

## **Planning and implementing mathematics lectures for first-year university students – tips and tricks**

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The present manuscript, is based on the author's pedagogical project report written in July 2012 in the scope of Adjunkt-pædagogikum, which is a one-year long Teaching and Learning in Higher Education programme for assistant professors and postdocs, organized by the Department of Science Education, Faculty of Science, University of Copenhagen in the academic year 2011-2012.

### **Introduction**

As a lecturer in mathematics for first-year university students one faces several challenging problems.

- **Big classes:** Many lectures are attended by nearly 200 students, and sometimes even more. This makes it difficult to reach out to individual members of the class and activate and motivate everyone.
- **Low motivation:** When lecturing for mathematics students, this is not a big problem, but basic mathematics courses are often mandatory in many educational programmes. This means that a lot of students have to take mathematics courses even if they do not really want to.
- **High level of abstraction:** Most people find university mathematics quite abstract. Nowadays, there are different ways of visualizing examples and certain mathematical concepts by using tools such as projectors and computer software. However, there are of course limits to what is possible, and to many people mathematics remains abstract.

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The first problem is not present to the same extent during exercise sessions (i.e. problem solving classes). The second problem is connected to the role that mathematics currently has in many educational programmes in engineering and natural sciences. The third problem lies in the nature of mathematics.

This project report is mainly concerned with methods and approaches that will activate and motivate students during mathematics lectures. Based on interviews with seven experienced, award-winning and popular mathematics lecturers from Lund University and the University of Copenhagen, we have compiled a toolbox of tips and tricks. The purpose of doing this, is that we believe that it will be useful to anyone (including the present author) who is trying to develop and refine his or her lecturing skills. Many of the tips and tricks can be applied to more general situations and are therefore not limited to mathematics lectures.

## **Tips and tricks**

As a lecturer it is important that to develop a teaching style of your own, that you feel comfortable with. No one is born to be a good or a bad lecturer. By constantly training and reflecting over your work, you will be able to improve your lecturing skills and become an appreciated lecturer.

The list<sup>3</sup> of tips and tricks below is not intended as a template of matters that any good lecturer needs to take into account, but rather as a buffet of themes and ideas that might serve as a good basis and an inspiration for self-reflection.

### **Golden rules**

We begin by presenting some important general principles.

#### **Tip 1: The real golden rule**

You should always ask yourself: If I were a student attending my own lecture, what type of presentation would I like to see?

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<sup>3</sup> Many of the tips and tricks are closely related, and in some cases there is even some overlap. This has been difficult to avoid.

**Tip 2: Preparation, preparation, preparation**

As a lecturer you have to exhibit self-confidence and there is simply no room for hesitation. Therefore, it is crucial that you are extremely well-prepared.

During the interviews it became clear that one thing that all interviewees had in common, was that they are always very well-prepared before every lecture. Perhaps this is the most important piece of advice in the list.

One interviewee said:

I am always extremely well-prepared. At the moment my lectures begin at 8:15. I typically wake up at 6:30 and between 6:30-7:00, before I have breakfast, I read through my lecture notes. If I have an afternoon lecture, I also spend 30 minutes reading through the lecture notes.

Another said:

After having planned a lecture, I typically prepare myself twice. The first time, I think about what I am going to say. The second time, I think about how I am going to say it.

**Tip 3: Never go over time**

It is very important to stop on (or before) time. To go over time is not only impolite, it might even be useless. For example, if a lecture is supposed to end at 12:00, then anything that is being said after 12:00 is likely to fall outside of the students' attention span.

**Tip 4: There are no stupid questions**

In order to encourage the students to ask questions and to take active part in your course, it is of great importance that they feel welcome to do so. Therefore, one should never under any circumstances do anything that could give a student the feeling that you think that he or she is stupid. A rule of thumb that one should bear in mind: There are NO stupid questions! Even if you think that a student asks a crazy question, you must not let it show! Always be polite and treat any question as if it were the most interesting one you had ever heard.

One interviewee said:

If you get a strange or stupid question in the lecture hall, then it might just be that 30 out of 100 students are asking themselves the same question. Therefore one has to take every question seriously!

### **Tip 5: The tone of your voice**

A monotonous voice can easily make the students feel less motivated and eventually sleepy. You should try to talk loudly, with a variation in both pitch and intensity, in order to avoid monotonicity. And remember never to mumble!

One interviewee described himself:

I am pretty intense. I talk loud and fast and move a lot. I run back and forth in front of the blackboards. All together, I think that this helps to maintain the attention of the students.

### **Tip 6: Show that you care**

If you show the students that you genuinely care about their learning, then they are far more likely to actually learn something. Let the students see that you enjoy explaining mathematics to them. It will make your job easier and the students will appreciate it!

### **Tip 7: Memorize the names of your students**

If you have a good memory, it is probably worth investing some time on this. This will also make the students feel that you care about them.

## **The role of the lecturer**

### **Tip 8: Let your enthusiasm and passion shine through**

Do you want the students to think that mathematics is fun and interesting? Then you have to let them know that you feel the same way. Therefore it is important that you always show a great deal of enthusiasm during your lectures. Things that you find trivial, and perhaps not very exciting, can still become exciting for the students if you give them the impression that it is exciting to you. This may sometimes require some good acting skills on your part, but it will be worth it because enthusiasm is contagious!

One interviewee said:

In the beginning, I thought that the most important thing was to present the material in a structured and logical way. While it is still very important, I have learnt that it is far more important to make the students excited and interested!

**Tip 9: Establish a good atmosphere during the first week**

If you work hard during the first week of your course, to create a good atmosphere in the class, then everything becomes much easier from that week on. A friendly atmosphere should make the students feel at home so that they feel that they are always welcome to ask questions for example. In fact, under optimal circumstances they will even be inclined to ask questions.

One interviewee cautioned:

The students may say that they genuinely enjoy your lectures, but that does not necessarily mean that they are learning mathematics. This is something that one needs to be aware of! It might be a good idea to throw out some control questions once in a while.

**Tip 10: Earn the trust and respect of your students**

In order to earn the trust and respect of your students, you should of course act like a decent person and be friendly and show respect for them. On a more professional level, you need to explain to the students, for example, why complex numbers are useful and why one needs to be able to solve a differential equation. It is crucial that the students get an idea of why they are supposed to learn what we want them to learn. If you do this successfully, the students will eventually trust you and teaching them new material will be easier.

**Tip 11: Use your authority**

When you stand in front of the blackboard, you should never challenge yourself, for example by indicating that your method or approach may not be the best one. You should never show hesitation or come across as indecisive in the lecture hall. Remember that the students have come to listen to YOU and that they thereby have given you the mandate to tell them what you want! When you meet students in smaller groups, you should of course be more humble. But in the lecture room, you are the one in charge.

**The structure of a lecture****Tip 12: Explain the purpose and the goals**

Whenever you begin a new lecture or introduce a new concept, it is of great importance that you carefully explain the purpose. If not, the students

might easily lose interest. Therefore, it is a good idea to start every lecture by explaining what you want to do today and why it is important. One could for example write down a menu of what today's lecture looks like. This will add to the excitement. (Some lecturers go so far as to write down time estimates on what time they will arrive at different sections of the programme.) You should strive to explain how your mathematics course fits into a bigger picture. At the beginning of a lecture, it is important to formulate a good question which seems natural and important enough to want to find an answer to, and which the students can think about. Why? Explain to the students!

A related tip is the following:

### **Tip 13: Always anchor your ideas in the class**

If you want to try new approaches, for example new pedagogical ideas, always make sure that you explain the purpose to the students and ask them what they think. If the students do not show their consent, it is possible that you will experience major difficulties later on.

### **Tip 14: Interrupt the flow**

The attention span of the students is quite limited. If you talk for too long, then eventually you will have lost the attention of most of the class. Therefore it is sometimes a good idea to make (informal) pauses once in a while, just in order to create a discontinuity that will stop the flow of information. It then becomes natural to allow for the students to ask questions. One could also for example tell a joke, describe a curiosity or display a funny example – anything in order to break the flow. By doing this, for example, every 20 minutes, the brains of the students will have the opportunity to relax for a minute or two, after which they will be ready for another session of 20 minutes.

### **Tip 15: Do something unexpected, if necessary**

As a lecturer you can quickly tell if the students are listening to you or not. All you need to do is to take a look across the lecture hall. If you notice that some students are being unfocused, are losing attention or – even worse – are sleeping, then you immediately need to take action. Sometimes you need to do something strange or unexpected, in order to regain their attention.

## **Presentation of the material**

One interviewee said:

There is a good reason for presenting our (mathematics) lectures mainly on blackboards – it makes the presentation slow enough for people to have time to think about what is actually being said. A PowerPoint presentation does not give the same impression.

### **Tip 16: How to handle the blackboard**

This advice might seem unnecessary, but it needs to be mentioned. You should always begin writing at the top left blackboard. Use the boards carefully and think through in what order you want to use them. Step aside, in order not to block the sight of your students. Try to avoid speaking into the blackboard while you are writing. Instead, turn around, talk to the students while looking them in their eyes and let them feel your presence!

One interviewee said:

I pay a lot of attention to the presentation of the lectures on the blackboards. I use different colours and try to make definitions and theorems appear in a certain way. I am thinking that when they write (copy and paste) this down into their notebooks, I want it to look almost like a book.

### **Tip 17: Grouping the blackboards together**

When you are in a big lecture hall you might even divide the blackboards into different groups. For example, you could use some of the blackboards to demonstrate examples and some of the blackboards to make calculations. By doing this, the students will see when some change is about to occur, that is when you walk from one group of blackboards to the next.

### **Tip 18: Create some drama around a mystery**

A couple of the interviewees compared the flow of a lecture to the dramaturgy of a movie. They start by describing some part of the theory or an example as a mystery that needs to be solved. One could even announce it as the mystery of today. During the lecture they try to build up tension and drama, and under optimal circumstances, the mystery is resolved just in time for a break or the end of the lecture. This requires a lot of preparation and planning, but if made properly it will be an important way of keeping the students excited and motivated during the lecture.

**Tip 19: Less is more**

A common mistake is to try to fit too much material into one lecture. A rule of thumb is to always include the material that is needed to proceed with the rest of the theory, but not more. One interviewee described that for a 2x45 minutes lecture, he only plans for 2x40 minutes of lecturing, in order to leave some room for questions. Another interviewee described that she always tries to choose three or four essential things that she thinks that the students should understand before the lecture is over, and then her focus will stay on those things. Essentially, the students will not be able to remember everything that you say. If you choose some topics and handle them with care, then hopefully the students will understand and remember them better. This might lead them to wanting to find out more from the textbook. Another rule of thumb is that: Quality is always better than quantity.

One interviewee said:

Knowing how difficult it is for many students to stay focused, you might want to end the lecture by saying something along the lines of “If you don’t remember everything, at least try to remember ... this and that.”

The following tip is closely related to the one above.

**Tip 20: No stress**

Stress resulting from the feeling of not being able to cover the material that one wanted to fit into one lecture, is common. There is no reason to feel stressed about this. You should make sure that the things that you do say, are being said well. Remember that your job as a lecturer is to tell a story that will help the students to find a red thread and navigate through the material of the course. For example, you might not want to spend too much time giving full proofs, but instead refer the students to the textbook. You have to make sure that you explain this to the students at the beginning of the course.

**Tip 21: Tell a funny story**

When presenting some new material that has some important historical connection, it might be appropriate to tell a funny story or to mention some curious facts or historical anecdotes. This will allow for more interest to arise.



Remember that your lecture should be pleasant to watch and listen to. By displaying the historical relevance of things that you are about to say, it will also create more excitement.

**Tip 22: Repetition**

The students tend to forget details if they do not see any immediate use made of them. Therefore it can be a good idea to repeat some of the main concepts during subsequent lectures. If they hear something two or three times, it is more likely to stick. This should of course only be applied to the most central concepts.

**Tip 23: Get textbooks from parallel courses**

Consult textbooks that the students use in the courses running in parallel to your mathematics course. Just by browsing through them, you will get an idea of what type of mathematics (if any!) is being used in those courses. If there is any connection between the mathematics that you teach, and what is being used in the parallel courses, it should be pointed out to the students. By explaining such connections, you will show them the position that your course has within their educational programme. Thereby, your course will seem more relevant and the students will hopefully become more motivated.

**Tip 24: No farfetched examples**

Never use any farfetched or artificial examples. The use of unnecessary or irrelevant examples can easily come across as strained, and the students will soon see through it. Remember that most of your students come to your lectures to learn how to better understand mathematical concepts. If you can find suitable examples (from adjacent areas such as physics) that will help to illuminate mathematical concepts, then they might be useful and valuable. Otherwise, leave them out!

**Tip 25: Finding the right level**

To find an appropriate level for the presentation in a lecture is always difficult. A lecture will always be a compromise; there will be some students

that find the presentation too hard, and some students that find it too easy. One should aim at placing the level somewhere close to the average. However, keep in mind that there is no such thing as a too simple explanation.

### **Tip 26: Online lecture notes**

Scan your lecture notes and post them onto the course homepage before your lecture. This has several advantages. The students will be able to print them or read them from their e-readers, and then not feel the need for taking notes of their own. This creates an opportunity for the students to try to follow what is being said in the lecture and not drown in their own notebooks. Another advantage is that if you find yourself in a situation where you are running out of time, you could easily refer the students to your lecture notes and simply skip things that are not of immediate importance. Obviously, the use of such lecture notes is optional and the students will of course still be able to write their own notes if they prefer to do so.

One interviewee explained:

I want the students to listen to me and stop focusing on writing everything down. They should be able to relax and really listen carefully to what I have to say.

Another one said:

There is a great danger with mathematics students going to a lecture and taking notes, thinking that they are being active, when they are in fact passive! They need to listen and wait for questions to arise within their heads.

Some lecturers are critical of posting lecture notes online, one argument being that most undergraduate courses are based on a textbook which is usually much better written than any set of lecture notes. However, a set of lecture notes, of course, gives an opportunity to give alternative explanations that are not to be found in the textbook.

### **Tip 27: Using pencasts**

Using a special type of smart pen (from LiveScribe) one can create so called pencasts. The pen is equipped with a laser reader and a microphone. By writing your notes on a special type of paper and simultaneously recording your voice, a video is generated. When watching the video, the students

will see how the things you write gradually grow in front of their eyes as they hear your explanation. This is a very powerful method. By placing such videos on the course homepage, you are able to explain some things in much more detail than you have time for during a lecture. Moreover, the students will be able to hit the pause button which is sometimes necessary in order to have time to think, but which is not possible during a lecture.

The goal should not be to put all of the lecture notes as pencasts on the course webpage, but certain concepts that are central and that might need more explanation, could easily be provided in this way.

One interviewee said:

Once, in a course of 200 students, I had a single pencast-video that was viewed 1000 times.

### Visualization aids

In addition to drawing on the blackboard, there are several aids that one can use.

#### Tip 28: Document camera

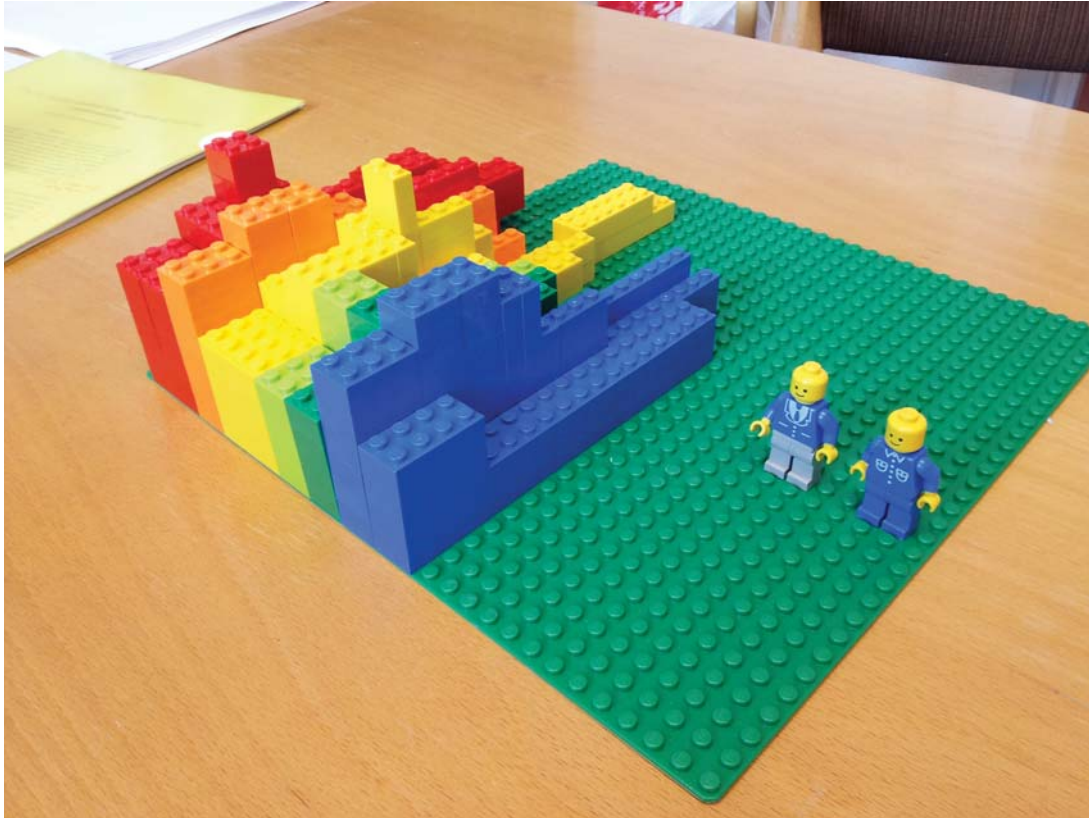
By placing an object in a document camera, one can immediately project an image of the object onto the wall. It is possible, for example, to display anything from the screen of a mobile phone, or a paper or even some bricks of Lego. More concretely, it is very useful if one wants to display something from the textbook. If you have a smartphone, then it will certainly be able to produce function graphs. Hence, you could display function graphs without bringing a computer. One interviewee explained how he explains Riemann sums when teaching integration of functions

$$\mathbb{R}^2 \rightarrow \mathbb{R}$$

using Lego and a document camera. (Fig. 1.1).

#### Tip 29: Maple software

If you are good at drawing 3D pictures, then you can do it by hand on the board. Otherwise, it is an excellent idea to use the computer software Maple to create 3D images. One can also create animations.



**Fig. 1.1.** Using lego bricks to explain Riemann-sums. N.B.: The lego men are supposed to be Riemann respectively Lebesgue.

### **Tip 30: Videos**

In some situations playing a video can create a lot of excitement among the students. For example, a video of the collapse of the Tacoma Bridge can serve as a motivation when discussing eigen frequency.

## **Discussion**

As mentioned in the section “Tips and tricks” above, the list of tips and tricks is intended as a buffet of themes and ideas that might serve as a good basis and inspiration for self-reflection for lecturers. The tips and suggestions from the interviewees overlapped with each other substantially, and the final list is the result of an interpretation and moulding process conducted by the present author. We think that each tip is valuable in its own right and deserves attention.

The approach taken by this report is very practical and, in the light of Biggs & Tang (2007), fits the description of the views of a level-2 teacher, i.e. a teacher who mainly focuses on what the teacher does. To ascend to level 3, i.e. to focus on what the student does and how the student actually learns during a lecture, is a gigantic undertaking, which would have required a massive investigation. (Remember that we are dealing with populations of approximately 200 students.) One has to keep in mind that a lecture is just one example of a situation when learning (hopefully) takes place. Typically, in the scope of a course, you also have exercise sessions, laboratory exercises, examination, homework, self-study, etc. The present author believes that only a small portion of learning actually takes place during mathematics lectures. It is later that the students are able to process what they heard and saw in the lecture. During exercise sessions and self-study sessions following a particular lecture is where the actual learning takes place.

Mathematics is not a spectator sport. You can't expect to learn mathematics without doing mathematics, any more than you could learn to swim without getting in the water.

In the beginning of his book, Hungerford (1996) makes the above statement to emphasize the importance of doing exercises, and I fully agree with this opinion. Nevertheless, lectures represent an important channel for introducing concepts and ideas, preparing the students for other learning activities and giving them a natural platform for asking questions. Another important role of lectures is to function as a source of inspiration for the students (compare this to Gibbs (1981)).

The current typical lecture format for mathematics lectures for first-year university students does not encourage students to do much actual mathematics during the lectures; they mainly listen passively and occasionally ask questions.

In fact, it is natural to ask if our current way of lecturing in mathematics could be made more efficient. This is why, at the end of each interview, the opinions and teaching methods (e.g. conceptual challenges) of Mazur (1997), were explained to the interviewees and followed up with the following question: "Do you think that Eric Mazur's methods for physics lecturing also could be applied to mathematics lecturing for first-year students?"

Roughly half of the interviewees responded: "Maybe. Why not? We should try to find out!" The other half responded "Absolutely not!", their main objection being that it simply takes too much time. One has to realize

that teaching mathematics to first-year university students is a large-scale enterprise in many universities. We have a limited amount of time that needs to be used wisely and planned carefully. It could be that Mazur's methods are not suitable for all types of courses or mathematics students, but it is my belief that it at least needs to be further investigated by doing experiments, beginning on a small scale.

## A The interviews

In total seven mathematics lecturers were interviewed; five at Lund University and two at the University of Copenhagen. They have all shown extraordinary skills when it comes to lecturing. Their experience as lecturers for first-year university students ranges from five to twenty-five years. The interviews were recorded with durations ranging from twenty-three to eighty-two minutes per interview. In total, the seven interviews took five hours and 13 minutes.

### Interview template

Getting started and in general:

- Practical things; anonymity, recording, purpose of the project.
- Relevant data: How long have you been lecturing mathematics to first-year students at university level?
- As a new lecturer, what did you think of as the biggest challenge with the teaching situation lecture?
- What lessons have you learnt during your time as a lecturer? Please be concrete.
- Today, what do you see as the biggest challenge in your role as a lecturer? Do you actively take any measures to affect or prevent this?

Before the lecture:

- Do you have any trick to make the students prepare before the lecture?
- Do you have any trick to make the students actually come to the lecture?

During the lecture:

- Describe a typical lecture, which is given by you.
- What measures do you take in order to keep the students alert, active (participating) and motivated, curious and wanting to learn more?
- Do you have any practical tips and tricks or examples of games that one could use?
- Do you use any kind of visualization aids?
- The ideas of Eric Mazur's is explained to the interviewee. Follow up question: Do you think that Eric Mazur's teaching methods for physics lecturing also could be applied to mathematics lecturing for first-year students?

All contributions to this volume can be found at:

[http://www.ind.ku.dk/publikationer/up\\_projekter/2012-5/](http://www.ind.ku.dk/publikationer/up_projekter/2012-5/)

The bibliography can be found at:

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