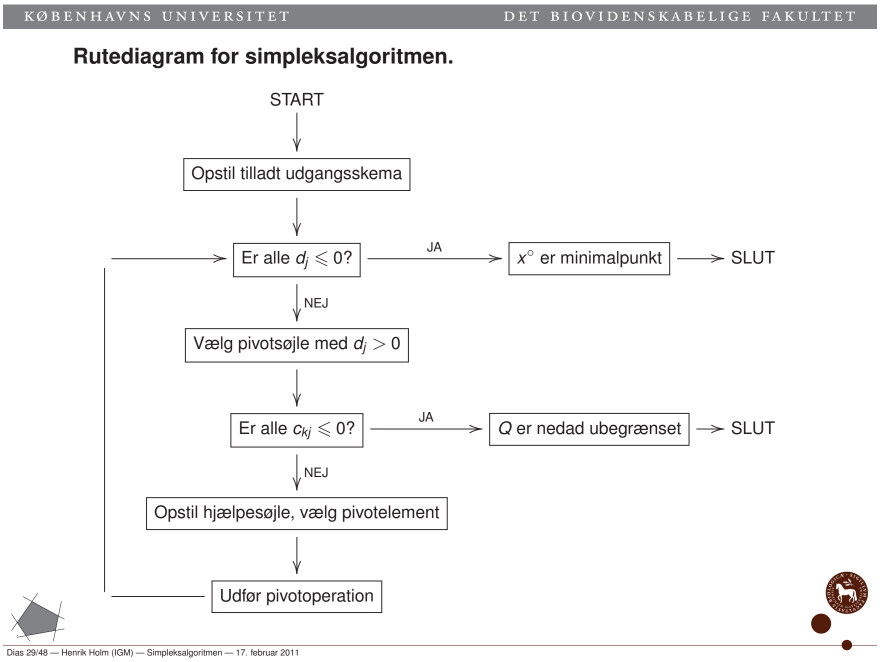
It is also important for the students to know the level of complexity I expect from their understanding. I was actually not particularly explicit about this; instead I hoped that my written solutions to exercises and projects would illustrate the thoroughness and depth of understanding I expected.

Teaching and Learning Activities (TLAs)

I aimed to ensure that the teaching and learning activities I designed for Mathematics and Optimization reflected the intended learning outcomes. To illustrate how, I describe below four different kinds of TLAs from the course.

(i) *Lectures*. I suppose that my lectures were quite traditional. Below is a sample slide from the course which directly address the second ILO stated above ("solve concrete optimization problems"): It demonstrates how to apply Dantzig's simplex algorithm to solve an LP-optimization problem.

[4] The SOLO (Structure of Observed Learning Outcomes) taxonomy was developed by Biggs and Collis (1982), and is described in Biggs & Tang (2007). The taxonomy describes level of increasing complexity in a student's understanding of a subject through five stages: 1–prestructural 2–unistructural, 3–multistructural, 4–relational, and 5–extended abstract.



(ii) *Mini-Exercises*. During lectures, I frequently paused and made the students do a five minutes mini-exercise. The purpose was to keep students active and to facilitate their understanding of the material just explained in the lecture. Below is a sample mini-exercise from the course, designed to practice how to do a *pivot operation* (which is the basic mechanism in the simplex algorithm).

Mini-Exercise B

Do the following pivot operation:

	x_1	x_2	
y_1	-1	0	3
y_2	4	$\boxed{2}$	4
	1	-3	-4

$\curvearrowright$?

(iii) *Problem sessions*. Lectures were succeeded by problem sessions where the students solved a number of problems under my supervision. Below is a sample exercise from the course which directly addresses the first ILO stated above ("selecting between optimization methods"): Part of the exercise is to determine which method to use to solve the given optimization problem (in this case, a variant of the Kuhn–Tucker method, and not, for example, the simplex algorithm).

Exercise K15

Solve the following optimization problem:

$$\begin{cases} Q(x_1,x_2) = 6x_1^2 + 9x_2^2 - 14x_1x_2 + 3x_1 = \text{Min!} \\ x_1^2 + 2x_2^2 + 2x_1x_2 - 4x_1 \leqslant -3 \\ 2x_1 + 3x_2 \leqslant 12 \\ x_1, x_2 \geqslant 0 \end{cases}$$

(iv) *Projects*. The students were divided into study groups of 2–4 members. Every week, each group answered a project in writing. It was no secret that some of the exercises in the projects were similar to those the students could encounter in the written exam. Below is a sample project exercise from the course which directly addresses the third ILO stated above ("give mathematical descriptions of real life optimization problems").

From Project C

Five types of feed for pigs (A, B, C, D, and E) contain two types of nutrients (I and II) in the following doses (units per kg):

Type of feed	A	B	C	D	E
Nutrient I	1	1	2	2	2
Nutrient II	1	2	1	0	4

The prices (in DKK) of the five types of feed are as follows:

Type of feed	A	B	C	D	E
Price per kg	3	8	9	8	14

A breeder has 250 pigs. Each pig must consume at least 0.6 units of nutrient I and 1.2 units of nutrient II per day.

Formulate a mathematical optimization problem which describes how to minimize the breeder's expenses for feed.

Complementarity in TLAs

The complexity and the nature of mini-exercises, exercises, and projects in the course Mathematics and Optimization described above were deliberately very different. For example, mini-exercises were operational whereas project exercises were more structural.

Sfard (1991) discusses how notions in mathematics can be conceived in two fundamentally different ways: structurally–as objects, and operationally–as processes. These two approaches, although ostensibly incompatible, are in fact complementary. Sfard argues that the processes of learning and of problem-solving consist in an intricate interplay between operational and structural conceptions of the same notions.

Exam

In any course, it is crucial that the exam reflects the intended learning outcomes (ILOs) as well as the teaching and learning activities (TLAs). In the course Mathematics and Optimization, I strove to make sure that this was the case. Below is a sample exercise from the written exam, which directly addresses the third ILO stated above ("give mathematical descriptions of real life optimization problems"), and which the students were trained to do in the planned TLAs (such as in Project C described above).

From Exam (Exercise 2)

A farmer can buy three types of NPK fertilizer (types I, II, and III), whose contents of nitrogen, phosphorus, and potassium (measured in appropriate units) per kg fertilizer is as follows.

	Nitrogen	Phosphorus	Potassium
Fertilizer type I	2	2	4
Fertilizer type II	2	4	2
Fertilizer type III	5	3	2

The farmer's crops need certain minimum doses of each of the three nutrients:

	Nitrogen	Phosphorus	Potassium
Minimum dosis	25	20	40

The price for 1 kg of fertilizer type I, II and III is 10 kr, 6 kr, and 10 kr, respectively.

(a) Formulate a mathematical optimization problem which describes how to minimize the farmer's expenses for fertilizer, considering that his crops must have the required minimum doses of each of the three nutrients.

Student Evaluations

As previously mentioned, the students evaluated various aspects of the course Mathematics and Optimization via LIFE's standard online questionnaire. On 5 April 2011, I followed up on this evaluation with an interview of the class. The interview was not recorded, but I took rather detailed handwritten notes. The main conclusions from the students' evaluations are presented below. In the last section, I will comment on them and discuss how they might improve future editions of the course.

I would say that, in general, the students found Mathematics and Optimization to be constructively aligned to a high degree. For example, in the online questionnaire all students agreed or completely agreed with the statement:

> *1.4. I find that the course activities reflected the learning outcomes/competences described in the course outline.*

Responses from the interview generally supported this. Furthermore, the students found the course to be very well organized and logically structured. However, there were three main points of critique:

The Textbook

The interview with the class revealed that the students do not read the textbook as suggested and described in the reader's guide on the weekly work sheets, cf. Appendix A. In fact, they seem to consult the textbook mostly for exercises and their solutions. To acquire the actual theory and the examples, it sufficed for them to read the slides from the lectures. The students told me that they found the textbook to be "too mathematical" and very hard to read. Actually, a colleague of mine (Søren Eilers from SCIENCE) pointed out to me some weeks ago that perhaps the textbook I use is not really suited for LIFE's students.

I encourage my students to skip all mathematical proofs in the textbook, but I certainly expect them to read the statements, algorithms, ideas, examples etc. Apparently, this was generally not the case.

The Projects

During the course, the students handed in seven written projects. The projects were meant to prepare them for the written exam at the end of the course—and I told this to the students already from day one. The projects were not graded on the 7-scale, but formative feedback was given instead.

On the positive side, the students liked the formative feedback on the projects, and no one expressed the desire for an actual grade. They also felt that the projects did, in fact, prepare them for the written exam, in the sense that exam exercises were easier than project exercises.

On the negative side, the students found the projects to be way too technical. Many of them spend hours on mathematical details from which they "learned nothing", and which—to the surprise of some students—were not even tested on the exam. Some students suggested fewer projects with more time reserved for counseling; others suggested simply to downgrade the technical level, as it was too high compared to what was needed to pass the exam.

The Contents

A few students felt that the contents of the course did not really reflect the course description and the introductory lecture (given on 1 February 2011). These students expected the course to be more "case based" and less theoretical. I got the impression that they liked the course, but were a bit surprised of the direction in which it went.

Conclusions

A common theme in the students' evaluations were the mathematical/technical level found in the textbook, projects etc. As mentioned previously, the course is based on the philosophy that it should relate mathematics to real life problems of relevance to the students. It seems like this philosophy is right on the money, but apparently my choice of topics, textbook etc. does not support this philosophy as well as I had hoped. The textbook is quite mathematical, and perhaps not as contemporary in style and exposition as it should be. Since the course ended, I have been looking at alternative textbooks which might better comply with the students' needs. Two alternatives which should definitely be investigated further are *Introduction to Applied Optimization* by Urmila Diwekar (2008), and *Optimization—Theory and Practice* by Wilhelm Forst and Dieter Hoffmann (2010).

In retrospect, I think that I might have downplayed the theoretical/technical aspects of the course in the description and in the introductory lecture – not to "cheat" anyone, of course, but rather to emphasize the more applicational aspects. I will consider to adjust next year's course description accordingly, so that the students will find no "surprises" in that department.

Since Mathematics and Planning – the predecessor of Mathematics and Optimization – was first held in 2007/2008, I have been wondering if the written exam is the optimal way to evaluate the students taking this course. Furthermore, it seems a bit excessive to produce a written exam for only 10-15 people. In the interview with my class, I specifically asked for their opinion on the examination form. Most students actually found the written exam to be well-suited for the course, but one student suggested that a “portfolio exam”—based on the students’ projects – might be more appropriate. I can certainly understand this student’s point of view, that is, to orally examine each student in one of his/her projects (randomly chosen). However, to do so would require some adjustments in the type of problems posed in the projects – some of these are simply way to technical to present orally in a meaningful manner. Adjusting the projects by replacing some of the technical aspects by more conceptual ones would also accommodate the students’ critique on this point.

A Sample Work Sheet

Matematik og optimering
2011
UGESEDDEL 3

Tirsdag den 15/2 2011 kl. 8–12: *LP–problemer og simpleksskemaer* Husk computer!

Program
8–10: Forelæsninger og miniøvelser
10–12: Øvelser

Læsevejledning til [HS]
Generelt kan beviser overspringes! Eksempler med "X" findes i [TVP].
Overheads kan hentes fra kursushjemmesiden.

Sidetal	Emner	Anbefalede eksempler
1–6	Introduktion	0.1, 0.2, 0.3, X1(a-b), X2(a)
7–11	Lineær algebra, konvekse mængder	
14–18	Formulering af LP–problem	2.1, 2.2, 2.3, X1(c-d), X2(b)
19–28	Hjørner og basisløsninger	2.6, 2.8, X4(a-b)
29–38	Basisform, opstilling af simpleksskema	3.1, 3.3, 3.5, 3.6, 3.7, X5(a-e)

Miniøvelser
Miniøvelser (med løsninger) kan hentes fra kursushjemmesiden.

Øvelser
opg. 1 (`LPgraphic`), øv. 3.1, øv. 3.2, opg. 7 (`LPgraphic`), opg. 12, opg. 10 (tjek med `Vertex`).
Skriftlig aflevering til torsdag den 17/2 2011: øv. 0.1 (`LPgraphic`), opg. 11(1)–(5).

Torsdag den 17/2 2011 kl. 8–12 og 13–17: *Simpleksalgoritmen* Husk computer!

Program
8–10: Forelæsninger og miniøvelser
10–12: Øvelser
13–14: Installation af, og øvelser med, `LP-KVL` (ved Thomas Vils Pedersen)
14–17: Gruppearbejde med Projekt B (hentes fra kursushjemmesiden)

Læsevejledning til [HS]
Generelt kan beviser overspringes! Eksempler med "X" findes i [TVP].
Overheads kan hentes fra kursushjemmesiden.

Sidetal	Emner	Anbefalede eksempler
42–49	Pivotoperationer	3.9, X4(f), X5(f)
50–58	Simpleksalgoritmen	4.1, 4.2, 4.3, 4.4, X4(g-h), X5(g)
39–41	Matrixformler for basisform	3.8
60–62	Den reviderede simpleksalgoritme	4.6, X4(i-j)

Miniøvelser
Miniøvelser (med løsninger) kan hentes fra kursushjemmesiden.

Øvelser
øv. 4.1, opg. 13, øv. 4.3 (kun første problem), øv. 4.5, opg. 18, opg. 23(1) (`LPgraphic`).
Benyt gerne `LP-KVL` hvor det er relevant.

Projekt
Projekt B (Simpleksalgoritmen og `LP-KVL`). Afleveres tirsdag den 22/2 2011 kl. 8.

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12. februar 2011

All contributions to this volume can be found at:

http://www.ind.ku.dk/publikationer/up_projekter/2011-4/

The bibliography can be found at:

http://www.ind.ku.dk/publikationer/up_projekter/kapitler/2011_vol4_nr1-2_bibliography.pdf/